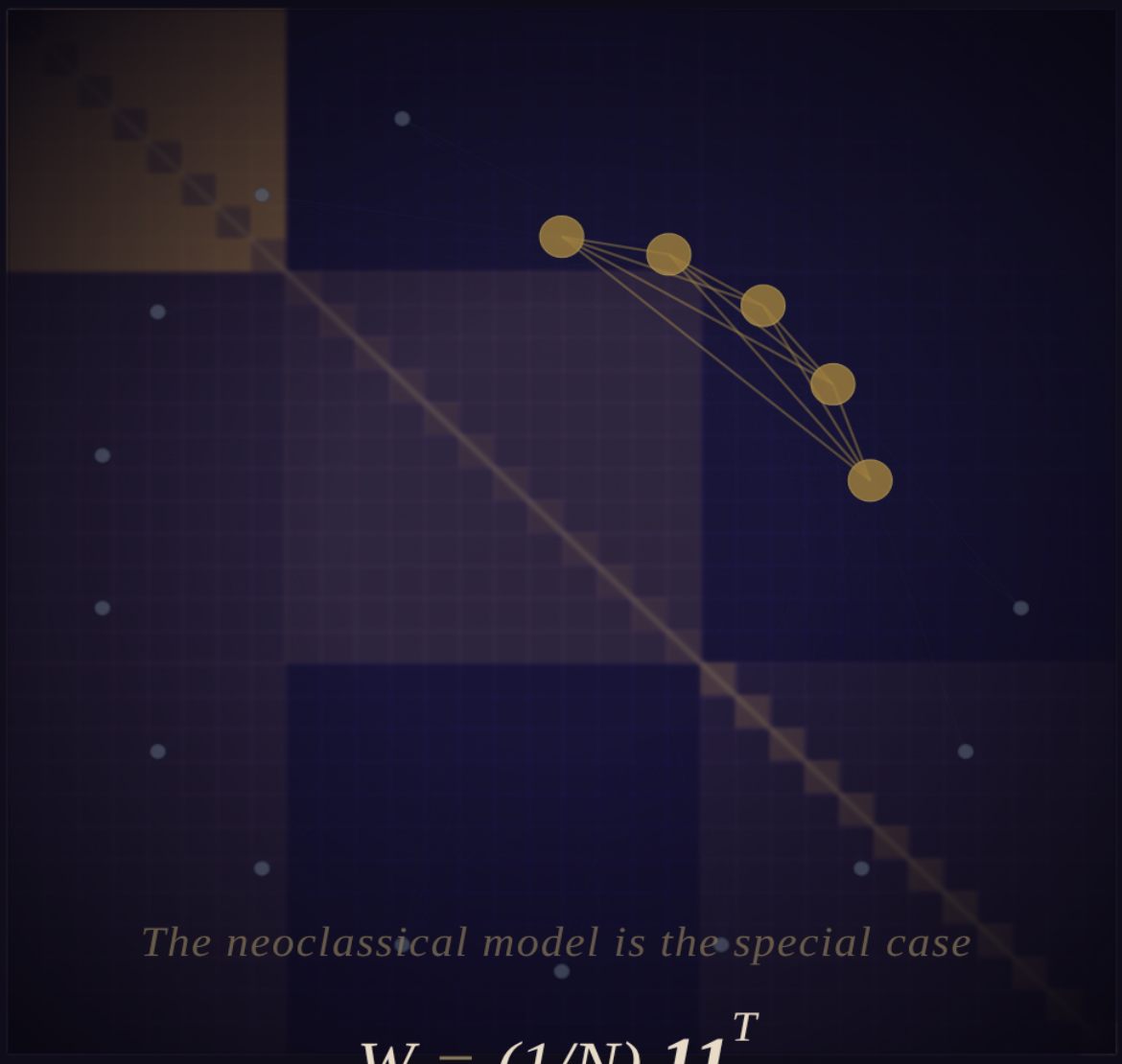
A TEXTBOOK



The neoclassical model is the special case

$$W = (1/N)\,\mathbf{1}\mathbf{1}^{T}$$

STRUCTURAL ECONOMICS

This textbook is the general case.

Werner L. Hernani-Limarino

STRUCTURAL ECONOMICS

The General Case

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DRAFT — September 2027

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Draft manuscript. Not yet published.

For my daughter — the park I care about most.

"O subdesenvolvimento não constitui uma etapa necessária do processo de formação das economias capitalistas modernas. É, em si, um processo particular, resultante da penetração de empresas capitalistas modernas em estruturas arcaicas."

— Celso Furtado, *Desenvolvimento e Subdesenvolvimento* (1961)

"The neoclassical model is the special case $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$.
Structural economics is the general case."

— This book

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Preface

From *Structural Economics: Theory and Computation*

Werner L. Hernani-Limarino

I learned economics in La Paz, Bolivia, in the late 1990s, at approximately 3,600 meters above sea level and approximately zero meters from reality.

The curriculum was standard. We optimized utility functions. We derived Marshallian demand. We proved the welfare theorems on the blackboard and were told that competitive markets, under the right conditions, produce efficient outcomes. The right conditions were listed: complete markets, perfect information, price-taking behavior, no externalities. We wrote them down. We passed the exam.

Outside the classroom, the city ran on a different operating system. The woman who sold salteñas on the corner near the university did not participate in a competitive market with free entry and symmetric information. She participated in a web of obligations, debts, kinship ties, and territorial agreements with other vendors that determined her location, her prices, her hours, and her customers. Her economic life was not organized by the price mechanism. It was organized by a structure of relationships that the price mechanism could not see, that our models could not represent, and that our welfare theorems could not evaluate.

The men who carried goods up from the Yungas on their backs did not face

a labor market in the textbook sense. They faced a system of compadrazgo and reciprocal labor exchange—ayni and mink'a—that allocated work, distributed surplus, and reproduced itself across generations through mechanisms that had nothing to do with marginal productivity and everything to do with social position. Their wages, such as they were, reflected not their marginal product but their location in a network of mutual dependence that was simultaneously economic, social, and moral.

Bolivia in the 1990s was living through the Washington Consensus. The structural adjustment programs prescribed by the IMF and World Bank had been implemented with unusual discipline: the New Economic Policy of 1985 had stabilized hyperinflation through monetary contraction, the state mining corporation had been dismantled, trade had been liberalized, the fiscal deficit had been closed. By every textbook criterion, the reforms were a success. Inflation was under control. The budget was balanced. The economy was "open."

By every criterion visible from the street, the reforms had reproduced the structure they were supposed to transform. The tin economy had been replaced by the gas economy, but the structural heterogeneity—a modern enclave connected to global commodity markets, coexisting with a vast traditional sector operating at a fraction of the modern sector's productivity—was identical to what Aníbal Pinto had described three decades earlier. The productivity gap between sectors had not narrowed. The informal economy had not formalized. The indigenous majority had not been integrated into the modern sector, because the modern sector had not been designed to integrate them, and the formation mechanism—the park of rural schools, monolingual curricula, and occupational invisibility—continued producing subjects whose aspirations were calibrated to the traditional sector even as the formal barriers to the modern sector fell.

I did not have the language for this at the time. I had utility functions. I had Lagrangians. I had the first welfare theorem. What I did not have was a way

to put the salteña vendor's network of obligations into the Lagrangian, or the compadrazgo system into the production function, or the park of a Quechua-speaking girl in Potosí into the preference parameter θ that my models took as given. The structure was everywhere. The theory was blind to it.

I went to the University of Pennsylvania to learn better theory. I learned better theory. I learned general equilibrium with more mathematical precision. I learned econometrics with identification strategies. I learned development economics from people who knew the literature. I also learned, gradually and then suddenly, that the better theory had the same blind spot as the worse theory, only more elegantly formalized. Arrow-Debreu general equilibrium is a theorem about economies where agents interact anonymously through prices. The interaction matrix is implicit, and it is symmetric: every agent has the same weight, the same access, the same power. In a doubly stochastic world, the welfare theorems hold. In La Paz, the world is not doubly stochastic. The theorems do not hold. And the gap between the theorem and the world is not a friction to be estimated. It is the structure.

This book is the result of twenty years of trying to close that gap.

The central idea is simple enough to state in one sentence: *every economic interaction is mediated by a weight matrix W, and the properties of W determine everything that matters—power, allocation, preferences, beliefs, welfare, and structural change.*

The standard neoclassical model is the special case where W is symmetric—where every agent has equal weight, where interactions are anonymous, where the structure is invisible because it has been assumed away. The welfare theorems hold for that case. The first welfare theorem says that competitive equilibrium is Pareto efficient. It is correct. It is also the statement that when $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$, no agent has structural power, no externality is unpriced, and no preference has been distorted by the formation mechanism. Remove any of these conditions—make W asymmetric, introduce network externalities, let preferences be shaped

by developmental exposure to W^I—and the theorem fails. Not because the theorem is wrong, but because the world is not the theorem's special case.

Structural economics is the study of what happens for general W.

This book was written because it does not exist. That sentence sounds grandiose. It is meant literally. I spent the first months of this project surveying the landscape—over eighty books, handbooks, and foundational papers across network economics, general equilibrium theory, endogenous preferences, bargaining theory, Latin American structuralism, and the political economy of structural change. The pieces are all there. The mathematics converges with remarkable precision. And nobody has assembled them.

The Leontief inverse $(I - A)^{-1}$, which maps final demand to gross output through inter-sectoral linkages, is the same mathematical object as the Bonacich centrality $(I - \delta G)^{-1}\mathbf{1}$, which maps individual actions to equilibrium outcomes through social network effects. The left Perron eigenvector of the information matrix W^I, which determines long-run beliefs in DeGroot learning, is the same object as the influence vector that determines which sectors matter for aggregate fluctuations in Acemoglu's production network framework. The spectral gap of the weight matrix, which governs the speed of convergence in social learning models, is the same parameter that governs the speed of intergenerational mobility in a properly formulated transition model. These are not analogies. They are the same theorems applied to different matrices.

Yet the literatures that use these tools do not speak to each other. The production network economists (Acemoglu, Baqaee, Farhi) build general equilibrium models with input-output matrices but ignore social networks and preference formation. The social network economists (Jackson, Bramoullé, Golub) develop elegant theories of learning, diffusion, and peer effects but work in partial equilibrium without production or welfare analysis. The endogenous preference theorists (Bowles, Bisin, Verdier) model cultural transmission through evo-

lutionary and Bayesian mechanisms but assume well-mixed populations rather than structured networks. The bargaining theorists (Manea, Corominas-Bosch) derive power from network position but do not embed the results in general equilibrium. And the Latin American structuralists—Prebisch, Furtado, Pinto, Sunkel—had the deepest structural intuition of any school in the history of economic thought but lacked the mathematical tools to formalize it.

The irony is almost too perfect. The CEPAL economists of the 1950s and 1960s were using Social Accounting Matrices—input-output tables extended to include household income and expenditure flows—as their primary analytical tool. These are weight matrices. The center-periphery structure that Prebisch described is a specific topology of the global production network W^P: the center has high column sums (everyone depends on it for technology and manufactures), the periphery has low column sums (nobody depends on it for critical inputs). The terms of trade deterioration is a consequence of the spectral properties of this matrix—the periphery's products have low network centrality, so productivity gains in the periphery are transferred to the center through prices. The structural heterogeneity that Pinto described—the coexistence of a modern enclave and a traditional sector within a peripheral economy—is a block-diagonal structure in the national W^P, with weak off-diagonal links between blocks. The Leontief inverse of a block-diagonal matrix has small off-diagonal elements: growth in one block does not propagate to the other. Pinto's "trickle-down failure" is a spectral property of a disconnected matrix.

None of this has been formalized. The most important school of structural economics that the Global South ever produced, and the most powerful mathematical tools for studying structure that the Global North ever developed, have existed for decades without anyone connecting them. The bridge is not hard to build. The mathematics is identical. What was missing was someone with a foot in both worlds—someone who learned Prebisch in Spanish and Perron-

Frobenius in English, who grew up in a structurally heterogeneous economy and was trained in the formalist tradition that could have diagnosed it, who watched the Washington Consensus prescribe symmetric remedies for an asymmetric world and wondered what the eigenvalues would have said.

The research survey identified seven specific gaps that this textbook aims to fill.

First: no general equilibrium model exists that embeds both production networks and social interaction effects on preferences. The production side uses one matrix; the demand side uses another; no model has both.

Second: no formal framework models economic preference formation on explicit network structures. DeGroot learning is mathematically a model of preference convergence governed by the spectral properties of W^I, but the economics literature has not framed it that way. The cultural transmission literature and the network games literature developed in parallel without integration.

Third—and this is the largest gap—no connection exists between CEPAL structuralism and modern network economics. The input-output matrices of the structuralist tradition are the same objects as the production networks of Acemoglu and Baqaee-Farhi. The Economic Complexity Index of Hidalgo and Hausmann—the second eigenvector of a stochastic matrix derived from the country-product network—formalizes structural heterogeneity through spectral methods conceptually identical to Pinto's framework but developed independently, without reference to CEPAL. The center-periphery structure that network scientists detect using core-periphery algorithms is the same structure that Prebisch described in 1950. The mathematical tools have been reinvented. The intellectual tradition they formalize has been forgotten.

Fourth: no general theorem maps network centrality to bargaining power. The Invisible Fist framework that I developed in a companion paper proposes the left Perron eigenvector as the power measure, but a clean, broadly accepted

mapping does not yet exist in the literature.

Fifth: no structural inequality or mobility measurement uses network methods. The intergenerational transition matrix is a stochastic matrix whose spectral properties—the second eigenvalue is the rate of mixing, which is the rate of mobility—naturally formalize structural mobility. Nobody has developed this.

Sixth: no heterodox-mainstream textbook has a unified mathematical formalism. The CORE Project integrates conceptually but not mathematically. This book integrates mathematically: one object (W), one theorem (Perron-Frobenius), one framework (network general equilibrium with endogenous power and preferences), many applications (gender, caste, race, class, global trade, development).

Seventh: "structural economics" does not exist as a named, unified research program. Unlike behavioral economics or development economics, there is no recognized field that integrates networks, institutions, endogenous preferences, power, and structural change under one umbrella. The pieces exist across five or more subfields. The synthesis does not.

This textbook is an attempt to build the synthesis.

A word about what this book is not. It is not a polemic against neoclassical economics. The welfare theorems are correct. Constrained optimization is the right tool for modeling individual choice. General equilibrium is the right framework for modeling market interactions. What the book argues is that these tools, applied with a symmetric W, describe a world that does not exist—and that the same tools, applied with the W that the world actually has, describe a world that does exist but that standard economics cannot see.

The move is not from mainstream to heterodox. It is from special case to general case. Newton is not refuted by Einstein. Newton is nested as the low-velocity limit of a more general theory. The competitive equilibrium is not refuted by the structural equilibrium. It is nested as the doubly-stochastic limit of

a more general framework. Every theorem in Mas-Colell, Whinston, and Green holds within its domain. This book extends the domain.

The move is also not from mathematics to narrative. If anything, this book has more linear algebra than the standard curriculum. The formation model requires preference recursions. The bargaining equilibrium requires Perron eigenvector computation. The welfare analysis requires spectral decomposition. The structural dynamics require fixed-point theorems for matrix-valued mappings. The measurement requires estimating weight matrices from data and computing their spectral properties. If you cannot invert a matrix, this book will be difficult. If you can, it will show you what the inverse means.

A word about where this book comes from. The intellectual debts are obvious and are acknowledged in the text: to Prebisch, Furtado, and Pinto for the structural vision; to Jackson, Bramoullé, and Golub for the network tools; to Acemoglu and Baqaee-Farhi for the production network general equilibrium; to Bowles for endogenous preferences; to Bourdieu for the habitus; to Marx for the original structural analysis; to Leontief for the matrix.

The personal debt is to Bolivia. To a country whose economy is a textbook illustration of structural heterogeneity and whose society is a textbook illustration of why the textbooks are wrong. To the salteña vendor whose network of obligations is the weight matrix that this book formalizes. To the compadrazgo system that is the social interaction effect that the standard model assumes away. To the girl in Potosí whose phone is changing the information matrix W^I faster than any curriculum reform could, and whose aspirations—formed in a park that Prebisch could not have imagined and that the Washington Consensus cannot control—will determine whether the structure reproduces or breaks.

And to my daughter, to whom this book is dedicated. She is the park I care about most. The aspiration I hope to form in her is not a career or a credential but the conviction that structures are not destiny—that the weight matrix can be

changed, that the eigenvalues can shift, and that the equilibrium is not the last word. If this book teaches her nothing else, let it teach her that. If it teaches the profession nothing else, let it teach the profession the same.

The neoclassical model is the special case $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$.

This textbook is the general case.

La Paz, Bolivia

March 2026

Visual Prologue

Seeing the Matrix

A guide to reading economic structures before computing them

> *"Before you can solve the matrix, you must see the matrix."*
> — This book

This prologue contains no equations. It teaches you to *read* the structure of an economy by looking at a picture — a heatmap of the weight matrix. By the end of these twelve pages, you will be able to glance at a heatmap and say: "That economy has an enclave. Growth won't trickle down. Power is concentrated." You will be able to say this before computing a single eigenvalue.

The equations come later. The seeing comes first.

What Is a Weight Matrix?

Imagine five people in a small economy: a Farmer, a Miller, a Baker, a Shopkeeper, and a Consumer. Each one depends on some of the others. The Farmer sells wheat to the Miller. The Miller sells flour to the Baker. The Baker sells bread to the Shopkeeper. The Shopkeeper sells to the Consumer. The Consumer works for the Farmer.

These dependencies can be written in a table. Each row is a person; each column is a person. The entry in row i, column j answers: "How much does person i depend on person j?" A large number means strong dependence. Zero means no connection.

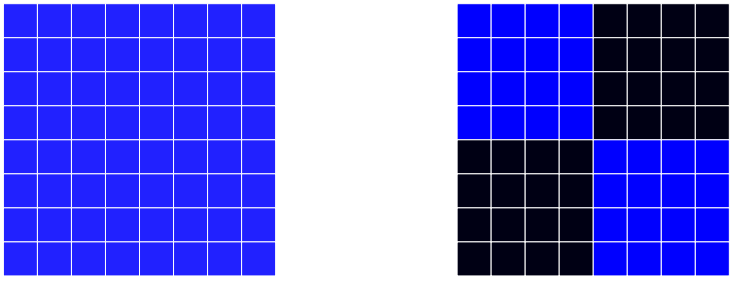
This table is the **weight matrix**. It is the most important object in this book. Every result in every chapter is a property of this matrix.

Now: instead of reading the table as numbers, render it as colors. A bright cell means a strong connection. A dark cell means a weak connection. A black cell means no connection at all.

This colored table is a **heatmap**. It is a photograph of the economy's structure.

Your First Heatmap

Here are two economies, each with eight people. The only difference is who depends on whom.



Economy A
Everyone depends on everyone equally.

Economy B
Two groups. Barely connected.

Economy A is a uniform blue square. Every cell has the same brightness. This means every person depends on every other person equally. No one is special. No one is peripheral. If something good happens to any person, everyone benefits equally. If something bad happens, everyone suffers equally.

Economy B has two bright squares — upper-left and lower-right — separated by a dark region. This means there are two groups. Within each group, people depend on each other (bright). Between the groups, almost no connection exists (dark). If something good happens in the upper-left group, the lower-right group barely notices. Growth in one group does not reach the other.

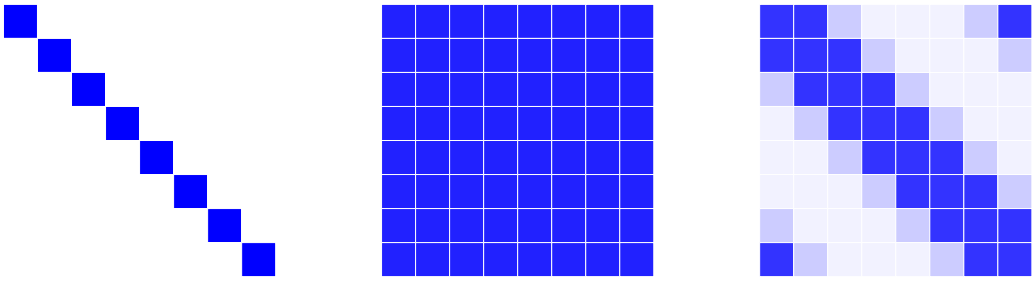
> How to read this **Uniform brightness** = everyone connected to everyone = anonymous market = the textbook ideal.
> **Two bright blocks, dark between them** = two separate economies sharing one country = structural heterogeneity = Bolivia's gas sector and traditional sector.
> You just read Aníbal Pinto's 1970 theory of structural heterogeneity. It took him 18 pages. It took you one glance.

The Diagonal and the Off-Diagonal

Every heatmap has two regions: the **diagonal** (top-left to bottom-right) and the **off-diagonal** (everything else).

The diagonal tells you how much each person depends on *herself*. A bright diagonal means self-reliance: each person's activity is mostly self-contained.

The off-diagonal tells you how much each person depends on *others*. Bright off-diagonal means strong interdependence.



Autarky
Bright diagonal, dark off-diagonal. Everyone alone.

Market
Uniform everywhere. Everyone connected equally.

Supply Chain
Bright band near diagonal. Each person connects to neighbors.

Autarky: Only the diagonal is bright. Nobody trades, nobody cooperates, nobody depends on anyone. Each person is an island. This is the starting point, not the goal.

Market: Everything is bright. Everyone depends on everyone through anonymous exchange. There is no special relationship, no privileged position. This is the textbook competitive equilibrium.

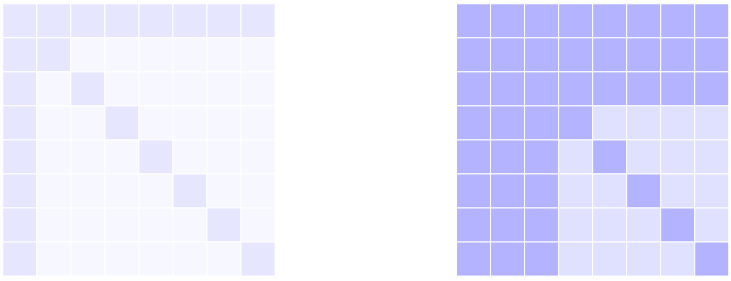
Supply chain: A bright band near the diagonal. Each person depends on her immediate neighbors in the production chain (the Miller depends on the Farmer and the Baker, not on the Consumer). Shocks travel slowly along the chain. Distance matters.

> The diagnostic question When you see a heatmap, ask: *How much of the brightness is on the diagonal versus off the diagonal?*
> All diagonal = autarky (no interdependence). All off-diagonal = maximum interdependence. The ratio tells you how connected the economy is.

Power: Who Has It, Who Doesn't

Not all positions in the matrix are equal. Some people are depended upon by many others; some are depended upon by few. The people who are depended upon by many — especially by other people who are themselves depended upon — have **structural power**.

You can *see* power in the heatmap: look at the **columns**. A bright column means that many people depend on that person. A dark column means few do.



Hub Economy **Center–Periphery**

One bright column. One person has all the power. *Bright upper-left fading to dim lower-right.*

Hub economy: One blazing column (person 1) and seven dim ones. Everyone depends on the hub; the hub barely depends on anyone. This is a platform economy (Amazon, Uber), a colonial trading company (the East India Company), or a company town where one employer controls everything. Almost all the power belongs to one node.

Center–periphery: The first three columns are bright (the "center" — they are depended upon by everyone). The last five columns are dim (the "periphery" — nobody much depends on them). The center depends on the center (bright upper-left block). The periphery depends on the center (bright lower-left rectangle). But the center does NOT depend on the periphery (dim upper-right rectangle). This asymmetry is the key: *the periphery needs the center more than the center needs the periphery.*

> Prebisch in one heatmap Raúl Prebisch spent his career arguing that the global economy has the center–periphery pattern: rich industrial countries form a bright upper-left block; poor commodity exporters form a dim lower-right block; and the critical asymmetry is in the off-diagonal rectangles — the periphery depends on the center (bright lower-left) but not vice versa (dim upper-right).
> Look at the heatmap again. That asymmetry — the difference between the lower-left and upper-right rectangles — IS the Prebisch thesis.

Ten Structures Every Economist Should Recognize

The world has a small number of recurring structural patterns. We catalog ten. Learn to recognize them by sight, and you can diagnose an economy the way a doctor diagnoses a disease — by pattern recognition, before running tests.

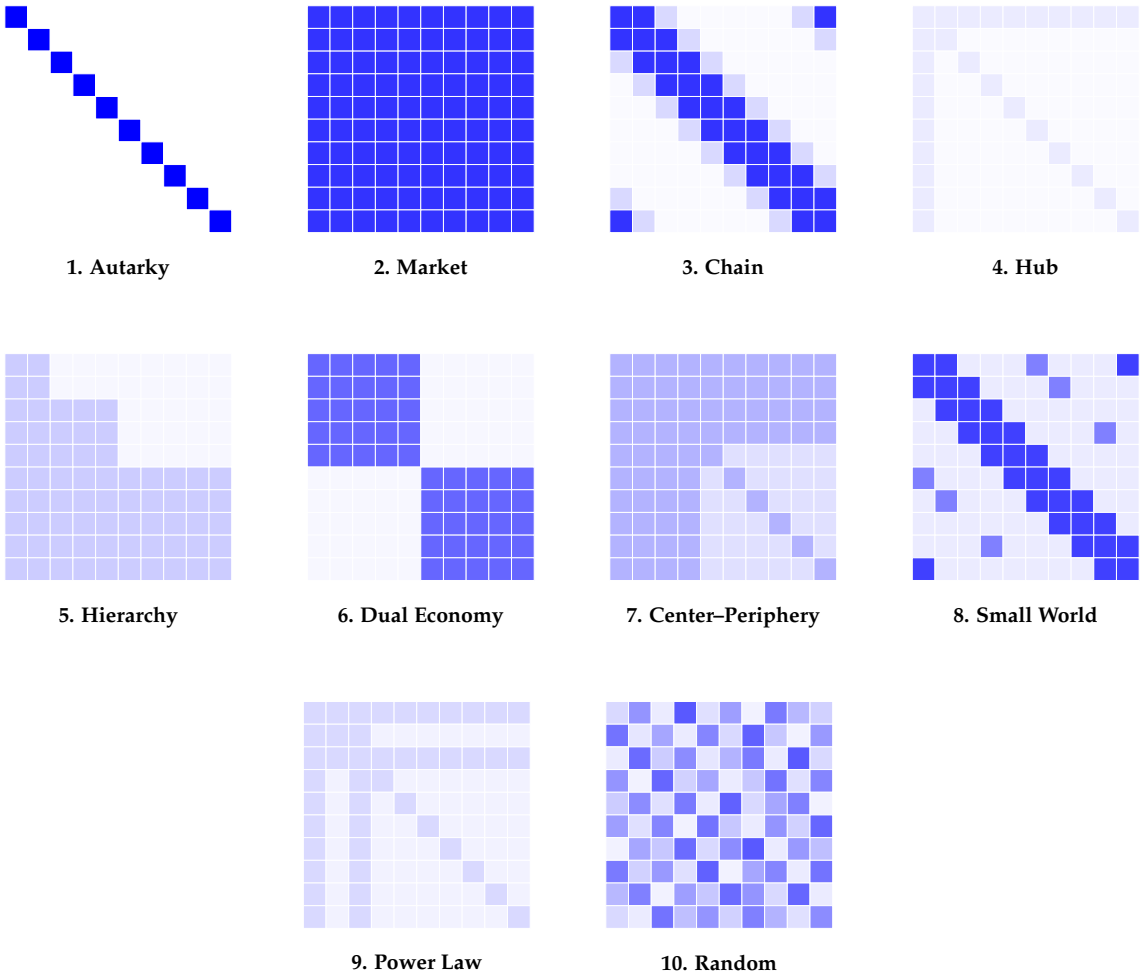


Figure 1: *

Ten canonical structures at $N = 10$. Each heatmap shows who depends on whom. Bright = strong dependence. Dark = weak or no dependence.

Reading guide to the ten structures

1. Autarky — Only the diagonal glows. Every person is self-sufficient. No trade, no cooperation, no interdependence. This is not an economy; it is ten isolated individuals. The "structure" is the absence of structure.

2. Market — Uniform blue everywhere. Everyone depends on everyone equally through anonymous exchange. No person is more important than any other. Power is perfectly equal. This is the idealized competitive market — the benchmark that textbooks assume and the real world never achieves.

3. Chain — A bright band running along the diagonal, with darkness above and below. Each person depends on her immediate neighbors in a production sequence. The Farmer connects to the Miller connects to the Baker. Shocks travel slowly along the chain. The person at the end of the chain barely feels what happens at the

beginning. Manufacturing assembly lines, agricultural value chains, and sequential bureaucracies have this shape.

4. Hub — One blazing column, one blazing row, everything else dark. All roads lead to one node. The hub has nearly all the power: everyone depends on her, and she depends on no one. Amazon in retail, YPFB in Bolivia's gas economy, the colonial governor. The most extreme concentration of power possible.

5. Hierarchy — A gradient from bright (upper-left) to dim (lower-right), with brightness concentrated in the first few columns. Power decreases with rank. The top of the hierarchy is depended upon by everyone below; the bottom is depended upon by no one above. Firms, bureaucracies, military chains of command, and caste systems have this shape.

6. Dual Economy — Two bright squares. Two economies sharing one country. The gas sector and the quinoa sector. The formal and informal economy. Silicon Valley and Appalachia. São Paulo and the Sertão. Each group is internally connected; the groups barely interact. This is Aníbal Pinto's structural heterogeneity: the "modern" and "traditional" sectors coexist without exchanging much. Growth in one group does not trickle to the other because the connections between them are nearly zero.

7. Center–Periphery — A bright upper-left block (the center depends on itself), a bright lower-left rectangle (the periphery depends on the center), a dim upper-right rectangle (the center does NOT depend on the periphery), and a dim lower-right area (the periphery's internal connections are weak). The asymmetry between the lower-left and upper-right rectangles is the signature of center–periphery: *dependence runs one way.* This is the global economy as Prebisch described it. It is also the relationship between a capital city and its provinces, a platform and its suppliers, or a colonial power and its colony.

8. Small World — A chain (bright diagonal band) plus a few random bright spots scattered off the diagonal. These are "shortcuts" — unexpected long-range connections in an otherwise local network. Your neighbor is the Miller. But your cousin in Buenos Aires is a shortcut. These few long-range links dramatically change how fast information and shocks travel through the economy. The phone in the girl's hand in Potosí is a shortcut. It connects her to signals from outside her local chain.

9. Power Law — A few very bright columns (hubs), many very dim columns (periphery). Unlike the single hub in pattern 4, there are two or three "super-connectors" and everyone else has very few connections. Wealth distributions, internet link structure, citation networks, and social media followers have this shape. Power is concentrated but not monopolized — there is a hierarchy of hubs.

10. Random — Salt and pepper. No pattern. Bright and dark cells scattered without structure. This is the null model — what an economy would look like if connections formed purely by chance. Any recognizable pattern (blocks, columns, bands) is *evidence of structure*. The deviation from random IS the structure.

What Each Structure Produces

The shape of the heatmap determines three things that matter for human wellbeing:

Three things the heatmap tells you **1. How power is distributed.**
Look at the columns. If one column is much brighter than the rest, power is concentrated. If all columns are equally bright, power is equal.
Equal power means the welfare theorems hold. Unequal power means they fail.
2. How shocks propagate.
Look at the off-diagonal brightness. If the off-diagonal is bright and uniform, a shock to any person spreads to everyone (the market: everybody shares gains and losses). If the off-diagonal is dark or block-structured, shocks stay local (the dual economy: gas price collapse hurts the gas sector but doesn't reach the quinoa farmer).
A connected economy shares risk. A fragmented economy concentrates it.
3. How fast beliefs converge.
In an information network, the heatmap tells you who listens to whom. If the heatmap is nearly uniform, everyone hears everyone, and beliefs converge quickly to a balanced average. If the heatmap has blocks, each group converges to its own beliefs, and the groups may disagree permanently.
A connected information network produces consensus. A fragmented one produces polarization.

The Heatmap Challenge

Exercise: Name That Structure Below are six unlabeled heatmaps. Each one corresponds to one of the ten structures described above. For each, identify: (a) which structure it is, (b) how power is distributed, and (c) what would happen to the bottom-right group if the top-left group received a large positive shock.

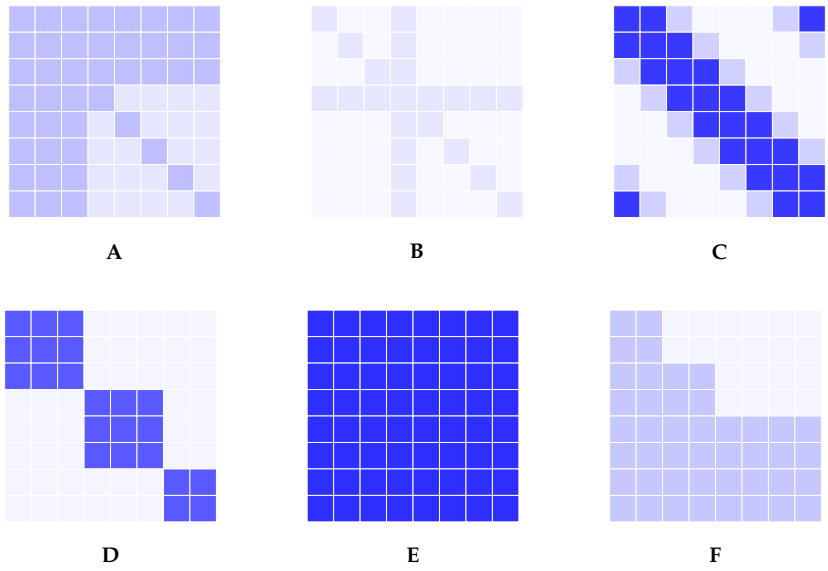


Figure 2: *

Six unlabeled structures. Answers are at the end of Chapter 1.

From Heatmap to Theory

This prologue has given you a visual vocabulary. You can now recognize ten structures by sight and say what each one implies for power, propagation, and convergence.

The rest of the book does three things:

Part I (Chapters 1–4) shows that the standard economic model assumes a specific heatmap — uniform blue, pattern 2 — and derives all its results from the properties of that pattern.

Parts II–III (Chapters 5–12) replace the uniform blue with the heatmap the world actually has — blocks, hubs, asymmetries, fragmentation — and derive what happens when you do. The results are different. Power is unequal. Growth is uneven. Preferences are produced. The welfare theorems fail.

Parts IV–VII (Chapters 13–28) ask: can the heatmap be changed? At what cost? By whom? With what consequences?

Every chapter will begin with a heatmap. Every result will be visualized. The interactive companion tool (accessible via the QR code on the back cover, or at the companion website) lets you build any heatmap, compute its properties, and watch the economy it describes come to life.

The one thing to remember An economy is a heatmap. The bright cells are the connections. The dark cells are the absences. The pattern is the structure. The structure determines who has power, who has wealth, who has opportunity, and who has none.

If you can see the pattern, you can see the structure. If you can see the structure, you can understand the economy.

Turn the page.

Part I

The Matrix Reformulation

Chapter 1

Economics in Matrix Notation

The Standard Model, Rewritten So the Structure Becomes Visible

> *"La realidad está destruyendo en la América Latina aquel pretérito esquema de la división internacional del trabajo que, después de haber adquirido gran vigor en el siglo XIX, seguía prevaleciendo doctrinariamente hasta muy avanzado el presente."*
>
> ["Reality is destroying in Latin America that outdated scheme of the international division of labor."]
> — Raúl Prebisch, *El desarrollo económico de la América Latina* (1949)

> *"The purpose of abstracting is not to be vague, but to create a new semantic level in which one can be absolutely precise."*
> — Edsger W. Dijkstra

This chapter does one thing: it rewrites the economics you already know in a notation that reveals the economics you have not been taught.

Every result here is standard. If you have taken intermediate microeconomics, you know the consumer's problem, the producer's problem, and the idea of market equilibrium. If you have taken a first course in linear algebra, you know what a matrix is and how to multiply them. This chapter puts the two together.

The payoff comes at the end. In Section 6, we will name an object that is present in every microeconomics textbook but that no microeconomics textbook names: the *interaction matrix* — the mathematical description of how agents in the economy relate to each other. The standard competitive model assumes a very specific interaction matrix, and that assumption is what makes the welfare theorems work. Structural economics is what happens when you replace that matrix with the one the world actually has.

But first, the standard model.

1 Who Gets What: The Allocation Matrix

Economics studies how scarce resources are distributed among people. Let us write this down in the simplest possible way.

1.1 A Small Economy

Suppose there are 3 people and 2 goods — call them Food and Clothing. Each person consumes some amount of each good. We can organize this information in a table:

	Food	Clothing
Ana	4	2
Boris	3	5
Carmen	1	3

Ana eats 4 units of Food and wears 2 units of Clothing. Boris eats 3, wears 5. Carmen eats 1, wears 3. This table is the entire economy. Every question economics asks — Is this allocation efficient? Fair? Sustainable? Better or worse than some alternative? — is a question about the numbers in this table.

Now write the table as a matrix:

$$X = \begin{pmatrix} 4 & 2 \\ 3 & 5 \\ 1 & 3 \end{pmatrix}. \tag{1}$$

Each *row* is a person. Each *column* is a good. The entry $X_{i\ell}$ tells us how much of good ℓ person i consumes.

Intuition: Reading the matrix two ways

Read the matrix by rows and you see individuals: Ana's bundle is $(4, 2)$, Boris's is $(3, 5)$, Carmen's is $(1, 3)$. Read it by columns and you see the distribution of each good: Food is split $(4, 3, 1)$ across three people, Clothing is split $(2, 5, 3)$. The row view is how microeconomics usually thinks — one person at a time. The column view is how a planner or a sociologist thinks — one resource at a time, distributed across a population. This book uses both.

Definition 1.1 (Allocation Matrix)**.** *For an economy with N agents and L goods, the* ***allocation matrix*** *is the $N \times L$ matrix*

$$X = \begin{pmatrix} x_1' \\ x_2' \\ \vdots \\ x_N' \end{pmatrix}, \tag{2}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{iL})' \in \mathbb{R}^L_+$ is agent i's consumption vector. Row i is person i's bundle. Column ℓ is the distribution of good ℓ across agents.

How much Food exists in total? Sum the first column: $4 + 3 + 1 = 8$. In matrix notation: $\mathbf{1}'Xe_1 = 8$, where $\mathbf{1} = (1, 1, 1)'$ is the vector of ones and $e_1 = (1, 0)'$ picks out the first column.

More generally, the total endowment of each good is the vector $\mathbf{1}'X$: a $1 \times L$ row vector whose ℓ-th entry is the total amount of good ℓ consumed.

1.2 Endowments: Where People Start

Before any trade or production happens, each person starts with some stuff. Ana might have 6 units of Food and 1 of Clothing (she is a farmer). Boris might have 1 of Food and 6 of Clothing (he is a tailor). Carmen might have 1 of each.

$$\Omega = \begin{pmatrix} 6 & 1 \\ 1 & 6 \\ 1 & 3 \end{pmatrix}. \tag{3}$$

The endowment matrix Ω has the same shape as X: $N \times L$. Entry $\omega_{i\ell}$ is how much of good ℓ person i starts with. The total supply of each good is $\mathbf{1}'\Omega$: here, $(8, 10)$ — 8 units of Food and 10 of Clothing exist in total.

Definition 1.2 (Feasibility)**.** *An allocation X is* ***feasible*** *if nobody consumes more of any good than exists in total:*

$$\mathbf{1}'X \leq \mathbf{1}'\Omega \qquad (\textit{component-wise for each good}). \tag{4}$$

With equality (no waste), this is $\mathbf{1}'X = \mathbf{1}'\Omega$: total consumption equals total endowment.

Check: in our example, $\mathbf{1}'X = (8, 10)$ and $\mathbf{1}'\Omega = (8, 10)$. The allocation is feasible.

2 The Consumer's Problem

2.1 What People Want

Each person has preferences over bundles. Ana might prefer more Food, Boris might prefer more Clothing, Carmen might want a balanced mix. We represent preferences by a utility function $U_i(x_i)$: a number that is higher when person i is happier with her bundle x_i.

Each person chooses the bundle that makes her happiest, subject to what she can afford. If prices are $p = (p_F, p_C)' = (1, 2)'$ (Food costs 1, Clothing costs 2), then Ana's budget is $p'\omega_{\text{Ana}} = 1 \cdot 6 + 2 \cdot 1 = 8$ dollars. She can buy any bundle (x_F, x_C) with $x_F + 2x_C \leq 8$.

Definition 2.1 (The Consumer's Problem)**.** *Agent i solves:*

$$\max_{x_i \geq 0} U_i(x_i) \quad \textit{subject to} \quad p'x_i \leq p'\omega_i. \tag{5}$$

*The solution $x_i^*(p, w_i)$ is agent i's **Marshallian demand**, where $w_i = p'\omega_i$ is her wealth.*

For the entire economy, stack the N problems. Define the wealth vector $\mathbf{w} = \Omega p$: an $N \times 1$ vector whose i-th entry is $w_i = p'\omega_i$. The budget constraints become:

$$Xp \leq \Omega p = \mathbf{w}. \tag{6}$$

Intuition: What this equation hides

Look at equation (5) carefully. Agent i's utility depends only on x_i — her *own* consumption. Nobody else's consumption appears. Ana does not care what Boris eats. Boris does not care what Carmen wears. Each person optimizes in isolation, connected to others only through the price vector p.

This is an assumption, not a fact. In the real world, people care intensely about what others consume, earn, and do. The keeping-up-with-the-Joneses effect, peer pressure, status competition, network externalities — all of these are cases where U_i depends on x_j for $j \neq i$. The standard model assumes all of these away.

We will make this assumption visible and then relax it.

2.2 A Worked Example: Cobb-Douglas

Suppose all three agents have the same preferences:

$$U_i(x_{i1}, x_{i2}) = x_{i1}^{1/2} x_{i2}^{1/2}. \tag{7}$$

This is the Cobb-Douglas utility function with equal weights on both goods. The Marshallian demands (derived by setting up the Lagrangian and solving the first-order conditions — see any intermediate micro text) are:

$$x_{i1}^* = \frac{w_i}{2p_1}, \qquad x_{i2}^* = \frac{w_i}{2p_2}. \tag{8}$$

Each person spends half her income on each good. Let $p = (1, 1)'$ (both goods cost 1). Then:

$$w_{\text{Ana}} = 1 \cdot 6 + 1 \cdot 1 = 7, \tag{9}$$

$$w_{\text{Boris}} = 1 \cdot 1 + 1 \cdot 6 = 7, \tag{10}$$

$$w_{\text{Carmen}} = 1 \cdot 1 + 1 \cdot 3 = 4. \tag{11}$$

The demands are:

$$X^* = \begin{pmatrix} 3.5 & 3.5 \\ 3.5 & 3.5 \\ 2.0 & 2.0 \end{pmatrix}. \tag{12}$$

Intuition: Two things to notice

First, Ana and Boris have *identical demands* even though they start with very different endowments (Ana has mostly Food, Boris has mostly Clothing). The market — through trade at prices p — erases the difference. This is the power of markets: they allow people to trade away from their endowments toward their preferred bundles.

Second, Carmen consumes less of *both* goods. Not because she has different preferences (all three have the same U_i), but because she is poorer. Her endowment has lower total value. The allocation matrix X^* reflects not just preferences but wealth — and wealth reflects endowments, which reflect history.

This is the first hint of the structural question: where do the endowments come from?

Now write the demand in matrix form. From (8), each person's bundle is $x_i^* =$

$\frac{w_i}{2}(1/p_1, 1/p_2)'$. Stack them:

$$X^* = \frac{1}{2}\mathbf{w} \cdot (1/p_1,\ 1/p_2) = \frac{1}{2}\begin{pmatrix}7\\7\\4\end{pmatrix}\begin{pmatrix}1 & 1\end{pmatrix}. \tag{13}$$

The allocation matrix is the *outer product* of the wealth vector $\mathbf{w}$ and the price-dependent expenditure-share vector $(1/2p_1, 1/2p_2)$. Every row is a scalar multiple of the same vector. People differ only in *how much* they consume, not in *what* they consume.

Key Structural Insight

In the Cobb-Douglas model, the allocation matrix X^* is a **rank-1 matrix**: it can be written as a column vector times a row vector. Rank 1 means all rows are proportional. This is the mathematical signature of an economy where people differ only in wealth, not in preferences or social position.

When we introduce social structure in Chapter 5, the allocation matrix will have higher rank. Different agents will consume different *kinds* of bundles, not just different *amounts* of the same bundle. The rank of X is a measure of structural differentiation.

2.3 Market Clearing: Does Everything Add Up?

Check feasibility: $\mathbf{1}'X^* = (3.5 + 3.5 + 2.0,\ 3.5 + 3.5 + 2.0) = (9, 9)$. But total endowment is $\mathbf{1}'\Omega = (8, 10)$. Food demand exceeds supply by 1; Clothing demand falls short by 1. The allocation is not feasible at these prices.

This means $p = (1, 1)$ is not an equilibrium price. The correct price must adjust to equate supply and demand. We need to find p^* such that $\mathbf{1}'X^*(p^*) = \mathbf{1}'\Omega$.

Setting up the market-clearing condition for Food (normalizing $p_1 = 1$):

$$\frac{w_1}{2} + \frac{w_2}{2} + \frac{w_3}{2} = 8 \quad \Rightarrow \quad \frac{(6 + p_2) + (1 + 6p_2) + (1 + 3p_2)}{2} = 8. \tag{14}$$

Solving: $\frac{8+10p_2}{2} = 8$, so $8 + 10p_2 = 16$, giving $p_2^* = 0.8$.

At $p^* = (1, 0.8)$: $w_{\text{Ana}} = 6.8$, $w_{\text{Boris}} = 5.8$, $w_{\text{Carmen}} = 3.4$. The equilibrium allocation is:

$$X^* = \begin{pmatrix}3.40 & 4.25\\2.90 & 3.63\\1.70 & 2.13\end{pmatrix}. \tag{15}$$

Verify: column sums are $(8.0, 10.0)$. Markets clear. This is the **Walrasian equi-**

librium.

3 The One-Good Economy: Welfare and Power

The multi-good case demonstrates trade. But to see *power* clearly, we need the simplest possible setting: one good, N agents, and the question of who gets how much.

3.1 Setup

N agents share a total endowment $\Omega > 0$ of a single good. Person i starts with $\omega_i \geq 0$ and $\sum_i \omega_i = \Omega$. Preferences are quadratic:

$$U_i(x_i) = x_i - \frac{\kappa}{2} x_i^2, \qquad \kappa > 0. \tag{16}$$

This utility function is concave: the first unit is worth approximately 1, each additional unit is worth less, and the marginal utility hits zero at $x_i = 1/\kappa$. The parameter κ controls how fast the returns diminish. Think of it as how quickly a person gets "full."

Example 3.1 (Three People, One Good)**.** *Let $N = 3$, $\kappa = 1$, $\Omega = 3$. Endowments: $\omega_1 = 2$, $\omega_2 = 0.5$, $\omega_3 = 0.5$.*

Person 1 is rich (starts with 2 out of 3). Persons 2 and 3 are poor (start with 0.5 each).

What is the best possible allocation? *A utilitarian planner maximizes total welfare $\sum_i U_i(x_i)$ subject to $\sum_i x_i = 3$. The first-order conditions require equal marginal utility: $1 - x_i = 1 - x_j$, so $x_i = x_j$ for all i, j. Equal division: $x_i^{FB} = 1$ for all i.*

Total welfare at the first-best: $W^{FB} = 3 \times (1 - 0.5) = 1.5$.

What happens without a planner? *With one good and no trade mechanism, each person consumes their endowment: $x_1 = 2$, $x_2 = x_3 = 0.5$.*

Welfare: $W^{CE} = (2 - 2) + (0.5 - 0.125) + (0.5 - 0.125) = 0 + 0.375 + 0.375 = 0.75$.

Person 1 is actually at zero utility! She has so much that the quadratic term wipes out all benefit (at $x = 2$ with $\kappa = 1$: $U = 2 - 2 = 0$). Meanwhile, persons 2 and 3 are at $U = 0.375$ each. The "rich" person is worse off than the poor ones — the allocation is absurdly inefficient.

The welfare loss: *$L = W^{FB} - W^{CE} = 1.5 - 0.75 = 0.75$. Fully half the potential welfare is destroyed by the unequal starting point.*

Proposition 3.2 (Welfare Loss from Inequality)**.** *In the one-good economy with quadratic utility* (16)*, the welfare loss from endowment inequality is:*

$$L = \frac{\kappa N}{2} \cdot Var(\omega_i), \tag{17}$$

where $Var(\omega_i) = \frac{1}{N}\sum_i \omega_i^2 - \left(\frac{\Omega}{N}\right)^2$ *is the variance of endowments. The loss is zero if and only if all endowments are equal.*

Intuition: What this says in words

The cost of inequality is proportional to three things: how curved preferences are (κ), how many people there are (N), and how unequally the endowment is distributed ($\text{Var}(\omega_i)$). More curvature means the rich waste more of what they have (diminishing returns bite harder). More people means more total waste. More inequality means more mismatch between who has stuff and who can use it.

This is the simplest possible welfare cost of inequality. In the structural model, the question becomes: where does $\text{Var}(\omega_i)$ come from? The answer will be: from the weight matrix.

Proof. First-best welfare: $W^{FB} = N \cdot U(\Omega/N) = N[\Omega/N - \frac{\kappa}{2}(\Omega/N)^2] = \Omega - \frac{\kappa\Omega^2}{2N}$.

Endowment welfare: $W^{CE} = \sum_i U_i(\omega_i) = \sum_i[\omega_i - \frac{\kappa}{2}\omega_i^2] = \Omega - \frac{\kappa}{2}\sum_i \omega_i^2$.

The difference:

$$L = W^{FB} - W^{CE} = \frac{\kappa}{2}\left(\sum_i \omega_i^2 - \frac{\Omega^2}{N}\right) = \frac{\kappa}{2} \cdot N \cdot \text{Var}(\omega_i). \tag{18}$$

The last step uses the identity $\sum_i \omega_i^2 - \Omega^2/N = N \cdot \text{Var}(\omega_i)$. □

Example 3.3 (Verifying the Formula)**.** *In Example 3.1: Var*$(\omega) = \frac{1}{3}(4 + 0.25 + 0.25) - 1^2 = 1.5 - 1 = 0.5$*. So* $L = \frac{1 \cdot 3}{2} \cdot 0.5 = 0.75$*. Matches.*

3.2 Bargaining: When Power Enters

Now suppose the three agents can negotiate how to split Ω, but they have unequal bargaining power. Agent 1 is a powerful landowner. Agents 2 and 3 are laborers with no outside options.

We model this through the **generalized Nash bargaining solution**: the allocation maximizes a weighted product of gains from trade.

Definition 3.4 (Bargaining Power)**.** *The **bargaining weight vector** is $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_N)'$ with $\lambda_i > 0$ and $\sum_i \lambda_i = 1$. Agent i's weight λ_i represents her share of total bargaining power.*

Definition 3.5 (Nash Bargaining Solution)**.** *The **Nash bargaining allocation** maximizes:*

$$\max_{\mathbf{x} \geq 0} \sum_{i=1}^{N} \lambda_i \ln\big(U_i(x_i) - U_i(\omega_i)\big) \quad \text{subject to} \quad \sum_i x_i = \Omega, \tag{19}$$

*where $U_i(\omega_i)$ is agent i's **disagreement utility** — what she gets if bargaining fails and she keeps her endowment.*

Intuition: What bargaining power does

The weight λ_i acts like a "voting share" in the allocation decision. An agent with high λ_i pulls the allocation toward her preferred outcome. In the extreme, if $\lambda_1 = 1$ and all others are zero, agent 1 gets everything she wants (subject to feasibility). If $\lambda_i = 1/N$ for all i, bargaining is symmetric and the allocation is "fair" in a specific sense.

The critical question for this book: where does $\boldsymbol{\lambda}$ come from? In the standard model, it is assumed (usually implicitly as $\lambda_i = 1/N$). In the structural model, $\boldsymbol{\lambda}$ is determined by the weight matrix W: your power comes from your position in the network of economic relationships. This is the subject of Chapters 5–7.

Example 3.6 (Bargaining with Asymmetric Power)**.** *Same setup as Example 3.1: $N = 3$, $\kappa = 1$, $\Omega = 3$, $\boldsymbol{\omega} = (2, 0.5, 0.5)$. But now let $\boldsymbol{\lambda} = (0.6, 0.2, 0.2)$ — agent 1 has three times the bargaining power of each other agent.*

The FOC for (19) gives (working through the calculus):

$$\frac{\lambda_i}{S_i} \cdot (1 - \kappa x_i) = \mu \quad \text{for each } i, \tag{20}$$

where $S_i = U_i(x_i) - U_i(\omega_i)$ is agent i's surplus from trade. Powerful agents (high λ_i) receive allocations that give them a larger share of the total surplus.

Numerically (solving the system): $x_1^B \approx 1.3$, $x_2^B \approx 0.85$, $x_3^B \approx 0.85$.

Compare the three allocations:

	Agent 1	*Agent 2*	*Agent 3*
Endowment	*2.00*	*0.50*	*0.50*
First-best (planner)	*1.00*	*1.00*	*1.00*
Bargaining ($\lambda_1 = 0.6$)	*1.30*	*0.85*	*0.85*

The powerful agent gets more than the equal-division first-best. The weak agents get less. But everyone is better off than at the endowment — the bargaining solution at least produces some redistribution (agent 1 gives up 0.7 units).

Welfare: $W^B = U_1(1.3) + U_2(0.85) + U_3(0.85) = (1.3 - 0.845) + 2(0.85 - 0.361) = 0.455 + 0.978 = 1.433$.

Compare: $W^{FB} = 1.5$, $W^B = 1.433$, $W^{CE} = 0.75$. *Bargaining recovers most of the gains from trade, but power asymmetry prevents full efficiency.*

The welfare loss from power: $L_{power} = W^{FB} - W^B = 0.067$. *Small compared to the welfare loss from no-trade* ($L_{no\text{-}trade} = 0.75$), *but real.*

Key Structural Insight

In the one-good economy, there are two sources of welfare loss. The first is *endowment inequality*: some people start with too much, others with too little. The second is *power inequality*: bargaining power determines who captures the gains from reallocation. The standard competitive model assumes power is symmetric ($\lambda_i = 1/N$), which eliminates the second source. This book studies what happens when it isn't.

4 Production: The Leontief System

We now move from consumption to production. The production side introduces the first genuinely structural matrix in standard economics: the *input-output matrix*.

4.1 A Story Before the Math

Consider a simple economy with three sectors: Farming, Milling, and Baking. To produce bread, you need flour. To produce flour, you need wheat. To produce wheat, you need…bread? Well, you need to feed the farmers.

This creates a web of dependencies:

- Baking needs Milling (for flour).
- Milling needs Farming (for wheat).
- Farming needs Baking (for food for the farmers).

Now suppose the country wants to *export* 100 loaves of bread. How much wheat must be grown? Not just enough for 100 loaves — also enough for the flour that those loaves require, and enough to feed the farmers who grow the wheat for

the flour, and enough for the millers who mill the wheat that feeds the farmers who…

The cascade of indirect requirements goes on forever. Each round is smaller than the last (you don't need as much farmer-bread as you need consumer-bread), so the total converges. But computing the total requires summing an infinite series.

This is exactly what the Leontief inverse does.

4.2 The Input-Output Matrix

Example 4.1 (A Three-Sector Economy)**.** *Let us assign numbers. To produce \$1 of output:*

- *Farming uses \$0.10 of its own output (seeds), \$0.05 from Milling, and \$0.10 from Baking.*
- *Milling uses \$0.20 from Farming, \$0.05 of its own output, and \$0.05 from Baking.*
- *Baking uses \$0.05 from Farming, \$0.20 from Milling, and \$0.10 of its own output.*

We organize this as a matrix where entry a_{ij} is the amount of good j needed to produce one unit of good i:

$$A = \begin{pmatrix} 0.10 & 0.05 & 0.10 \\ 0.20 & 0.05 & 0.05 \\ 0.05 & 0.20 & 0.10 \end{pmatrix}. \tag{21}$$

Row i tells you what sector i needs from each sector. Column j tells you who uses sector j's output.

Definition 4.2 (Technical Coefficients Matrix)**.** *The **input-output matrix** A is the $N \times N$ matrix where $a_{ij} \geq 0$ is the amount of good j required to produce one unit of good i.*

The material balance for each sector says: what is produced (y_i) must equal what is used as inputs by all sectors ($\sum_j a_{ji} y_j$, the total demand for i's output as intermediate inputs) plus what is consumed by final users (d_i, household consumption, exports, government):

$$y_i = \sum_{j=1}^{N} a_{ji} y_j + d_i. \tag{22}$$

Wait — read that carefully. The sum $\sum_j a_{ji} y_j$ uses a_{ji}, not a_{ij}. This is because sector j uses a_{ji} units of good i to produce y_j units. The total intermediate demand for good i is the sum over all sectors of how much each sector uses.

In matrix form, stacking all N equations:

$$\mathbf{y} = A'\mathbf{y} + \mathbf{d} \qquad \Leftrightarrow \qquad (I - A')\mathbf{y} = \mathbf{d}. \tag{23}$$

(Some textbooks define A with rows and columns transposed; we follow the convention where a_{ij} = "sector i needs good j," so the material balance uses A'. The reader should always check the convention.)

4.3 Why Infinite Rounds? The Cascade

Before solving the system, let us see what it means.

Example 4.3 (The Cascade for 100 Loaves)**.** *Suppose final demand is* $\mathbf{d} = (0, 0, 100)'$*: we want 100 units of Baking output and nothing else.*

Round 0 (direct): *We need 100 units of Baking. Output needed:* $(0, 0, 100)$.

Round 1 (first indirect): *To produce 100 units of Baking, we need* $100 \times (0.05, 0.20, 0.10) = (5, 20, 10)$ *units of inputs. So Farming must produce 5 more, Milling 20 more, Baking 10 more. Total after round 1:* $(5, 20, 110)$.

Round 2: *The 5 extra Farming output needs inputs:* $5 \times (0.10, 0.05, 0.10) = (0.5, 0.25, 0.5)$. *The 20 extra Milling needs:* $20 \times (0.20, 0.05, 0.05) = (4, 1, 1)$. *The 10 extra Baking needs:* $10 \times (0.05, 0.20, 0.10) = (0.5, 2, 1)$. *Round 2 total:* $(5, 3.25, 2.5)$.

Each round is smaller. By round 5, the additions are tiny. The total converges to the solution of $(I - A')\mathbf{y} = \mathbf{d}$.

Intuition: Why the series converges

Each round of indirect effects is smaller because $a_{ij} < 1$ for all i, j: no sector uses more than one unit of any input to produce one unit of output. The total input requirement per unit of output (the column sum of A, or equivalently the row sum of A') is less than 1 for each sector. This means the cascade "leaks" value at each round — some of each round's output becomes final product rather than being recycled as input. The leak ensures convergence. Technically, convergence requires that the **spectral radius** of A — the largest eigenvalue in magnitude — is less than 1. We explain eigenvalues in Appendix 8. For now: if the economy doesn't require more than a dollar of inputs to produce a dollar of output, the cascade converges.

4.4 The Leontief Inverse: Solving the System

Theorem 4.4 (The Leontief Inverse)**.** *If $\rho(A) < 1$ (the economy is **productive** — see Appendix 8 for what this means), then:*

$$\mathbf{y} = (I - A')^{-1}\mathbf{d}. \tag{24}$$

*The matrix $L = (I - A')^{-1}$ is the **Leontief inverse**. Its entry ℓ_{ij} gives the total output of sector i required to deliver one unit of good j to final demand, counting all direct and indirect effects.*

Intuition: What the Leontief inverse says in words

If you want one more loaf of bread (one more unit of final demand for Baking), how much extra wheat must be grown? Not just the direct input — also the wheat to feed the millers, the wheat to feed the farmers who grow the extra wheat, and so on. The Leontief inverse answers this question exactly: $\ell_{\text{Farming, Baking}}$ is the total Farming output needed for one unit of Baking final demand, including all indirect rounds.

The proof uses the *Neumann series*, which is the matrix version of the geometric series. We develop this in Appendix 8. Here we state the key formula:

$$(I - A')^{-1} = I + A' + (A')^2 + (A')^3 + \cdots = \sum_{k=0}^{\infty} (A')^k. \tag{25}$$

Round 0 is I (direct demand). Round 1 is A' (first-order indirect demand). Round 2 is $(A')^2$ (second-order). The Leontief inverse sums all rounds.

Example 4.5 (Computing the Leontief Inverse)**.** *For the matrix in Example 4.1, computing $(I - A')^{-1}$ numerically (do this with Python — see Exercise 1.7):*

$$L = (I - A')^{-1} \approx \begin{pmatrix} 1.16 & 0.27 & 0.14 \\ 0.10 & 1.13 & 0.25 \\ 0.09 & 0.10 & 1.16 \end{pmatrix}. \tag{26}$$

Reading column 3 (Baking): to deliver 1 unit of bread to final demand requires 1.16 units of Baking output (the 1 unit of bread plus 0.16 units recycled as input to other sectors that supply Baking), 0.25 units of Milling output, and 0.14 units of Farming output.

*The **output multiplier** of each sector is the column sum of L:*

Sector	*Output multiplier*
Farming	$1.16 + 0.10 + 0.09 = 1.35$
Milling	$0.27 + 1.13 + 0.10 = 1.50$
Baking	$0.14 + 0.25 + 1.16 = 1.55$

Baking has the highest multiplier: stimulating bread demand generates the most total economic activity, because bread production is deeply embedded in the input chain (it needs flour, which needs wheat, which needs food). Farming has the lowest: it has fewer backward linkages.

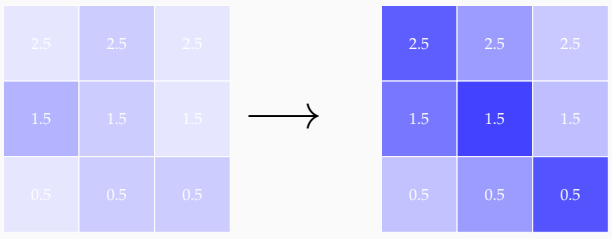
Key Structural Insight

The Leontief inverse is the first **structural multiplier** in this book. It shows that the effect of a change in one sector propagates through the entire economy, amplified by the network of inter-sectoral dependencies. The structure of A — which sectors depend on which — determines the pattern and magnitude of propagation. Two economies with identical technologies but different A matrices will respond differently to the same demand shock. The matrix A *is* the production structure.

In Chapter 16, we will apply this to the global economy, with countries as "sectors" and international trade as the input-output matrix. The center-periphery structure that Prebisch described is a property of the column sums and spectral structure of the global A.

Visualization:

The Leontief Inverse as a Heatmap



A **(inputs)** $(I - A)^{-1}$ **(total effects)**

The input matrix A (left) is dim: weak direct dependencies. The Leontief inverse $(I - A)^{-1}$ (right) is brighter everywhere — indirect effects amplify direct ones. The diagonal is always brightest. If the off-diagonal is bright, growth propagates. If dark (block-diagonal A), growth stays localized.

5 General Equilibrium: Markets Clear

5.1 The Walrasian Equilibrium

We now combine consumption and production. The economy has N agents, L goods, prices p, and an allocation X.

Definition 5.1 (Walrasian Equilibrium)**.** *A **Walrasian equilibrium** is a price vector p^* and an allocation X^* such that:*

1. *Every agent optimizes: x_i^* solves (5) at prices p^*.*
2. *Markets clear: $\mathbf{1}'X^* = \mathbf{1}'\Omega$ (total demand equals total supply for each good).*

The equilibrium p^* is a *fixed point*: it is the price at which the demands generated by p^* are exactly consistent with p^*. Under standard conditions (continuous preferences, positive endowments), such a price exists — this is the content of the Arrow-Debreu existence theorem, proved using Brouwer's fixed-point theorem.

We will not reproduce the proof here (see MWG Chapter 17). What matters for us is not the existence but the *structural content* of the existence argument: it requires that agents interact only through p. If agents interacted through other channels — peer effects, network externalities, bargaining relationships — the standard proof would need to be modified. We provide the modified proof in Chapter 7.

6 The Implicit Weight Matrix

We are now ready for the chapter's punchline.

6.1 How Do Agents Interact in the Standard Model?

Consider the economy we have developed: N agents, each maximizing $U_i(x_i)$ subject to $p'x_i \leq w_i$, with market-clearing prices p^*. How does agent i affect agent j?

The answer: only through p^*. Agent i's demand for goods affects aggregate demand, which affects the equilibrium price, which affects j's budget set and optimal choice. The interaction is:

(a) **Anonymous**: the effect does not depend on who i is — only on i's total demand.

(b) **Symmetric**: i's effect on j is the same magnitude as j's effect on i (both go through p).

(c) **Indirect**: i does not enter j's utility function. The effect passes through the market.

(d) **Equal-weighted**: in a competitive market, no single agent affects p disproportionately. Each agent's demand has weight $1/N$ in the aggregate.

These four properties describe a very specific pattern of interaction. We can write it as a matrix.

6.2 Naming the Matrix

Definition 6.1 (The Competitive Interaction Matrix)**.** *The **competitive interaction matrix** is:*

$$W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}' = \frac{1}{N}\begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}. \tag{27}$$

Every entry is $1/N$: each agent affects every other agent equally.

This matrix has remarkable properties, which we now catalog.

Proposition 6.2 (Properties of the Competitive Interaction Matrix)**.** $W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$ *satisfies:*

1. ***Doubly stochastic**: every row sums to 1, and every column sums to 1. Nobody gives or receives more total influence than anyone else.*
2. ***Symmetric**: $W^{CE} = (W^{CE})'$. The effect of i on j equals the effect of j on i.*
3. ***Rank 1**: all rows are identical. The matrix can be written as the outer product of two vectors: $W^{CE} = \frac{1}{N}\mathbf{1} \cdot \mathbf{1}'$. This means W^{CE} contains no information about individual positions — every agent looks the same.*
4. ***Equal centrality**: for any reasonable centrality measure, all agents have the same centrality. (We will make "centrality" precise in Chapter 6 using the Perron eigenvector.)*
5. ***Maximal spectral gap**: W^{CE} has eigenvalue 1 (with eigenvector $\mathbf{1}$) and eigenvalue 0 (with multiplicity $N-1$). The gap between the largest and second-largest eigenvalue is $1-0=1$, the maximum possible for any matrix with these properties. (See Appendix 8 for eigenvalues.)*

Proof. (1) $\sum_j W^{CE}_{ij} = \sum_j 1/N = 1$. Similarly for column sums.

(2) $(W^{CE})' = (1/N \cdot \mathbf{1}\mathbf{1}')' = 1/N \cdot \mathbf{1}\mathbf{1}' = W^{CE}$.

(3) Every row is $(1/N, 1/N, \ldots, 1/N)$, so all rows are proportional (in fact, identical). Rank $= 1$.

(4) See Chapter 6 for the formal definition. Informally: if every row of W is the same, the "importance" of every agent is the same.

(5) $W^{CE}\mathbf{1} = \frac{1}{N}\mathbf{1}(\mathbf{1}'\mathbf{1}) = \frac{1}{N}\mathbf{1} \cdot N = \mathbf{1}$, so $\mathbf{1}$ is an eigenvector with eigenvalue 1. For any $\mathbf{x}$ perpendicular to $\mathbf{1}$ (i.e., $\mathbf{1}'\mathbf{x} = 0$): $W^{CE}\mathbf{x} = \frac{1}{N}\mathbf{1}(\mathbf{1}'\mathbf{x}) = 0$. So eigenvalue 0 has multiplicity $N - 1$. □

Intuition: What W^{CE} does to a vector

Apply W^{CE} to any vector $\mathbf{x}$:

$$W^{CE}\mathbf{x} = \frac{1}{N}\mathbf{1}(\mathbf{1}'\mathbf{x}) = \bar{x} \cdot \mathbf{1}, \tag{28}$$

where $\bar{x} = \frac{1}{N}\sum_i x_i$ is the mean.

The competitive interaction matrix **replaces every entry with the mean**. It destroys all information about individual positions and keeps only the aggregate. This is exactly what the competitive market does: individual identities are washed out by the price mechanism, and only the aggregate (total demand, total supply) determines the outcome.

6.3 The Autarky Matrix

The other extreme is no interaction at all:

$$W^A = I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}. \tag{29}$$

Each agent interacts only with herself. No trade, no externalities, no social contact. Robinson Crusoe, N times.

6.4 The Space of All Possible Structures

Between $W = I$ (autarky) and $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$ (anonymous competitive market) lies a vast space of possible interaction structures. Most real economies live somewhere in this space:

	Autarky $W = I$	**General** W	**Competitive** $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$
Interaction	None	Structured	Anonymous
Power	Trivially equal	Asymmetric	Equal
Information	Private	Filtered by W^I	Symmetric
Welfare theorems	Trivially hold	Fail	Hold

Key Structural Insight

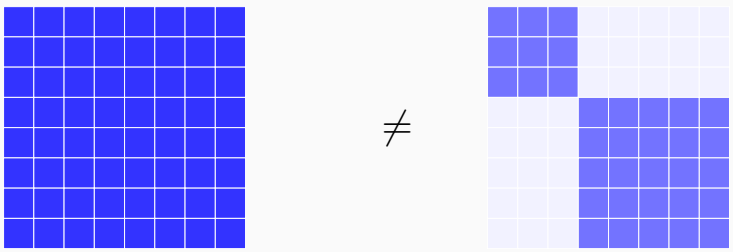
The program of this book is now visible. The standard competitive model lives at the right edge of this table. It assumes the most special possible interaction matrix: symmetric, rank 1, maximal spectral gap, equal centrality. *Every* standard result — the welfare theorems, the efficiency of competitive equilibrium, the irrelevance of social structure — is a consequence of this assumption.

Structural economics replaces W^{CE} with a general W: asymmetric, high-rank, with a spectral gap that may be small, and centrality that varies across agents. Power becomes endogenous (Chapter 6). Preferences become endogenous to the network (Chapter 10). The welfare theorems fail (Chapter 2). And the question shifts from "is the equilibrium efficient?" to "what is the structure, who does it benefit, and can it be changed?"

The standard model is not wrong. It is a special case. This book is the general case.

Visualization:

W^{CE} vs. the real world



$W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$ $\neq$ $W_{\textbf{Bolivia}}$

Equal power. Welfare theorems hold. Unequal power. Welfare theorems fail.

Left: the competitive weight matrix (Prologue pattern 2, "Market"). Right: Bolivia's production structure (pattern 6, "Dual Economy"). The difference between these two heatmaps is the subject of this book.

7 Summary

This chapter established five results, all of them standard economics expressed in matrix notation:

1. The **allocation matrix** X organizes who gets what. Read by rows: individual bundles. Read by columns: distribution of each good. The rank of X measures structural differentiation.

2. The **consumer's problem** determines each row of X independently under the standard model. Agent i's utility depends only on x_i. This is a structural assumption: the weight agent i places on agent j's consumption is zero.

3. The **welfare loss from inequality** in the one-good economy is $L = \frac{\kappa N}{2}\text{Var}(\omega_i)$. With bargaining power $\boldsymbol{\lambda}$, the allocation depends on who has power, not just who has endowments.

4. The **Leontief inverse** $(I - A')^{-1}$ is the first structural matrix in standard economics. It encodes all direct and indirect inter-sectoral dependencies. Its properties (column sums, spectral radius) characterize the production structure.

5. The **implicit interaction matrix** of the competitive model is $W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$: doubly stochastic, symmetric, rank 1, maximal spectral gap, equal centrality. The welfare theorems hold for this matrix. Structural economics studies general W.

No new economics was introduced. But the notation reveals an object — the weight matrix W — that was always present but never named. The next chapters will show what changes when W is not $\frac{1}{N}\mathbf{1}\mathbf{1}'$.

8 Exercises

1.1 Reading the allocation matrix. For $N = 4$ agents and $L = 3$ goods (Food, Clothing, Housing), the allocation matrix is:

$$X = \begin{pmatrix} 5 & 3 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 5 \\ 4 & 2 & 2 \end{pmatrix}. \tag{30}$$

(a) What is agent 3's consumption bundle? What is agent 3 likely "good at" (what might she produce)?

(b) What is the total consumption of Housing? Which agent consumes the most Housing?

(c) Is this allocation feasible if total endowments are $\mathbf{1}'\Omega = (12, 10, 10)$?

(d) Compute the rank of X. Is this a "structurally differentiated" economy in the sense discussed above?

1.2 Cobb-Douglas equilibrium. Three agents with $U_i = x_{i1}^{1/3} x_{i2}^{2/3}$ and endowments $\omega_1 = (4, 1)'$, $\omega_2 = (1, 3)'$, $\omega_3 = (3, 2)'$.

(a) Derive the Marshallian demand functions.

(b) Find the equilibrium price p_2^* (normalize $p_1 = 1$) by solving the market-clearing condition.

(c) Compute the equilibrium allocation X^* and verify feasibility.

1.3 Welfare loss computation. For $N = 5$, $\kappa = 2$, $\Omega = 5$:

(a) Compute L for $\boldsymbol{\omega} = (1, 1, 1, 1, 1)$. (Answer: 0.)

(b) Compute L for $\boldsymbol{\omega} = (3, 1, 0.5, 0.25, 0.25)$.

(c) Compute L for $\boldsymbol{\omega} = (5, 0, 0, 0, 0)$ (one person has everything).

(d) Plot L as a function of the Gini coefficient of $\boldsymbol{\omega}$.

1.4 Bargaining with different power. For $N = 2$, $\kappa = 1$, $\Omega = 2$, $\boldsymbol{\omega} = (1.5, 0.5)$:

(a) Find the first-best allocation and welfare.

(b) Find the bargaining allocation for $\boldsymbol{\lambda} = (0.5, 0.5)$ (equal power). Compare to the first-best.

(c) Find the bargaining allocation for $\boldsymbol{\lambda} = (0.8, 0.2)$ (asymmetric power). Who benefits?

(d) For what $\boldsymbol{\lambda}$ does the bargaining allocation equal the first-best?

1.5 The cascade in action. For the input-output matrix:

$$A = \begin{pmatrix} 0.2 & 0.1 \\ 0.3 & 0.1 \end{pmatrix}. \tag{31}$$

(a) Compute the first five terms of the Neumann series: $I, A', (A')^2, (A')^3, (A')^4$. (Use a calculator or Python.)

(b) Sum them: $\sum_{k=0}^{4}(A')^k$. Compare with the exact $(I - A')^{-1}$. How close are they?

(c) What is the spectral radius $\rho(A)$? (See Appendix 8.) Verify $\rho(A) < 1$.

(d) Compute output multipliers (column sums of $(I - A')^{-1}$). Which sector has stronger backward linkages?

1.6 Properties of W^{CE}.

(a) Show that W^{CE} is idempotent: $(W^{CE})^2 = W^{CE}$. (Hint: compute $\frac{1}{N}\mathbf{1}\mathbf{1}' \cdot \frac{1}{N}\mathbf{1}\mathbf{1}'$.)

(b) Show that for any vector $\mathbf{x}$, $W^{CE}\mathbf{x} = \bar{x}\mathbf{1}$. Interpret: the competitive market replaces individual variation with the average.

(c) Prove: W^{CE} has eigenvalue 1 with eigenvector $\mathbf{1}$, and eigenvalue 0 with multiplicity $N - 1$. Find the eigenvectors for eigenvalue 0 when $N = 3$.

(d) For $N = 3$, consider the matrix $W = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 2 & 0.5 & 0.5 \\ 0 & 1.5 & 1.5 \end{pmatrix}$. Is this doubly stochastic? Compute $W\mathbf{x}$ for $\mathbf{x} = (3, 6, 9)'$ and compare with $W^{CE}\mathbf{x}$. What is different?

1.7 Bolivia's input-output table (data exercise). Download the Bolivian input-output table from INE or the OECD ICIO database. Aggregate to 5–10 sectors.

(a) Construct A (normalize each column by gross output).

(b) Compute the Leontief inverse $(I - A')^{-1}$.

(c) Compute output multipliers for: Mining/Gas, Agriculture, Manufacturing, Services.

(d) Which sector has the highest multiplier? Lowest? Relate to Pinto's structural heterogeneity: is Bolivia's economy "structurally connected" or "block-diagonal"?

(e) *Challenge*: Is the gas sector connected to the rest of the economy, or is it an enclave?

1.8 From story to matrix. Doña María sells api morado on Calle Comercio in La Paz. To produce api, she buys corn from a wholesaler in El Alto, sugar from a shop on Calle Max Paredes, cinnamon from a market distributor,

and charcoal from a supplier in the altiplano. The wholesaler buys from farmers in the Yungas.

(a) Define 5 agents: María, Wholesaler, Sugar Shop, Cinnamon Distributor, Charcoal Supplier. Draw the directed dependency graph.

(b) Write the 5×5 adjacency matrix A ($A_{ij} = 1$ if agent i uses agent j's output as input, 0 otherwise).

(c) Is A symmetric? What does the asymmetry mean economically?

(d) Suppose the Wholesaler disappears. Set row and column 2 of A to zero. Which other agents are directly affected? Which are indirectly affected (via the Leontief cascade)?

(e) Compare this adjacency matrix to W^{CE} for $N = 5$. What structural information does A contain that W^{CE} destroys?

1.9 From sectors to countries. Suppose the "agents" in the Leontief system are 3 countries:

$$A = \begin{pmatrix} 0.05 & 0.30 & 0.30 \\ 0.01 & 0.05 & 0.05 \\ 0.01 & 0.05 & 0.05 \end{pmatrix}. \tag{32}$$

Country 1 is the "center" (Column 1 is small: few depend on it. Row 1 is large: it needs a lot from others). Wait — check this. Actually Column 1 shows who needs Country 1: $0.05, 0.01, 0.01$. Country 1's output is barely used by others. But Row 1 shows Country 1 needs a lot from Countries 2 and 3 (0.30 each).

(a) Which country is the "center" and which are "periphery"? (Hint: look at *column sums* of $(I - A')^{-1}$ — who has the highest output multiplier?)

(b) If final demand rises by 1 unit in Country 2 only, how much total output is generated in each country?

(c) Argue informally that the center-periphery structure is a property of the asymmetry of A.

Mathematical Appendices

Appendix 1A: Matrices as Economic Objects

This appendix covers the matrix concepts used in Chapter 1, starting from scratch.

What is a matrix? A matrix is a rectangular table of numbers. An $N \times L$ matrix has N rows and L columns. In this book, matrices represent *relationships*: the entry in row i, column j describes the relationship between agent (or sector) i and agent (or sector) j.

Matrix	What it describes
Allocation X ($N \times L$)	How much of each good each person consumes
Endowment Ω ($N \times L$)	How much of each good each person starts with
Input-output A ($N \times N$)	How much of each sector's output each sector needs
Interaction W ($N \times N$)	How much each agent affects each other agent

Special vectors. Two vectors appear constantly:

- $\mathbf{1} = (1, 1, \ldots, 1)'$: the vector of all ones. $\mathbf{1}'x = \sum_i x_i$ sums the entries of x.
- e_k: the k-th standard basis vector (1 in position k, 0 elsewhere). $e_k'x = x_k$ picks out the k-th entry.

Matrix multiplication as "flow through a system." If A is $N \times N$ and $\mathbf{x}$ is $N \times 1$, then $A\mathbf{x}$ is $N \times 1$ with $(A\mathbf{x})_i = \sum_j a_{ij}x_j$. Think of x_j as the "amount of stuff at node j" and a_{ij} as "how much of node j's stuff flows to node i." The product $A\mathbf{x}$ is the result of flowing everything through the system once.

Flow through twice: $A(A\mathbf{x}) = A^2\mathbf{x}$. Flow k times: $A^k\mathbf{x}$. The Leontief inverse $(I - A)^{-1} = I + A + A^2 + \cdots$ sums the result of flowing through all possible numbers of times.

The outer product $\mathbf{1}\mathbf{1}'$. This is an $N \times N$ matrix of all ones:

$$\mathbf{1}\mathbf{1}' = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & \cdots & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}. \tag{33}$$

Dividing by N gives $W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$, where every entry is $1/N$. This matrix has a beautiful property: $W^{CE}\mathbf{x} = \bar{x}\mathbf{1}$ — it replaces every entry of $\mathbf{x}$ with the mean. It is the mathematical representation of "anonymous, symmetric interaction."

The identity matrix I. The $N \times N$ identity matrix has 1 on the diagonal and 0 elsewhere. $I\mathbf{x} = \mathbf{x}$ for any $\mathbf{x}$: it does nothing. It represents "no interaction" — each agent affects only herself.

Row sums and column sums. For an $N \times N$ matrix M:

- Row sums: $M\mathbf{1}$ is an $N \times 1$ vector whose i-th entry is $\sum_j m_{ij}$ (total outflow from i).
- Column sums: $\mathbf{1}'M$ is a $1 \times N$ vector whose j-th entry is $\sum_i m_{ij}$ (total inflow to j).

A matrix is **row-stochastic** if all row sums equal 1: $M\mathbf{1} = \mathbf{1}$. It is **column-stochastic** if all column sums equal 1: $\mathbf{1}'M = \mathbf{1}'$. It is **doubly stochastic** if both. The competitive interaction matrix W^{CE} is doubly stochastic.

Appendix 1B: Eigenvalues and the Spectral Radius

The key idea. An eigenvalue is a number that tells you how a matrix *stretches* things. If $A\mathbf{v} = \lambda\mathbf{v}$ for some nonzero vector $\mathbf{v}$, then λ is an eigenvalue and $\mathbf{v}$ is the corresponding eigenvector. Applying A to $\mathbf{v}$ just rescales it by λ — the direction stays the same, only the magnitude changes.

Why economists care. Eigenvalues tell you whether a system explodes, converges, or oscillates:

- $|\lambda| < 1$: applying A repeatedly shrinks the component along $\mathbf{v}$. The system converges.
- $|\lambda| = 1$: the component stays the same. Steady state.
- $|\lambda| > 1$: the component grows without bound. Explosion.

The spectral radius. The **spectral radius** of A is $\rho(A) = \max_i |\lambda_i|$ — the largest eigenvalue in magnitude. It determines the long-run behavior of the system:

- $\rho(A) < 1$: $A^k \to 0$ as $k \to \infty$. The cascade of indirect effects dies out. The Leontief inverse exists.

- $\rho(A) = 1$: the system is on the boundary. Convergence is borderline.
- $\rho(A) > 1$: A^k explodes. The system requires more inputs than it produces. The economy is "unproductive."

For the input-output matrix, $\rho(A) < 1$ means the economy is productive: it produces more output than it uses as input.

Computing eigenvalues. For a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the eigenvalues solve $\det(A - \lambda I) = 0$:

$$\lambda^2 - (a + d)\lambda + (ad - bc) = 0. \tag{34}$$

The solutions are $\lambda = \frac{(a+d) \pm \sqrt{(a+d)^2 - 4(ad-bc)}}{2}$. For larger matrices, use numerical software.

Example. For $A = \begin{pmatrix} 0.2 & 0.1 \\ 0.3 & 0.1 \end{pmatrix}$ (from Exercise 1.5): trace $= 0.3$, determinant $= 0.02 - 0.03 = -0.01$. Eigenvalues: $\lambda = \frac{0.3 \pm \sqrt{0.09+0.04}}{2} = \frac{0.3 \pm 0.36}{2}$. So $\lambda_1 = 0.33$ and $\lambda_2 = -0.03$. Spectral radius: $\rho(A) = 0.33 < 1$. The economy is productive.

The spectral gap. For an $N \times N$ matrix with eigenvalues $|\lambda_1| \geq |\lambda_2| \geq \cdots \geq |\lambda_N|$, the **spectral gap** is $\Gamma = |\lambda_1| - |\lambda_2|$: the distance between the largest and second-largest eigenvalue. In this book, the spectral gap measures *structural asymmetry*:

- Large Γ (close to ρ): the system is dominated by one "mode" — fast convergence, symmetric structure. The competitive interaction matrix has $\Gamma = 1$ (maximum).
- Small Γ (close to 0): the system has competing modes — slow convergence, persistent asymmetries. Star networks and hub-spoke structures have small spectral gaps, allowing power to concentrate.

We will develop these connections formally in Chapter 6. For now: eigenvalues measure how a system amplifies or dampens, and the spectral gap measures how symmetric the system is. Both are properties of the matrix W, which is the social structure.

Appendix 1C: The Neumann Series

Start with scalars. You know the geometric series: if $|r| < 1$, then

$$1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r}. \tag{35}$$

Rearranging: $(1-r)(1 + r + r^2 + \cdots) = 1$, so $1 + r + r^2 + \cdots$ is the *inverse* of $(1-r)$.

Now matrices. Replace the scalar r with a matrix A and the number 1 with the identity matrix I:

$$I + A + A^2 + A^3 + \cdots = (I - A)^{-1}, \tag{36}$$

provided the series converges. The condition for convergence is $\rho(A) < 1$ (the spectral radius is less than 1) — the matrix analog of $|r| < 1$.

Why it converges. Each power A^k has entries bounded by $\rho(A)^k$ (roughly). Since $\rho(A) < 1$, these powers decay geometrically: $A^k \to 0$ as $k \to \infty$. The partial sums $\sum_{k=0}^{K} A^k$ converge to a finite matrix, which is $(I - A)^{-1}$.

Verification. Multiply both sides of (36) by $(I - A)$:

$$(I - A)(I + A + A^2 + \cdots) = (I + A + A^2 + \cdots) - (A + A^2 + A^3 + \cdots) \tag{37}$$

$$= I. \tag{38}$$

The infinite series telescopes, leaving only I. This confirms that $\sum_{k=0}^{\infty} A^k$ is indeed the inverse of $(I - A)$.

Economic interpretation. In the Leontief system:

- I: direct demand (round 0). Each unit of final demand requires one unit of direct output.
- A': first-round indirect demand. Producing the direct output requires A' units of intermediate inputs.
- $(A')^2$: second-round indirect demand. Producing the first-round inputs requires further inputs.
- $(A')^k$: k-th round. The cascade continues.

- $(I - A')^{-1}$: the total of all rounds. The complete direct + indirect requirement.

A numerical example. For $A' = \begin{pmatrix} 0.2 & 0.3 \\ 0.1 & 0.1 \end{pmatrix}$:

	$(1,1)$	$(1,2)$	$(2,1)$	$(2,2)$
I	1.000	0.000	0.000	1.000
$I + A'$	1.200	0.300	0.100	1.100
$I + A' + (A')^2$	1.270	0.360	0.130	1.140
$I + A' + \cdots + (A')^4$	1.283	0.370	0.134	1.147
$(I - A')^{-1}$ (exact)	1.284	0.370	0.135	1.149

By round 4, the partial sum is within 0.2% of the exact inverse. The cascade converges fast when $\rho(A)$ is small.

Structuralist Case Study: Prebisch's 1949 Manifesto and the Weight Matrix

In 1949, Raúl Prebisch — an Argentine economist who had served as head of his country's central bank — published what Albert Hirschman later called the "Latin American Manifesto": *El desarrollo económico de la América Latina y algunos de sus principales problemas*. The essay served as the introduction to CEPAL's first *Estudio económico de América Latina* and laid the intellectual foundation for half a century of structuralist economics.

Prebisch's argument has three steps, each of which corresponds to a property of the weight matrix developed in this chapter.

Step 1: The global economy has a structure. Prebisch rejected the Ricardian doctrine that free trade benefits all participants symmetrically. He argued that the world economy is divided into an industrialized "center" and a primary-commodity-exporting "periphery," connected by trade linkages that are fundamentally *asymmetric*: the periphery exports raw materials with low value-added and imports manufactures with high value-added. In the notation of this chapter: the global input-output matrix A_{global} has a core-periphery topology — the center's column sums in the Leontief inverse $(I - A')^{-1}$ are large (high backward linkages), while the periphery's column sums are small. The structure is not $\frac{1}{N}\mathbf{1}\mathbf{1}'$.

Step 2: The structure generates asymmetric outcomes. Prebisch argued that the fruits of technical progress are distributed unequally: center countries retain the productivity gains from industrialization (through higher wages negotiated by strong unions), while periphery countries transfer theirs to the center through

deteriorating terms of trade. In our notation: the Perron eigenvector of the global W assigns high weight to the center and low weight to the periphery — $\pi_{\text{center}} \gg \pi_{\text{periphery}}$. The bargaining allocation (Chapter 7) gives the center more than its "fair share" of the surplus. The welfare loss from this asymmetry is proportional to $\text{Var}(\pi_i)$ — the variance of the power distribution.

Step 3: The doctrine that defends the structure is produced by the structure. Prebisch opened his essay by declaring that "reality is destroying that outdated scheme of the international division of labor." The "scheme" he attacked was classical trade theory, which claimed that free trade was optimal for all participants. His point: the theory itself was produced at the center and propagated to the periphery through academic training, policy advice, and institutional pressure. In the language of Chapter 5: the global information network W^I has a center-periphery topology — the periphery absorbs economic doctrine from the center (the Perron eigenvector of W^I assigns dominant weight to center economists), and the resulting "consensus" reflects center interests. The Washington Consensus, forty years later, is the DeGroot equilibrium of this W^I.

Each step corresponds to a formal object in this book:

Prebisch's argument	**Formal object**	**C**
The global economy has a structure	$W \neq \frac{1}{N}\mathbf{1}\mathbf{1}'$	1
The structure generates asymmetric outcomes	$\boldsymbol{\pi}(W) \neq \frac{1}{N}\mathbf{1}$	6
The structure produces the doctrine	W^I has center-periphery topology	5,
Terms of trade deteriorate	Column sums of $(I - A')^{-1}$ are asymmetric	3,
The welfare theorems fail	W is not doubly stochastic	2

Prebisch wrote all of this in prose. The equations that formalize his argument are the content of Chapters 1–8. The structuralists built the cathedral in words. This book provides the engineering drawings. But the blueprint was always theirs.

Further Reading

Standard microeconomics: Mas-Colell, Whinston, and Green (1995), Chapters 1–3 (consumer theory), 5 (production), 15–17 (general equilibrium).

The Leontief system: Leontief (1986), *Input-Output Economics*. Miller and Blair (2009), *Input-Output Analysis*, 2nd edition. Sargent and Stachurski (2024), *Economic Networks*, Chapters 1–4.

Network perspective on production: Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), "The Network Origins of Aggregate Fluctuations," *Econometrica*.

Linear algebra for economists: Strang (2016), *Introduction to Linear Algebra*, 5th edition. Simon and Blume (1994), *Mathematics for Economists*. Sargent and Stachurski's QuantEcon lectures (`quantecon.org`).

The structuralist tradition: Prebisch (1949), *El desarrollo económico de la América Latina y algunos de sus principales problemas*, CEPAL (the full text, in Spanish, is available at `repositorio.cepal.org`). Hirschman (1958), *The Strategy of Economic Development*, Chapters 6–7. Taylor (1983), *Structuralist Macroeconomics*, Chapter 1. Bielschowsky (2009), "Sixty years of CEPAL: structuralism and neo-structuralism," *CEPAL Review* 97.

Answers to the Prologue Challenge

A: Center–Periphery (Pattern 7). Persons 1–3 form the "center" (bright upper-left). The lower-left rectangle is brighter than the upper-right: dependence runs one way.

B: Hub / Star (Pattern 4). One blazing column at position 4 — not position 1. You had to read the columns.

C: Chain / Ring (Pattern 3). Bright band along the diagonal, wrapping at corners.

D: Triple Economy (Pattern 6, extended). Three bright squares: persons 1–3, 4–6, 7–8.

E: Market / Complete (Pattern 2). Uniform brightness. $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$.

F: Hierarchy (Pattern 5). Brightness gradient from upper-left to lower-right.

Chapter 2

Welfare Theorems as Matrix Properties

Why Efficiency Requires Symmetry — and Where It Breaks

> *"O subdesenvolvimento não é uma etapa necessária do processo de formação das economias capitalistas modernas. É um processo particular, resultante da penetração de empresas capitalistas modernas em estruturas arcaicas."*
>
> ["Underdevelopment is not a necessary stage in the formation of modern capitalist economies. It is a particular process, resulting from the penetration of modern capitalist enterprises into archaic structures."]
> — Celso Furtado, *Desenvolvimento e Subdesenvolvimento* (1961)

Chapter 1 revealed that the standard competitive model assumes a specific weight matrix: $W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$. This chapter asks: what happens to the welfare theorems when the weight matrix is something else?

The First Welfare Theorem says: the competitive equilibrium is Pareto efficient. The Second says: any efficient allocation can be achieved by suitable transfers. Both are beautiful results. Both are theorems about a matrix — and both require that matrix to be doubly stochastic.

When W departs from doubly stochastic, both theorems weaken. This chapter derives the conditions for each, quantifies the welfare loss from departures, and identifies three structural reasons for failure: asymmetric power, unpriced externalities, and block disconnection. The spectral gap of W bounds the welfare loss from above, giving us a single-number diagnostic of structural health.

1 The First Welfare Theorem

1.1 The Standard Statement and Proof

Theorem 1.1 (First Welfare Theorem)**.** *If: (i) every agent maximizes utility given prices, (ii) markets clear, and (iii) preferences are locally nonsatiated, then the competitive equilibrium allocation is Pareto efficient.*

Proof. Let (p^*, X^*) be a competitive equilibrium. Suppose it is not Pareto efficient: there exists a feasible allocation $\hat{X}$ with $U_i(\hat{x}_i) \geq U_i(x_i^*)$ for all i and $U_j(\hat{x}_j) > U_j(x_j^*)$ for some j.

For agent j: since x_j^* maximizes U_j on the budget set and $\hat{x}_j$ is strictly preferred, $\hat{x}_j$ must not be affordable: $p^{*\prime}\hat{x}_j > p^{*\prime}\omega_j$.

For all other agents $i \neq j$: by local nonsatiation, $U_i(\hat{x}_i) \geq U_i(x_i^*)$ implies $p^{*\prime}\hat{x}_i \geq p^{*\prime}\omega_i$ (if $\hat{x}_i$ were strictly cheaper, i could find something even better in her budget set by continuity).

Summing over all agents:

$$\sum_{i=1}^{N} p^{*\prime}\hat{x}_i > \sum_{i=1}^{N} p^{*\prime}\omega_i. \tag{1}$$

But feasibility requires $\sum_i \hat{x}_{i\ell} = \sum_i \omega_{i\ell}$ for all goods ℓ. Multiplying by p_ℓ^* and summing: $\sum_i p^{*\prime}\hat{x}_i = \sum_i p^{*\prime}\omega_i$. Contradiction. □

Intuition: Where does structure enter the proof?

The proof uses one critical feature of the competitive mechanism: every agent faces the *same* price vector p^*. The price aggregates individual demands anonymously — the identity of the agent does not affect the price she faces. Agent i's effect on p is proportional to her demand, not to her "position" in any network.

This is the structural content: the price mechanism is symmetric. In weight-matrix language, the interaction is $W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$: every agent enters the price mechanism with equal weight. If some agents had more influence on prices (market power, bargaining leverage, information advantage), the proof would fail — the inequality in the last step would not follow.

1.2 The Matrix Restatement

To state the structural condition precisely, consider the one-good economy from Chapter 1 with utility externalities. Agent i's utility now includes the effect of others:

$$V_i(\mathbf{x}) = U_i(x_i) + \gamma \sum_{j \neq i} W_{ij} U_j(x_j), \tag{2}$$

where $W_{ij} \geq 0$ is the weight i places on j's utility and $\gamma \geq 0$ is the externality intensity.

A social planner maximizes total social welfare $\sum_i V_i(\mathbf{x})$ subject to $\sum_i x_i = \Omega$.

Expanding:

$$\sum_{i=1}^{N} V_i = \sum_{i=1}^{N} U_i(x_i) + \gamma \sum_{i=1}^{N} \sum_{j \neq i} W_{ij} U_j(x_j). \tag{3}$$

Rearranging the double sum (collect terms by U_j): the coefficient on $U_j(x_j)$ is

$$\alpha_j = 1 + \gamma \sum_{i \neq j} W_{ij}. \tag{4}$$

The term $\sum_{i \neq j} W_{ij}$ is the column sum of W at column j (excluding the diagonal). It measures how much agent j is *cared about* by others — her total incoming externality weight.

Definition 1.2 (Externality Centrality)**.** *Agent j's **externality centrality** is $c_j = \sum_{i \neq j} W_{ij}$: the total weight others place on j's utility. The planner's effective weight on j is $\alpha_j = 1 + \gamma c_j$.*

Theorem 1.3 (First Welfare Theorem — Matrix Version)**.** *Consider the one-good economy with externalities (2). The competitive equilibrium (where each agent maximizes $U_i(x_i)$ ignoring externalities) is Pareto efficient with respect to social utilities V_i if and only if:*

$$\alpha_j = \alpha_k \quad \text{for all } j,k \qquad \Longleftrightarrow \qquad c_j = c_k \quad \text{for all } j,k. \tag{5}$$

That is: all agents must have equal externality centrality (equal column sums in W).

Proof. The planner's first-order condition is $\alpha_j U_j'(x_j) = \mu$ for all j, where μ is the Lagrange multiplier on the resource constraint. The market's FOC is $U_j'(x_j) = \mu'$ for all j (agents ignore externalities). These coincide if and only if α_j is the same for all j — otherwise the planner would set different marginal utilities for different agents, while the market equalizes them. □

Proposition 1.4 (The Doubly Stochastic Condition)**.** *If W is row-stochastic ($\sum_j W_{ij} = 1$ for all i), then the FWT condition $c_j = c_k$ for all j,k is equivalent to: W **is doubly stochastic** ($\sum_i W_{ij} = 1$ for all j).*

Proof. Row-stochastic: $\sum_j W_{ij} = 1$. Summing over i: $\sum_i \sum_j W_{ij} = N$, so $\sum_j c_j = \sum_j (\sum_i W_{ij} - W_{jj}) = N - \mathrm{tr}(W)$. If all c_j are equal: $c_j = (N - \mathrm{tr}(W))/N$ for all j, meaning all column sums $\sum_i W_{ij}$ are equal. Combined with $\sum_j (\sum_i W_{ij}) = N$ (total of all entries = N from row-stochasticity), each column sum must be 1. Hence doubly stochastic. □

Intuition: What doubly stochastic means for efficiency

Row-stochastic means each agent distributes her total attention/dependency across others, summing to 1. This is a normalization — it says your total outgoing weight is fixed.

Column-stochastic means each agent receives the same total weight from others. Nobody is "more cared about" than anyone else. Nobody generates more externality.

Doubly stochastic means both: the giving is normalized AND the receiving is equal. This is the symmetry condition the welfare theorems need.

When column sums are unequal, the planner wants to give more to the agents with high column sums (they generate lots of positive externalities). The market, ignoring externalities, gives everyone the same marginal utility. The market under-provides to the high-externality agents and over-provides to the low-externality agents.

Two special cases satisfy the condition trivially:

Corollary 1.5 (Two Sufficient Conditions)**.** *The FWT holds if:*

1. $\gamma = 0$ *(no externalities). Then* $\alpha_j = 1$ *for all* j *regardless of* W*.*
2. $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$ *(competitive benchmark). Then* $c_j = (N-1)/N$ *for all* j*.*

2 Three Failures

The FWT fails when W departs from doubly stochastic. We distinguish three structural mechanisms.

2.1 Failure 1: Asymmetric Power

When column sums of W are unequal, the Perron eigenvector $\boldsymbol{\pi}$ departs from $\frac{1}{N}\mathbf{1}$. The agents with the highest column sums have the most structural power (Chapter 6 develops this fully). In the bargaining equilibrium (Chapter 7), the allocation tilts toward the powerful:

$$x_i^* \approx \frac{\Omega}{N} + \frac{1}{\kappa}\left(\pi_i - \frac{1}{N}\right). \tag{6}$$

The welfare loss from this power asymmetry in the LQ economy:

Proposition 2.1 (Welfare Loss from Power Asymmetry)**.**

$$L_{power} = \frac{\kappa N}{2} \operatorname{Var}(\pi_i) \cdot \Phi^2(\Omega, N, \kappa), \tag{7}$$

where $\operatorname{Var}(\pi_i) = \frac{1}{N}\sum_i (\pi_i - 1/N)^2$ *and* $\Phi > 0$ *is a scaling factor.*

The loss is proportional to the variance of the Perron eigenvector. When $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$ (equal power), $\operatorname{Var}(\pi_i) = 0$ and the loss vanishes. The more concentrated the power distribution, the larger the welfare loss.

Example 2.2 (Power Asymmetry with $N = 4$)**.** *Four agents,* $\Omega = 20$*,* $\kappa = 0.5$*. Compare three weight matrices:*

Structure	π_1	π_2	π_3	π_4	$\operatorname{Var}(\pi_i) \times 10^3$
Competitive ($\frac{1}{4}\mathbf{11}'$)	*0.25*	*0.25*	*0.25*	*0.25*	*0*
Mild asymmetry	*0.30*	*0.28*	*0.22*	*0.20*	*1.55*
Strong asymmetry (star)	*0.55*	*0.15*	*0.15*	*0.15*	*25.0*

Bargaining allocations (from (6)):

	x_1^*	x_2^*	x_3^*	x_4^*	L_{power}
Competitive	*5.00*	*5.00*	*5.00*	*5.00*	*0*
Mild	*5.10*	*5.06*	*4.94*	*4.90*	*0.008*
Star	*5.60*	*4.80*	*4.80*	*4.80*	*0.128*

The star network produces a welfare loss 16 times larger than the mildly asymmetric case. The powerful agent ($\pi_1 = 0.55$) *captures 12% more than equal division despite identical endowments and preferences. This is inequality produced entirely by structure.*

2.2 Failure 2: Unpriced Externalities

Even if $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$ (the market interaction is symmetric), externalities that the market does not price generate inefficiency.

The planner's allocation equates $\alpha_j U_j'(x_j)$ across agents. The market's allocation equates $U_j'(x_j)$. When α_j varies (unequal externality centrality), these differ.

Proposition 2.3 (Welfare Loss from Unpriced Externalities)**.** *In the LQ economy with* $\gamma > 0$*:*

$$L_{externality} = \gamma^2 \cdot \operatorname{Var}(c_j) \cdot \Psi(\Omega, N, \kappa), \tag{8}$$

where $c_j = \sum_{i \neq j} W_{ij}$ *is agent* j*'s externality centrality,* $\operatorname{Var}(c_j) = \frac{1}{N}\sum_j (c_j - \bar{c})^2$*, and* $\Psi > 0$*.*

This loss exists even with equal power. It is proportional to the *variance of column sums* in W^U — how unevenly the caring is distributed. If everybody is cared about equally (c_j constant), the loss is zero even with large γ.

Example 2.4 (The Unpriced Bond). $N = 3$, $\gamma = 0.5$, $\kappa = 1$, $\Omega = 3$.

Case A: *Symmetric externalities.* $W^U = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{pmatrix}$. *Column sums:* $(1, 1, 1)$. *Equal.* $L_{ext} = 0$.

Case B: *Asymmetric.* $W^U = \begin{pmatrix} 0 & 0.6 & 0 \\ 0.5 & 0 & 0 \\ 0.0 & 0.5 & 0 \end{pmatrix}$. *Column sums:* $(0.5, 1.1, 0)$. *Agent 2 is cared about by both others; Agent 3 is cared about by nobody.* $\mathrm{Var}(c_j) = 0.203$. $L_{ext} = 0.5^2 \times 0.203 \times \Psi = 0.051\Psi$.

The planner gives more to Agent 2 (who generates large positive spillovers). The market ignores this and equalizes marginal utilities. The market under-provides for Agent 2 and over-provides for Agent 3.

2.3 Failure 3: Block Disconnection

When W is approximately block-diagonal with k blocks, the economy fragments into nearly independent components. The market may achieve efficiency *within* each block, but gains from cross-block reallocation are unrealized.

Proposition 2.5 (Block-Diagonal Welfare Loss). *Let W have k blocks with sizes $N_1, \ldots, N_k$, mean endowments $\bar{\omega}_1, \ldots, \bar{\omega}_k$, and cross-block weights $\varepsilon \to 0$. The welfare loss from disconnection is:*

$$L_{block} = \frac{\kappa}{2} \sum_{b=1}^{k} N_b (\bar{\omega}_b - \bar{\omega})^2 + O(\varepsilon). \tag{9}$$

*This is the **between-group variance** of endowments, weighted by $\kappa/2$.*

Proof. With $\varepsilon = 0$, each block b solves its own welfare problem independently, achieving within-block equal division at $\bar{\omega}_b$. The first-best (full-economy equal division) would give everyone $\bar{\omega}$. The difference in welfare:

$$L = \sum_b N_b \left[U(\bar{\omega}) - U(\bar{\omega}_b) \right] = \sum_b N_b \left[(\bar{\omega} - \bar{\omega}_b) - \frac{\kappa}{2} (\bar{\omega}^2 - \bar{\omega}_b^2) \right] \tag{10}$$

$$= -\frac{\kappa}{2} \sum_b N_b (\bar{\omega}^2 - \bar{\omega}_b^2) = \frac{\kappa}{2} \sum_b N_b (\bar{\omega}_b - \bar{\omega})^2, \tag{11}$$

using $\sum_b N_b (\bar{\omega} - \bar{\omega}_b) = 0$ (block means average to the population mean) and expanding the quadratic. □

Intuition: Between-group inequality as structural inefficiency

The block-diagonal welfare loss equals the between-group variance of endowments. If all blocks have the same average ($\bar{\omega}_b = \bar{\omega}$), the loss is zero even with total disconnection — there is nothing to redistribute across blocks. But if blocks differ in average endowment (one is rich, another poor), disconnection prevents the equalization that would improve welfare.

This is the structural diagnosis: the welfare cost of fragmentation is not fragmentation per se, but fragmentation *in the presence of between-group inequality*. An economy with two equally poor sectors loses nothing from disconnection. An economy with one gas sector and one subsistence sector loses enormously.

Example 2.6 (Two Blocks with Different Means)**.** $N = 8$, $\kappa = 0.5$. *Block 1 (agents 1–3): gas/formal sector,* $\bar{\omega}_1 = 8$. *Block 2 (agents 4–8): traditional sector,* $\bar{\omega}_2 = 3.2$. *Overall mean:* $\bar{\omega} = \frac{3\times 8+5\times 3.2}{8} = 5$.

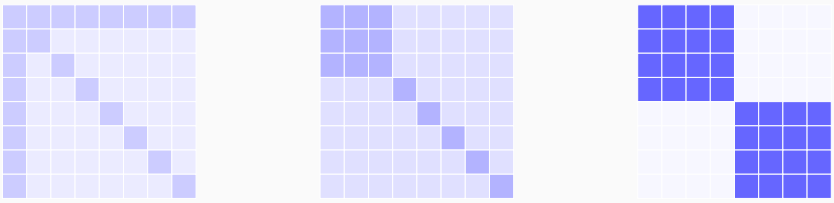
Between-group variance: $\frac{1}{8}[3(8-5)^2 + 5(3.2-5)^2] = \frac{1}{8}[27 + 16.2] = 5.4$.

Block-diagonal welfare loss: $L_{block} = \frac{0.5}{2} \times 8 \times 5.4 = 10.8$.

Now connect the blocks: increase cross-block weights to 0.15 (structural policy). The economy partially equilibrates across blocks. The welfare gain is bounded above by $L_{block} = 10.8$ *— the maximum achievable improvement from structural integration.*

Visualization:

Three failures, three heatmaps



Asymmetric Power — $\mathrm{Var}(\pi_i)$ large

Clustered Externalities — $\mathrm{Var}(c_j)$ large

Block Disconnection — $\Gamma \approx 0$

Three departures from W^{CE}, each causing the FWT to fail through a different channel. The real world typically has all three simultaneously.

3 The Second Welfare Theorem

3.1 The Standard Statement

Theorem 3.1 (Second Welfare Theorem)**.** *Under convexity: any Pareto-efficient allocation x^* can be supported as a competitive equilibrium after suitable lump-sum transfers of endowments.*

The standard interpretation: equity and efficiency are separable. A benevolent planner can achieve any desired distribution by (1) computing the required endowment transfers, (2) executing them, and (3) letting the market handle the allocation. The market handles efficiency; the planner handles equity.

This separation is the intellectual foundation of "growth first, redistribute later": get the prices right (efficiency), then transfer income (equity).

3.2 The Transfer Trap

The SWT assumes that after the transfer, the market operates symmetrically on the new endowments. When the market mechanism is mediated by a non-doubly-stochastic W, this assumption fails.

Theorem 3.2 (The Transfer Trap)**.** *In the bargaining equilibrium with power $\boldsymbol{\pi}(W)$:*

1. *A lump-sum transfer T changes endowments: $\boldsymbol{\omega}' = \boldsymbol{\omega} + T\mathbf{1}$ (where T is the transfer matrix with $\mathbf{1}'T = 0$).*

2. *The post-transfer allocation is determined by the same W (transfers do not change the weight matrix).*

3. *The new bargaining allocation is:*

$$x_i' \approx \frac{\Omega}{N} + \frac{1}{\kappa}\left(\pi_i - \frac{1}{N}\right) + (\textit{terms involving } \omega_i'). \tag{12}$$

   The power correction $(\pi_i - 1/N)/\kappa$ persists regardless of the transfer.

4. *Consequently: no transfer T can achieve the first-best equal-division allocation unless $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$ (i.e., unless W is doubly stochastic).*

Proof sketch. The bargaining allocation solves $\max \sum_i \pi_i \ln S_i(\mathbf{x})$ subject to feasibility. The surplus $S_i = U_i(x_i) - U_i(\omega_i')$ depends on the new endowment ω_i', but the weight π_i does not. The FOC $\frac{\pi_i}{S_i}U_i'(x_i) = \mu$ generates allocations that deviate from equal division by a term proportional to $(\pi_i - 1/N)$. Since $\boldsymbol{\pi}$ is determined by W alone, no change in $\boldsymbol{\omega}$ can eliminate this term. □

Example 3.3 (Transfer vs. Structural Policy). $N = 4$, $\Omega = 20$, $\boldsymbol{\omega} = (5,5,5,5)$ *(equal endowments). Block-diagonal W with* $\boldsymbol{\pi} = (0.32, 0.33, 0.17, 0.18)$ *(the gas-enclave structure from Chapter 1).*

Bargaining allocation with no transfer: $\mathbf{x}^* = (5.56, 5.64, 4.36, 4.44)$. *Agents 1–2 (gas/government block) get 11% more; agents 3–4 (traditional block) get 12% less. Welfare loss:* $L = 0.18$.

Policy A — Transfer. *Move 2 units from Agent 2 to Agent 3:* $\boldsymbol{\omega}' = (5,3,7,5)$. *New bargaining allocation:* $\mathbf{x}' = (5.48, 4.80, 5.24, 4.48)$. *Agent 3 gained 0.88, not 2.00. The structure "clawed back"* $2.00 - 0.88 = 1.12$, *or 56% of the transfer.*

Policy B — Structural. *Increase cross-block entries of W from 0.01 to 0.10. New Perron eigenvector:* $\boldsymbol{\pi}' = (0.27, 0.28, 0.22, 0.23)$. *Same endowments* $(5,5,5,5)$. *New allocation:* $\mathbf{x}' = (5.16, 5.24, 4.76, 4.84)$. *The gap between the richest and poorest agent drops from 1.28 to 0.48 — a 63% reduction — without any transfer.*

Policy C — Both. *Transfer 1 unit from Agent 2 to Agent 3, AND increase cross-block entries to 0.10. New allocation:* $\mathbf{x}' = (5.12, 4.48, 5.52, 4.88)$. *The transfer now "sticks" better because the structural clawback is smaller (the new W is less asymmetric).*

	x_1	x_2	x_3	x_4	*Max gap*
No policy	*5.56*	*5.64*	*4.36*	*4.44*	*1.28*
Transfer only	*5.48*	*4.80*	*5.24*	*4.48*	*1.00*
Structural only	*5.16*	*5.24*	*4.76*	*4.84*	*0.48*
Both	*5.12*	*4.48*	*5.52*	*4.88*	*1.04*
First-best	*5.00*	*5.00*	*5.00*	*5.00*	*0*

Structural policy alone outperforms the transfer in reducing inequality because it changes the mechanism, not just the inputs.

Key Structural Insight

The Second Welfare Theorem is a theorem about $\boldsymbol{\omega}$: it says you can achieve any efficient allocation by choosing the right endowments. But it assumes the mechanism that converts endowments into allocations is symmetric ($W = \frac{1}{N}\mathbf{1}\mathbf{1}'$). When the mechanism is asymmetric, the mechanism itself distorts the allocation, and no transfer can fully undo the distortion. The distortion is proportional to $\mathrm{Var}(\pi_i)$, which is a property of W, not of $\boldsymbol{\omega}$.

Three policy levers exist: (1) transfers change $\boldsymbol{\omega}$, (2) structural policy changes W, (3) formation interventions change W^I and eventually preferences. The SWT assumes only lever (1) is needed. When $W \neq \frac{1}{N}\mathbf{1}\mathbf{1}'$, lever (2) is essential.

4 The Welfare Loss Decomposition

Combining the three failures, the total welfare loss from structural departures:

Theorem 4.1 (Structural Welfare Loss)**.** *In the LQ economy with externalities ($\gamma > 0$), the welfare loss relative to the first-best decomposes as:*

$$L(W, \gamma) = \underbrace{\frac{\kappa N}{2} \operatorname{Var}(\pi_i)\Phi^2}_{L_{power}} + \underbrace{\gamma^2 \operatorname{Var}(c_j)\Psi}_{L_{externality}} + \underbrace{\gamma \operatorname{Cov}(\pi_i, c_i)\Xi}_{L_{interaction}}, \tag{13}$$

where $\Phi, \Psi, \Xi > 0$ are functions of Ω, N, κ.

The interaction term $L_{\text{interaction}}$ is proportional to $\operatorname{Cov}(\pi_i, c_i)$: the covariance between power and externality centrality. When this is positive — when the agents who have the most structural power are also the ones most cared about by other powerful agents — the two distortions amplify each other.

Example 4.2 (Decomposition for Two Structures)**.** $N = 4$, $\gamma = 0.3$, $\kappa = 0.5$, $\Omega = 20$.

Connected economy: *W with $\boldsymbol{\pi} = (0.26, 0.25, 0.25, 0.24)$ and column sums $(1.02, 1.01, 0.99, 0.98)$:*

$$L_{power} = 0.001, \quad L_{ext} = 0.000, \quad L_{int} = 0.000, \quad L = 0.001.$$

Enclave economy: *W with $\boldsymbol{\pi} = (0.32, 0.33, 0.17, 0.18)$ and column sums $(1.24, 1.30, 0.72, 0.74)$:*

$$L_{power} = 0.043, \quad L_{ext} = 0.016, \quad L_{int} = 0.012, \quad L = 0.071.$$

The enclave economy's welfare loss is 71 times larger. The interaction term accounts for 17% of the total — the gas-government elite (π high) care about each other (c high for those agents), compounding both distortions.

5 The Spectral Gap as Structural Health

Proposition 5.1 (Spectral Gap Bounds the Power Loss)**.** *For an $N \times N$ row-stochastic matrix W with spectral gap $\Gamma = 1 - |\lambda_2(W)|$:*

$$\operatorname{Var}(\pi_i) \leq \frac{1}{N}\left(\frac{1-\Gamma}{\Gamma}\right). \tag{14}$$

Hence:

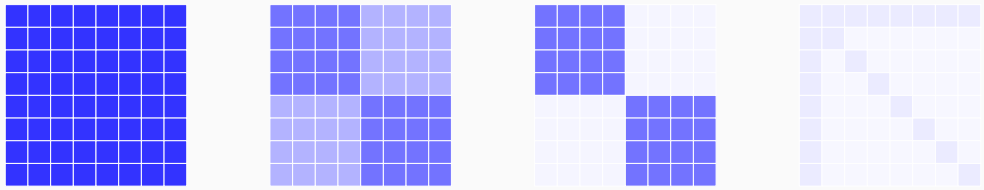
$$L_{power} \leq \frac{\kappa\Phi^2}{2}\left(\frac{1-\Gamma}{\Gamma}\right). \tag{15}$$

Proof sketch. The variance of the Perron eigenvector entries is bounded by the eigenvalue ratio through the Courant-Fischer characterization. When $\Gamma = 1$ (single dominant eigenvalue, $|\lambda_2| = 0$), $\boldsymbol{\pi}$ must be uniform: Var $= 0$. As $\Gamma \to 0$ (competing eigenvalues), $\boldsymbol{\pi}$ can concentrate arbitrarily. □

This result makes the spectral gap a single-number diagnostic. Compute Γ from any weight matrix (an input-output table, a trade network, a social interaction matrix) and you immediately have an upper bound on the structural welfare loss.

Visualization:

Four structures, four welfare losses



Competitive
$\Gamma = 1.0$
$L = 0$

Weakly Dual
$\Gamma \approx 0.4$
$L = 0.04$

Enclave
$\Gamma \approx 0.08$
$L = 0.18$

Monopoly
$\Gamma \approx 0.05$
$L = 0.42$

Same economy, same endowments, same preferences. Only W changes. As Γ shrinks, L grows. The heatmap IS the diagnosis.

6 Summary

1. The **First Welfare Theorem** requires W to be doubly stochastic (equal externality centrality across agents). It fails for three structural reasons: asymmetric power ($\text{Var}(\pi_i) > 0$), unpriced externalities ($\text{Var}(c_j) > 0$), and block disconnection ($\Gamma \approx 0$).

2. The **Second Welfare Theorem** requires that transfers change $\boldsymbol{\omega}$ while the mechanism W remains symmetric. When W is not doubly stochastic, the structure "claws back" part of any transfer. Structural policy (changing W) outperforms redistribution (changing $\boldsymbol{\omega}$) in reducing power-driven inequality.

3. The **welfare loss** decomposes as $L = L_{\text{power}} + L_{\text{ext}} + L_{\text{interaction}}$. The power component is $\text{Var}(\pi_i)$. The interaction component ($\text{Cov}(\pi_i, c_i) > 0$) compounds the other two when the powerful are also the most cared about.

4. The **spectral gap** Γ bounds the welfare loss: $L_{\text{power}} \leq \frac{\kappa\Phi^2}{2}\frac{1-\Gamma}{\Gamma}$. It is the single most informative structural diagnostic.

5. **Transfers change ω. Structural policy changes W. Only the latter changes the power structure π.**

7 Exercises

2.1 Doubly stochastic check. For each matrix, determine if it is doubly stochastic. If not, find the Perron eigenvector $\boldsymbol{\pi}$ and compute $\text{Var}(\pi_i)$:

(a) $W = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(b) $W = \begin{pmatrix} 0.6 & 0.3 & 0.1 \\ 0.1 & 0.6 & 0.3 \\ 0.3 & 0.1 & 0.6 \end{pmatrix}$

(c) $W = \begin{pmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{pmatrix}$

(d) $W = \begin{pmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.4 \end{pmatrix}$

2.2 The transfer clawback. Using Example 3.3: let τ be the transfer from Agent 2 to Agent 3. Compute the bargaining allocation for $\tau = 0.5, 1, 2, 3, 4$. Plot the "clawback fraction" (share of transfer not received by Agent 3) as a function of τ. Is it constant or does it depend on the transfer size?

2.3 Welfare decomposition. For $N = 3$, $\gamma = 0.5$, $\kappa = 1$, $\Omega = 3$:

(a) Let $W = \frac{1}{3}\mathbf{1}\mathbf{1}'$. Verify $L_{\text{power}} = L_{\text{ext}} = L_{\text{int}} = 0$.

(b) Let $W = \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{pmatrix}$. Compute $\boldsymbol{\pi}$, $\text{Var}(\pi_i)$, column sums c_j, $\text{Var}(c_j)$, and $\text{Cov}(\pi_i, c_i)$.

(c) Is $L_{\text{interaction}}$ positive or negative? What does the sign mean?

2.4 Spectral gap across topologies. For $N = 8$, construct: (a) W^{CE}, (b) a ring, (c) two disconnected blocks of 4, (d) a star. For each, compute Γ, $\text{Var}(\pi_i)$,

and the welfare-loss bound from Proposition 5.1. Plot L_{bound} vs. Γ. Use Python.

2.5 Block disconnection with data. Using Bolivia's 2016 I-O table (from Exercise 1.7), aggregate to 2 sectors: Hydrocarbons + Mining vs. Everything Else. Compute the between-group variance of output per worker. How large is L_{block} relative to the within-group component?

2.6 El Alto vs. Zona Sur. Suppose El Alto and Zona Sur are two blocks of a 2×2 weight matrix with cross-block entries $\varepsilon = 0.02$. Mean consumption: $\bar{\omega}_{\text{Alto}} = 800$ Bs/month, $\bar{\omega}_{\text{Sur}} = 4500$ Bs/month. Block sizes: $N_{\text{Alto}} = 5$, $N_{\text{Sur}} = 3$ (out of $N = 8$).

(a) Compute the between-group welfare loss L_{block} (use $\kappa = 0.001$).

(b) How much would L_{block} decrease if cross-block entries rose to $\varepsilon = 0.15$? (You will need to solve the bargaining equilibrium numerically.)

(c) What real-world intervention would increase ε?

2.7 Furtado's thesis formalized. Furtado claimed that underdevelopment is not a stage but a stable structural condition. Formalize: let W_t evolve according to $W_{t+1} = f(W_t, \mathbf{x}^*(W_t))$ (the structure at time $t+1$ depends on the current structure and the allocation it produces). A stable underdevelopment equilibrium is a fixed point W^* with $\boldsymbol{\pi}(W^*) \neq \frac{1}{N}\mathbf{1}$ and $L(W^*) > 0$.

(a) Construct a simple 2×2 example where the block-diagonal W^* is a fixed point: the allocation produced by W^* reinforces the cross-block disconnection. (Hint: if the traditional sector has low income, it cannot invest in cross-block linkages.)

(b) Under what conditions is the fixed point stable (small perturbations return to W^*)?

(c) What does "breaking out of underdevelopment" mean in this framework?

Mathematical Appendices

Appendix 2A: Doubly Stochastic Matrices

Definition. An $N \times N$ nonnegative matrix W is **row-stochastic** if $W\mathbf{1} = \mathbf{1}$, **column-stochastic** if $\mathbf{1}'W = \mathbf{1}'$, and **doubly stochastic** if both.

The Birkhoff-von Neumann theorem. The set of $N \times N$ doubly stochastic matrices is the convex hull of the $N!$ permutation matrices. That is: every doubly stochastic matrix can be written as $W = \sum_k \alpha_k P_k$ where P_k are permutation matrices, $\alpha_k \geq 0$, and $\sum_k \alpha_k = 1$. This means doubly stochastic matrices are "averages" of perfect matchings.

Key property. If W is doubly stochastic and irreducible, the left Perron eigenvector is $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$.

Proof. $\boldsymbol{\pi}'W = \frac{1}{N}\mathbf{1}'W = \frac{1}{N}\mathbf{1}' = \boldsymbol{\pi}'$. By uniqueness of the Perron eigenvector (Perron-Frobenius, Chapter 6), this is the only left eigenvector with eigenvalue 1. □

Measuring departure. A useful scalar measure:

$$\Delta(W) = \sqrt{\sum_{j=1}^{N}\left(\sum_{i=1}^{N} W_{ij} - 1\right)^2} = \|\mathbf{1}'W - \mathbf{1}'\|_2. \tag{16}$$

$\Delta = 0$ iff W is column-stochastic. For row-stochastic W: $\Delta = 0$ iff doubly stochastic.

Closest doubly stochastic matrix. Given a row-stochastic W, the closest doubly stochastic matrix (in Frobenius norm) is $W^{DS} = \arg\min_{D \in \mathcal{DS}} \|W - D\|_F$, which can be computed by Sinkhorn's algorithm: alternately normalize rows and columns until convergence.

Appendix 2B: Variance Decomposition

Total variance. For $\omega_1, \ldots, \omega_N$ with mean $\bar{\omega}$:

$$\mathrm{Var}(\omega) = \frac{1}{N}\sum_{i=1}^{N}(\omega_i - \bar{\omega})^2. \tag{17}$$

Group decomposition. Partition agents into k groups with sizes $N_1, \ldots, N_k$ and group means $\bar{\omega}_1, \ldots, \bar{\omega}_k$:

$$\mathrm{Var}(\omega) = \underbrace{\sum_{b=1}^{k} \frac{N_b}{N}(\bar{\omega}_b - \bar{\omega})^2}_{\text{Between-group}} + \underbrace{\sum_{b=1}^{k} \frac{N_b}{N} \mathrm{Var}_b(\omega)}_{\text{Within-group}}. \tag{18}$$

The between-group component is the structural welfare loss from block disconnection (Proposition 2.5). The within-group component is the loss addressable by within-block redistribution.

Connection to $\mathrm{Var}(\pi_i)$. The Perron eigenvector variance plays the same role for *power* inequality that the endowment variance plays for *wealth* inequality. Both generate welfare losses proportional to their variance. But endowment inequality can be addressed by transfers; power inequality cannot.

Structuralist Case Study: Furtado's Paraíba and the Fixed Point of Underdevelopment

Celso Furtado was born in 1920 in Pombal, Paraíba — deep in the Sertão, the semi-arid interior of northeastern Brazil. The Northeast was Brazil's original economic center (sugar, then cotton), but by the twentieth century it had become the country's poorest region while São Paulo industrialized. Furtado did not need to theorize structural heterogeneity. He grew up inside the dim lower-right block.

His 1961 book *Desenvolvimento e Subdesenvolvimento* advanced a thesis that maps directly onto the welfare-theoretic framework of this chapter. The argument has three parts.

Part 1: Underdevelopment is not a stage. The classical view (Rostow's "stages of growth") held that every economy passes through the same sequence — traditional, transitional, modern — and that the underdeveloped economies are simply earlier in the queue. Furtado rejected this. His claim: underdevelopment is not a phase that precedes development but a *specific structural condition* produced by the same historical process that produced development elsewhere. In the language of this chapter: the weight matrix W of the peripheral economy is not a transient state on the way to $W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$. It is a *fixed point* — a stable equilibrium of the structural dynamical system (Exercise 2.7). The welfare theorems do not "eventually hold." They fail permanently, because the matrix that makes them fail reproduces itself.

Part 2: Modern enterprises penetrate archaic structures without transforming them. Furtado's key mechanism: when capitalist firms enter a peripheral economy, they do not replace the pre-existing structure. They graft onto it. The modern enterprise and the archaic structure coexist within the same country, connected by a few asymmetric linkages (labor recruitment, raw material extraction) but otherwise operating independently. This is the block-diagonal W from Proposition 2.5: two blocks with weak cross-block entries. The modern block has high productivity and high $\bar{\omega}$; the archaic block has low productivity and low $\bar{\omega}$. The between-group variance is enormous. The welfare loss from disconnection is $L_{\text{block}} = \frac{\kappa}{2} \sum_b N_b(\bar{\omega}_b - \bar{\omega})^2$. In Paraíba, this was not an abstraction.

Part 3: The structure reproduces the conditions for its own persistence. The modern sector's profits are repatriated or spent on imported consumption goods (the center-periphery leakage). The archaic sector's surplus is too small to finance the investments that would create cross-block linkages. The education system, calibrated to the modern sector's needs, produces graduates who migrate to São Paulo rather than transforming the Sertão. Each of these mechanisms is a feedback loop: the allocation produced by W reinforces W. This is the Transfer Trap (Theorem 3.2) applied dynamically: not only does the current W distort the current allocation, but the current allocation makes the future W no less block-diagonal than the present one.

Furtado's argument	**Formal object**	**This chapter**
Underdevelopment is not a stage	W^* is a stable fixed point	Sec. 2, Ex. 2.7
Modern grafted onto archaic	Block-diagonal W with weak cross-links	Prop. 2.5
Between-region inequality	$L_{\text{block}} \propto \sum_b N_b(\bar{\omega}_b - \bar{\omega})^2$	Eq. (9)
Structure reproduces itself	$W_{t+1} = f(W_t, \mathbf{x}^*(W_t))$, W^* stable	Thm. 3.2
Transfers don't transform	$\boldsymbol{\pi}$ unchanged by $\Delta\boldsymbol{\omega}$	Thm. 3.2

In 1959, Furtado was appointed head of SUDENE (Superintendência do Desenvolvimento do Nordeste) — an agency created specifically to change W^P_{Brazil} by building cross-block linkages between the Northeast and the industrializing South. He was removed after the 1964 military coup. The coup did not change the welfare theorems. It changed who had the power to change W.

Further Reading

Welfare theorems: Mas-Colell, Whinston, and Green (1995), *Microeconomic Theory*, Chapter 16. Arrow (1951), "An Extension of the Basic Theorems of Classical Wel-

fare Economics," in *Proceedings of the Second Berkeley Symposium*. Debreu (1959), *Theory of Value*, Chapter 6.

Structural failures of efficiency: Greenwald and Stiglitz (1986), "Externalities in Economies with Imperfect Information and Incomplete Markets," *QJE* 101(2). Acemoglu (2003), "Why Not a Political Coase Theorem?" *Journal of Comparative Economics* 31(4).

The structuralist critique: Furtado (1961), *Desenvolvimento e Subdesenvolvimento*, Rio de Janeiro: Fundo de Cultura (the key text, available through Centro Celso Furtado: `centrocelsofurtado.org.br`). Furtado (1959), *Formação Econômica do Brasil* — the historical companion, tracing the evolution of Brazil's W^P from colonial plantation to ISI. Taylor (1983), *Structuralist Macroeconomics*, Chapter 1.

Sinkhorn's algorithm: Sinkhorn (1964), "A Relationship Between Arbitrary Positive Matrices and Doubly Stochastic Matrices," *Annals of Mathematical Statistics* 35(2). Used in Appendix 2A for computing the closest doubly stochastic matrix.

Chapter 3

Growth and Distribution in Matrix Form

Why Growth Makes Some Countries Rich and Others More Unequal

> *"Los enormes beneficios que derivan de la creciente productividad no han llegado a la periferia en medida comparable a la que han logrado disfrutar las poblaciones de los grandes países industriales. De ahí las diferencias tan acentuadas en los niveles de vida de las masas de éstos y de aquéllos y las notorias discrepancias entre sus respectivas fuerzas de capitalización."*
>
> ["The enormous benefits derived from increasing productivity have not reached the periphery in a measure comparable to that enjoyed by the peoples of the great industrial countries. Hence the great differences in standards of living, and the manifest discrepancies in their respective capacities for capital accumulation."]
> — Raúl Prebisch, *El desarrollo económico de la América Latina* (1949)

> *"Che boludo, it's obvious."*
>
> — What Prebisch would have said about this chapter

Prebisch saw it without the math: growth in the periphery does not work the same way as growth in the center, and the difference is structural. The production structure of the center—diversified, interconnected, industrialized—propagates productivity gains internally, raising wages and living standards broadly. The production structure of the periphery—concentrated in primary commodities, weakly linked internally, dependent on imported technology—leaks productivity gains outward through deteriorating terms of trade, leaving internal inequality unchanged or worse.

This chapter gives Prebisch his eigenvalues.

We develop growth theory in matrix form, starting from the standard Solow model (one sector, one country, growth converges to a steady state) and progres-

sively introducing the structural elements that make growth in the periphery different from growth in the center. The key mathematical object is the production network A—the input-output matrix from Chapter 1—now embedded in a dynamic growth model where the structure of A determines not just who produces what today but how productivity gains propagate across sectors over time.

The central result: whether growth reduces or increases inequality depends on the *spectral properties* of A. When A is well-connected (strong inter-sectoral linkages, large spectral gap), growth propagates and converges. When A is block-diagonal (disconnected sectors, small spectral gap), growth concentrates in the modern sector and does not reach the traditional sector. Pinto's structural heterogeneity is a spectral property.

1 The Standard Story: One Sector, One Country

1.1 Solow in One Paragraph

The Solow growth model says: a country accumulates capital, capital produces output, output is split between consumption and saving, and saving becomes next period's capital. The economy converges to a steady state where capital per worker is constant and output grows only through technological progress.

The model has one sector, one representative agent, and one production function: $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$, where K is capital, L is labor, A is technology, and $\alpha \in (0, 1)$ is the capital share. The savings rate s is exogenous. Capital accumulates as $K_{t+1} = sY_t + (1-\delta)K_t$.

In per-effective-worker terms ($k = K/(AL)$), the dynamics reduce to one equation:

$$k_{t+1} = \frac{sk_t^{\alpha} + (1-\delta)k_t}{(1+g)(1+n)}, \tag{1}$$

where g is the growth rate of technology and n is population growth. The steady state k^* satisfies $sf(k^*) = (\delta + g + n)k^*$.

Intuition: What Solow gets right and what it misses

Solow gets convergence right: poor countries with low k have high marginal product of capital, invest at the same rate s, and grow faster until they catch up. This predicts convergence—and conditional convergence (controlling for s, δ, n) is one of the most robust findings in empirical growth.

What Solow misses is everything structural. There is one sector, so there is no structural heterogeneity. There is one savings rate, so there is no differential accumulation by group. There is one production function, so there are no inter-sectoral linkages, no Leontief multipliers, no question of whether growth in mining propagates to agriculture. The country is a point, not a structure. Prebisch would say: this describes Sweden, not Bolivia.

1.2 Solow with Two Groups: The OLG Version

To introduce structure, we move from the Solow model to the overlapping-generations (OLG) framework, where savings decisions are made by *different groups* with *different parameters*.

Example 1.1 (A Two-Group Economy)**.** *Consider an economy with two groups: M (men) and F (women), each comprising half the population. Each agent lives two periods: young (work, earn, save) and old (consume savings).*

A young agent of group $g \in \{M, F\}$ has utility over young and old consumption:

$$U^g = \ln(c^{g,y}) + \theta^g \ln(c^{g,o}), \tag{2}$$

*where $\theta^g > 0$ is the **patience parameter**—how much group g values future consumption relative to present consumption.*

The budget constraints are $c^{g,y} + s^g = w_t$ (young: consume or save out of wage income) and $c^{g,o} = (1 + r_{t+1})s^g$ (old: consume return on savings). The optimal savings rate is:

$$\sigma(\theta^g) = \frac{\theta^g}{1 + \theta^g}. \tag{3}$$

A group with higher θ (more patient) saves a larger fraction of income. If $\theta^M = \theta^F$ (same patience), both groups save the same share and the economy behaves like standard Solow. If $\theta^M > \theta^F$—if men are more patient than women, or more precisely, if the structure has produced men who value future consumption more—then men save more, accumulate more, and the wealth gap widens over time.

Aggregate capital next period is:

$$K_{t+1} = \frac{1}{2}\left[\sigma(\theta_t^M) + \sigma(\theta_t^F)\right](1-\alpha)K_t^\alpha. \tag{4}$$

This is the Solow model with a twist: the aggregate savings rate depends on the *composition* of the population. If the two groups have different patience parameters, the savings rate—and therefore the growth path—depends on who has what θ.

Intuition: Where does θ come from?

In the standard OLG model, θ^g is given. Men and women are either equally patient or not, and the model does not ask why. In the formation model (Chapter 10), θ^g is produced by the park: the developmental environment determines how much each group values future consumption, investment, and career building. Women raised in a normative environment where female economic participation is invisible or discouraged will have lower θ^F—not because of biology but because the signal structure of the park made future consumption appear less attainable or less valuable.
For now, θ^g is a parameter. Later, it becomes an output of W^I.

2 Multi-Sector Growth: The Production Network

Now we introduce multiple sectors. This is where the Leontief system from Chapter 1 becomes dynamic.

2.1 A Story: Gas and Quinoa

Bolivia has two economies. The first is the gas economy: capital-intensive, high-productivity, connected to global commodity markets, responsible for roughly 30% of GDP and 50% of export revenues, employing approximately 1% of the labor force. The second is the quinoa economy (and potatoes, and llamas, and informal commerce): labor-intensive, low-productivity, disconnected from global markets, employing the vast majority of the population.

A dollar of investment in the gas sector produces output that is sold internationally, generates fiscal revenue, and feeds back into the gas sector (maintenance, exploration, expansion). Very little of it flows to quinoa farmers. A dollar of investment in quinoa production feeds local consumption, local transport, local markets—but generates negligible fiscal revenue and zero export earnings.

This is structural heterogeneity. And the question is: if the gas sector grows at 5% per year for a decade, does the quinoa economy grow?

The answer depends on the structure of A—the input-output matrix. If the gas sector buys inputs from the domestic economy (backward linkages) and sells intermediate products to domestic manufacturers (forward linkages), then gas growth propagates. If the gas sector imports its inputs and exports its output, with no domestic linkages, then gas growth is an enclave: it enriches the gas sector and the treasury without touching the quinoa economy.

Prebisch Connection: The enclave economy

This is exactly Prebisch's center-periphery argument applied within a country. The gas sector is the "center" of the Bolivian economy—high productivity, strong international linkages, favorable terms of trade (when commodity prices are high). The traditional sector is the "periphery"—low productivity, weak linkages, dependent on domestic demand. Pinto's contribution was to see that this center-periphery structure exists *inside* peripheral countries, not just between countries. Bolivia is peripheral in the global economy *and* internally heterogeneous. The gas sector's relationship to the quinoa sector replicates, at national scale, the center's relationship to the periphery at global scale.

2.2 The Multi-Sector Solow Model

Consider N sectors, each with its own capital stock K_{it}, labor force L_i, and production function. Sector i produces:

$$Y_{it} = K_{it}^{\alpha_i} L_i^{1-\alpha_i} + \gamma \sum_{j=1}^{N} W_{ij}^{P} Y_{jt}, \tag{5}$$

where the second term captures production spillovers: sector i's output benefits from sector j's output, weighted by W_{ij}^{P} (the input-output coefficient).

In matrix form, stacking all sectors:

$$\mathbf{Y}_t = \mathbf{f}(\mathbf{K}_t, \mathbf{L}) + \gamma W^{P} \mathbf{Y}_t, \tag{6}$$

where $\mathbf{f}$ is the vector of Cobb-Douglas production functions. Solving for $\mathbf{Y}_t$ (as in Chapter 1):

$$\mathbf{Y}_t = (I - \gamma W^{P})^{-1} \mathbf{f}(\mathbf{K}_t, \mathbf{L}). \tag{7}$$

This is the Leontief system made dynamic: at each period t, output is determined by capital (which accumulates) filtered through the production network (which amplifies through the Leontief inverse).

Capital accumulates sector by sector:

$$K_{i,t+1} = s_i Y_{it} + (1 - \delta_i) K_{it}, \tag{8}$$

where s_i is sector i's savings (reinvestment) rate and δ_i is depreciation. In matrix form:

$$\mathbf{K}_{t+1} = S\mathbf{Y}_t + (I - \Delta)\mathbf{K}_t, \tag{9}$$

where $S = \text{diag}(s_1, \ldots, s_N)$ and $\Delta = \text{diag}(\delta_1, \ldots, \delta_N)$.

Substituting the output equation:

$$\mathbf{K}_{t+1} = S(I - \gamma W^P)^{-1}\mathbf{f}(\mathbf{K}_t, \mathbf{L}) + (I - \Delta)\mathbf{K}_t. \tag{10}$$

Key Structural Insight

Equation (10) is the multi-sector growth equation. It says: next period's capital depends on this period's capital, transformed by two structural objects:

- $(I - \gamma W^P)^{-1}$: the Leontief inverse, which amplifies each sector's output through inter-sectoral linkages.
- S: the savings matrix, which determines how much of each sector's output is reinvested.

The *structure* of growth—which sectors grow, how fast, and whether growth spreads across sectors—is governed by the interaction of these two matrices.

2.3 The Steady State

At the steady state $\mathbf{K}^*$, capital is constant: $\mathbf{K}^* = S(I - \gamma W^P)^{-1}\mathbf{f}(\mathbf{K}^*, \mathbf{L}) + (I - \Delta)\mathbf{K}^*$. For Cobb-Douglas production ($f_i = K_i^{\alpha} L_i^{1-\alpha}$ with common α), this simplifies to:

$$\Delta\mathbf{K}^* = S(I - \gamma W^P)^{-1}\mathbf{f}(\mathbf{K}^*, \mathbf{L}). \tag{11}$$

Finding $\mathbf{K}^*$ is a fixed-point problem. For the linear case ($\alpha = 1$, which makes $f_i = K_i$), the system becomes:

$$\mathbf{K}^* = [I - (I - \Delta)^{-1} S(I - \gamma W^P)^{-1}]^{-1} \cdot \mathbf{0}, \tag{12}$$

which is degenerate (trivial steady state at zero without technological progress). The interesting dynamics are in the transition path, which is governed by the eigenvalues of the *growth mapping* $G = (I - \Delta) + S(I - \gamma W^P)^{-1} F'(\mathbf{K}^*)$, where F' is the Jacobian of the production function.

3 When Does Growth Reduce Inequality? The Kuznets Question

3.1 The Standard Answer

Simon Kuznets (1955) proposed that inequality first rises and then falls with development—the "inverted-U" hypothesis. The mechanism: early industrialization pulls workers from a low-inequality agricultural sector into a high-inequality industrial sector, raising between-sector inequality. Eventually, as most workers have moved, within-sector inequality dominates and falls.

The inverted-U has been one of the most tested hypotheses in economics. The evidence is mixed at best: cross-country data show no robust inverted-U, and the within-country trajectories vary enormously. Some countries (South Korea, Taiwan) grew rapidly and reduced inequality. Others (Brazil in the 1960s-70s, Bolivia, Nigeria) grew and saw inequality rise. Others (many sub-Saharan African countries) did not grow and saw inequality rise anyway.

The matrix framework explains why: it depends on W^P.

3.2 The Structural Answer: It Depends on W^P

Example 3.1 (Two Economies, Same Technology, Different W^P)**.** *Consider two economies, each with $N = 2$ sectors: Modern (M) and Traditional (T). Both have the same technology:*

$$Y_M = K_M^{0.4} L_M^{0.6}, \qquad Y_T = K_T^{0.3} L_T^{0.7}. \tag{13}$$

Modern is more capital-intensive ($\alpha_M = 0.4 > \alpha_T = 0.3$). Both sectors have savings rate $s = 0.2$ and depreciation $\delta = 0.05$.

Economy A (Connected): *Strong inter-sectoral linkages.*

$$W_A^P = \begin{pmatrix} 0.10 & 0.25 \\ 0.20 & 0.10 \end{pmatrix}. \tag{14}$$

Modern uses Traditional inputs ($W_{M,T}^P = 0.25$): factories buy agricultural raw materials. Traditional uses Modern inputs ($W_{T,M}^P = 0.20$): farmers buy manufactured fertilizer and equipment.

Economy B (Disconnected): *Weak inter-sectoral linkages.*

$$W_B^P = \begin{pmatrix} 0.10 & 0.02 \\ 0.02 & 0.10 \end{pmatrix}. \tag{15}$$

The sectors barely interact. Modern imports its inputs and exports its output. Traditional produces for local consumption.

Now give both economies a productivity shock in the Modern sector: A_M doubles.

Example 3.2 (Continued: The Propagation)**.** *The Leontief inverse for Economy A:*

$$L_A = (I - \gamma W_A^P)^{-1} \approx \begin{pmatrix} 1.05 & 0.27 \\ 0.21 & 1.05 \end{pmatrix} \quad (\textit{with } \gamma = 1). \tag{16}$$

A productivity shock in Modern ($\Delta Y_M = 1$) generates total output increase of 1.05 *in Modern (direct + own-sector feedback) and* 0.21 *in Traditional (cross-sector spillover). The Traditional sector benefits because it sells inputs to Modern.*

The Leontief inverse for Economy B:

$$L_B = (I - \gamma W_B^P)^{-1} \approx \begin{pmatrix} 1.01 & 0.02 \\ 0.02 & 1.01 \end{pmatrix}. \tag{17}$$

The same shock in Modern generates 1.01 *in Modern and only* 0.02 *in Traditional. The spillover is negligible. Growth in Modern stays in Modern.*

After 10 periods of compounding, Economy A shows both sectors growing (Traditional pulled up by its linkages to Modern). Economy B shows Modern growing and Traditional stagnant—the productivity gap widens with growth.

Proposition 3.3 (Growth Propagation and the Spectral Gap)**.** *Consider the multi-sector growth model* (10) *with N sectors and production network W^P. A productivity shock in sector j propagates to sector i with magnitude proportional to the (i, j)-th entry of the Leontief inverse $(I - \gamma W^P)^{-1}$.*

The average cross-sector propagation is bounded by:

$$\frac{1}{N(N-1)} \sum_{i \neq j} \ell_{ij} \leq \frac{\gamma \|W^P\|}{1 - \gamma \rho(W^P)}, \tag{18}$$

where ℓ_{ij} are off-diagonal entries of the Leontief inverse and $\rho(W^P)$ is the spectral radius. The propagation is small when $\gamma\rho(W^P)$ is small (weak linkages) and large when $\gamma\rho(W^P)$ is close to 1 (strong linkages, near the productivity boundary).

Intuition: What this says in words

Whether growth in one sector helps another sector depends on how connected they are in the production network. If the Leontief inverse has large off-diagonal entries (sectors are strongly linked), growth propagates. If the off-diagonal entries are small (sectors are weakly linked), growth concentrates. The spectral radius of W^P governs the overall strength of intersectoral transmission.

Proof. The Leontief inverse is $(I - \gamma W^P)^{-1} = \sum_{k=0}^{\infty} \gamma^k (W^P)^k$. The off-diagonal entries are $\ell_{ij} = \sum_{k=1}^{\infty} \gamma^k (W^P)^k_{ij}$ (the $k = 0$ term is I, contributing only to the diagonal). Each power $(W^P)^k_{ij}$ represents paths of length k from j to i, and satisfies $|(W^P)^k_{ij}| \leq \rho(W^P)^k$ for the spectral radius. Summing the geometric bound:

$$\ell_{ij} \leq \sum_{k=1}^{\infty} \gamma^k \rho(W^P)^k = \frac{\gamma \rho(W^P)}{1 - \gamma \rho(W^P)}. \tag{19}$$

Averaging over the $N(N-1)$ off-diagonal entries gives the bound. For the norm bound, use $|(W^P)^k_{ij}| \leq \|W^P\|^k$ and $\|(W^P)^k\| \leq \|W^P\|^k$. □

4 Structural Heterogeneity: The Block-Diagonal Economy

4.1 Pinto's Diagnosis

Aníbal Pinto observed in the 1960s that Latin American economies were not uniformly "underdeveloped." They contained a modern sector with productivity levels comparable to the center, coexisting with a traditional sector with productivity levels comparable to the 19th century. The gap between sectors—within the same country—was often larger than the gap between the modern sector and the developed world.

This was not a transitional phase. It was a stable configuration. Growth did not close the gap; it widened it. The modern sector absorbed capital and technology, grew rapidly, and became more productive. The traditional sector absorbed labor, grew slowly or not at all, and stagnated. The structural disconnection between them prevented the propagation that would have equalized productivity.

We now formalize this.

Definition 4.1 (Structural Heterogeneity)**.** *An economy with production network W^P has **structural heterogeneity of degree** k if W^P is approximately block-diagonal with k*

blocks:

$$W^P \approx \begin{pmatrix} W_{11}^P & \varepsilon_{12} \\ \varepsilon_{21} & W_{22}^P \end{pmatrix}, \tag{20}$$

where W_{11}^P and W_{22}^P are the within-block (within-sector) linkage matrices and $\varepsilon_{12}, \varepsilon_{21}$ are the cross-block linkages, with $\|\varepsilon_{12}\|, \|\varepsilon_{21}\| \ll \|W_{11}^P\|, \|W_{22}^P\|$.

Degree $k = 1$: the economy is structurally integrated (one block). Degree $k = 2$: the economy has a dual structure (Pinto's case). Degree $k > 2$: multiple disconnected sectors.

Example 4.2 (Bolivia: A Degree-2 Economy)**.** *Bolivia's production structure can be stylized as two blocks:*

Block 1 (Modern): *Gas extraction, mining, large-scale agriculture (soybeans), financial services. High capital intensity, high productivity, strong linkages to global markets, weak linkages to the domestic traditional sector.*

Block 2 (Traditional): *Subsistence agriculture (quinoa, potatoes, llamas), informal commerce, small-scale manufacturing, domestic services. Labor-intensive, low productivity, strong internal linkages (farmers sell to local markets, local markets sell to local consumers), negligible linkages to the modern sector.*

The stylized W^P:

$$W_{Bolivia}^P \approx \begin{pmatrix} 0.15 & 0.03 \\ 0.02 & 0.20 \end{pmatrix}. \tag{21}$$

*Within the modern sector, linkages are moderate (*0.15*): gas companies buy services from financial firms, mining needs transport. Within the traditional sector, linkages are strong (*0.20*): the farmer sells to the market seller who buys from the small manufacturer. But cross-sector linkages are tiny (*0.03 *and* 0.02*): the gas sector imports its equipment from Brazil, not from Bolivian workshops; the quinoa farmer does not sell to the gas company cafeteria.*

The Leontief inverse:

$$L \approx \begin{pmatrix} 1.18 & 0.04 \\ 0.03 & 1.25 \end{pmatrix}. \tag{22}$$

A productivity shock in the gas sector (column 1) generates 1.18 units of output in the modern sector and 0.03 in the traditional sector. Almost none of the growth propagates. The enclave multiplier is 1.21; of this, 97.5% stays in the modern sector.

Compare with a hypothetical connected Bolivia where cross-sector linkages are 0.15 *instead of* 0.03*:*

$$W_{connected}^P = \begin{pmatrix} 0.15 & 0.15 \\ 0.15 & 0.20 \end{pmatrix}, \quad L_{connected} \approx \begin{pmatrix} 1.26 & 0.23 \\ 0.21 & 1.33 \end{pmatrix}. \tag{23}$$

Now the same gas shock generates 0.21 units in the traditional sector—seven times more propagation. The multiplier rises to 1.47, with 14% reaching the traditional sector.

Prebisch Connection: The multiplier gap is the development gap

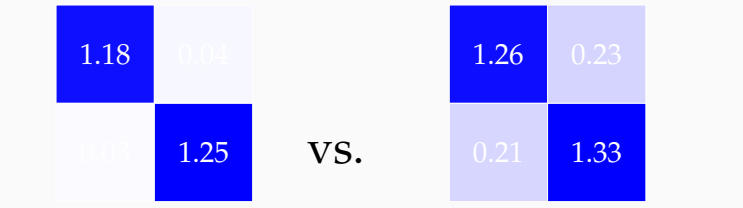
The difference between the two Bolivias above is not technology, not savings rates, not human capital, not institutions in the Acemoglu-Robinson sense. It is the *structure of inter-sectoral linkages*—the topology of W^P. Both economies have the same production functions, the same capital, the same labor. They differ only in how connected the sectors are. And this difference produces, over time, radically different development trajectories.

This is Prebisch's core argument, formalized: the terms of trade deterioration is a consequence of the periphery's weak structural linkages. When the center grows, its interconnected production structure amplifies the growth internally. When the periphery grows, its disconnected structure leaks the growth externally (through commodity export revenues that purchase imported manufactures) without amplifying it internally.

The multiplier gap—the difference in off-diagonal entries of the Leontief inverse between a connected and a disconnected economy—*is* the development gap. It is computable. It is a property of W^P. And it can be changed.

Visualization:

Bolivia's two possible Leontief inverses



$(I - \gamma W^P)^{-1}_{\textbf{enclave}}$ Off-diagonal ≈ 0. Growth stays.

$(I - \gamma W^P)^{-1}_{\textbf{connected}}$ Off-diagonal bright. Growth propagates.

Same economy, same production functions, same capital, same labor. The only difference is the off-diagonal entries of W^P (0.03 vs. 0.15). Left: the Leontief inverse is nearly diagonal — a gas shock generates 0.03 in the traditional sector. Right: the same shock generates 0.21 — seven times more. The dark off-diagonal squares on the left are the trickle-down problem in two pixels.

4.2 The Spectral Signature of Heterogeneity

Proposition 4.3 (Spectral Properties of Block-Diagonal Economies)**.** *Let W^P be approximately block-diagonal with $k = 2$ blocks as in (20). Then:*

1. *The eigenvalues of W^P are approximately the union of the eigenvalues of W_{11}^P and W_{22}^P, plus perturbation terms of order $\|\varepsilon\|$.*
2. *The spectral gap $\Gamma(W^P) = \rho(W^P) - |\lambda_2(W^P)|$ is small when the two blocks have similar spectral radii: $\Gamma \approx |\rho(W_{11}^P) - \rho(W_{22}^P)| + O(\|\varepsilon\|)$.*
3. *The Perron eigenvector of W^P is approximately block-localized: it assigns most weight to the block with the larger spectral radius (the more productive/connected sector) and little weight to the other.*

Intuition: What this says in words

A block-diagonal economy has a small spectral gap. And from Chapter 1 (Proposition 1B.1 in the appendix), a small spectral gap means: slow convergence, persistent asymmetries, and the possibility of extreme power concentration. In terms of growth: a block-diagonal production structure means that shocks propagate slowly between blocks, productivity differences persist across sectors, and the benefits of growth accrue primarily to the block with the higher spectral radius (the modern sector).
A connected economy has a large spectral gap. Growth propagates fast, sectoral differences converge, and the benefits are distributed broadly.
The spectral gap of W^P is a single number that summarizes Pinto's entire structural heterogeneity argument.

Proof sketch. (1) is a standard result in matrix perturbation theory (see Appendix 3A). If $W^P = \text{diag}(W_{11}^P, W_{22}^P) + E$ where E is the off-diagonal block, then by the Bauer-Fike theorem, the eigenvalues of W^P lie within $\|E\|$ of the eigenvalues of the block-diagonal part.

(2) follows because the two largest eigenvalues of the block-diagonal part are $\rho(W_{11}^P)$ and $\rho(W_{22}^P)$. If these are close, Γ is small. The perturbation E shifts both by order $\|E\|$.

(3) The Perron eigenvector of a block-diagonal matrix assigns weight only within the block that achieves ρ. The perturbation E "leaks" some weight to the other block, but the leakage is proportional to $\|E\|/\Gamma$—small when $\|E\|$ is small. □

5 Does Growth Trickle Down? A Formal Answer

5.1 The Trickle-Down Hypothesis in Matrix Form

The "trickle-down" hypothesis says: growth in the modern sector benefits the traditional sector through market linkages—increased demand for traditional products, higher wages that spread through labor mobility, technology transfer. In our framework, trickle-down is the claim that the off-diagonal entries of the Leontief inverse are large enough that modern-sector growth significantly raises traditional-sector output.

Proposition 5.1 (The Trickle-Down Condition)**.** *Growth in sector j (the modern sector) "trickles down" to sector i (the traditional sector) if and only if:*

$$\ell_{ij} = [(I - \gamma W^P)^{-1}]_{ij} > \tau, \tag{24}$$

where $\tau > 0$ is a threshold representing "economically meaningful" propagation. This is equivalent to requiring that there exists a chain of production linkages from j to i with sufficient cumulative strength.

Intuition: When does trickle-down fail?

Trickle-down fails when ℓ_{ij} is small—when the Leontief inverse has small off-diagonal entries. This happens in three cases:

1. **Weak direct linkages**: W^P_{ij} is small (the traditional sector does not sell inputs to the modern sector and does not buy its outputs).
2. **Weak indirect linkages**: there is no chain of intermediate sectors connecting j to i with sufficient strength. All paths through the network from j to i pass through weak links.
3. **The enclave**: the modern sector is connected to the *global* economy but disconnected from the *domestic* economy. Its linkages point outward (imports, exports) rather than inward (domestic input purchases, domestic sales).

Bolivia's gas sector fails on all three counts. It does not buy inputs from the domestic traditional sector (case 1). There are no intermediate sectors linking gas to quinoa (case 2). Its linkages are to Brazilian pipeline operators and international commodity markets, not to Bolivian workshops (case 3).

5.2 The Transfer Perversity Result

Here is where the formation model connects to the growth model. Suppose the government, observing that gas growth does not trickle down, intervenes with a fiscal transfer: tax the modern sector and redistribute to the traditional sector. (This is, in essence, what Bolivia has done since 2006 with gas revenue-funded transfers like Renta Dignidad and Bono Juancito Pinto.)

Proposition 5.2 (Transfer Perversity Under Structural Heterogeneity)**.** *In a block-diagonal economy with structural heterogeneity, an unconditional transfer τ from the modern sector to the traditional sector:*

1. ***Raises traditional-sector income*** *(direct effect: $+\tau$).*
2. ***Raises traditional-sector capital*** *(through higher savings: $\Delta K_T = s_T \tau$).*
3. ***Lowers the modern-sector wage premium*** *(by raising traditional-sector productivity relative to the gap that motivates sector-switching).*
4. ***Weakens the signal for modern-sector investment*** *(the return differential between sectors narrows).*
5. ***Makes the traditional sector more attractive relative to the modern sector****, reducing the incentive for structural transformation.*

The net effect can be perverse: the transfer improves welfare within the traditional sector while making it less likely that the economy escapes structural heterogeneity.

Intuition: Why transfers can entrench the trap

The transfer does not change W^P. It does not add linkages between sectors. It does not make the gas sector buy inputs from Bolivian workshops. It does not make quinoa farmers invest in modern-sector skills. It takes money from the modern sector and gives it to the traditional sector, which spends it on traditional-sector goods (because that is what the traditional-sector park trained its population to want), which raises traditional-sector income, which makes the traditional sector more comfortable, which reduces the already-weak incentive to invest in the structural transformation that would connect the two sectors.

This is not an argument against transfers. Transfers reduce poverty and they should be done. It is an argument that transfers are not structural policy. Structural policy changes W^P—it builds the linkages, the roads, the value chains, the input-output connections that make growth propagate. Transfers change ω (who has what). Structural policy changes A (who is connected to whom).

6 The Three Scales

We can now see the three-scale structure that Chapter 16 will develop fully:

Global scale. Countries are nodes. W^P_{global} is the international input-output matrix. The center (high column sums, high Perron eigenvector weight) captures gains from trade. The periphery (low column sums, low weight) transfers productivity gains outward. This is Prebisch.

National scale. Sectors are nodes. W^P_{national} is the domestic input-output matrix. If block-diagonal, the economy has structural heterogeneity. Modern-sector growth stays in the modern sector. Traditional-sector stagnation is self-reproducing. This is Pinto.

Individual scale. People are nodes. W^I is the information/signal network that shapes aspirations and preferences. If W^I channels traditional-sector people's attention toward traditional-sector signals, their aspirations are calibrated to the traditional sector even when the modern sector is accessible. This is the formation model (Chapter 10).

Key Structural Insight

The same mathematical framework—weight matrices and their spectral properties—operates at all three scales. The spectral gap of W^P_{global} determines whether global growth reduces or increases international inequality. The spectral gap of W^P_{national} determines whether domestic growth reduces or increases internal inequality. The spectral gap of W^I determines whether normative change reaches the formation-constrained population.

At each scale, the answer to "does growth trickle?" is: it depends on the off-diagonal entries of the Leontief inverse. And those entries are a function of W^P, which is the structure.

Prebisch saw this at the global scale. Pinto saw it at the national scale. Neither had the matrix. Now we do.

7 Summary

This chapter established five results:

1. The **standard Solow model** assumes one sector and one savings rate. It predicts convergence. The real world has multiple sectors with heterogeneous productivity and differential savings, and convergence is conditional on the structure of inter-sectoral linkages.

2. The **multi-sector growth model** (10) embeds the Leontief system in a dynamic framework. Growth propagation depends on the production network W^P through the Leontief inverse $(I - \gamma W^P)^{-1}$.

3. The **Kuznets question**—does growth reduce inequality?—has a structural answer: it depends on W^P. Connected economies (large spectral gap, large off-diagonal Leontief entries) propagate growth across sectors. Disconnected economies (small spectral gap, block-diagonal W^P) concentrate growth in the modern sector.

4. **Structural heterogeneity** (Pinto) is a block-diagonal W^P with weak cross-block linkages. It is diagnosed by the spectral gap: small Γ means persistent sectoral inequality. Bolivia's gas/traditional economy is a degree-2 case.

5. **Transfers do not change** W^P. They alleviate poverty within the traditional sector without building the inter-sectoral linkages that would make growth propagate. Structural policy changes the matrix. Transfer policy changes the endowment vector.

8 Exercises

3.1 Solow with two groups. For the OLG model with $\alpha = 0.3$, $\theta^M = 2$, $\theta^F = 0.5$:

(a) Compute the savings rates $\sigma(\theta^M)$ and $\sigma(\theta^F)$.

(b) Compute the aggregate savings rate $\bar{s} = \frac{1}{2}[\sigma(\theta^M) + \sigma(\theta^F)]$.

(c) Find the steady-state capital K^* as a function of $\bar{s}$.

(d) How much lower is K^* compared to an economy where both groups have $\theta = 2$? This gap is the macroeconomic cost of the formation mechanism.

3.2 Propagation in a 3-sector economy. For:

$$W^P = \begin{pmatrix} 0.10 & 0.20 & 0.05 \\ 0.15 & 0.10 & 0.15 \\ 0.05 & 0.10 & 0.10 \end{pmatrix}. \tag{25}$$

(a) Compute the Leontief inverse $(I - W^P)^{-1}$.

(b) Sector 2 receives a productivity shock. How much does each sector's output change?

(c) Which sector has the highest output multiplier? Why?

(d) Compute $\rho(W^P)$ and the spectral gap. Is this economy "connected" or "heterogeneous"?

3.3 The enclave. Modify the 3-sector economy: set $W^P_{31} = W^P_{32} = 0.01$ and $W^P_{13} = W^P_{23} = 0.01$ (sector 3 is an enclave).

(a) Recompute the Leontief inverse. How much does a shock to sector 3 propagate to sectors 1 and 2?

(b) Compare with the connected case. By what factor does propagation decline?

(c) Compute the spectral gap. Has it changed?

(d) Relate to Bolivia's gas sector.

3.4 Transfer vs. structural policy. Using Bolivia example (Example 4.2):

(a) Suppose the government transfers 10% of modern-sector output to the traditional sector. What happens to traditional-sector income? To the cross-sector productivity gap?

(b) Now suppose the government spends the same 10% building linkages: ε_{12} and ε_{21} in W^P increase from 0.03 to 0.10. Recompute the Leontief inverse. What happens to the cross-sector propagation?

(c) Compare the two policies after 10 periods. Which one closes the productivity gap?

3.5 Prebisch in the global I-O table. Download the OECD Inter-Country Input-Output (ICIO) table. Extract W^P for 5 countries: USA, Germany, China, Bolivia, Nigeria.

(a) Compute column sums of W^P (who depends on whom). Which countries have the highest column sums?

(b) Compute the Leontief inverse. Which countries have the highest output multipliers?

(c) A productivity shock hits Bolivia (agricultural sector). How much propagates to the USA vs. to Nigeria?

(d) Is the global economy "connected" or "block-diagonal"? Compute the spectral gap.

(e) *Challenge*: Compute the left Perron eigenvector of the global W^P. Which countries have the most "structural power"? Is this consistent with the center-periphery hypothesis?

3.6 Kuznets revisited. Simulate the multi-sector growth model for $T = 50$ periods with $N = 2$ sectors, starting from $K_M = K_T = 1$, with a productivity advantage in the modern sector ($A_M = 2, A_T = 1$):

(a) Case A: W^P connected (off-diagonal = 0.15). Plot the ratio Y_M/Y_T over time.

(b) Case B: W^P disconnected (off-diagonal = 0.02). Plot the same ratio.

(c) In which case does the Kuznets inverted-U hold? In which does inequality grow monotonically?

(d) At what value of cross-sector linkage strength does the trajectory switch from convergence to divergence?

3.7 The formation connection (preview of Chapter 10). In the two-group OLG model, suppose θ^F is not fixed but evolves according to:

$$\theta^F_{t+1} = 0.8\theta^F_t + 0.2\bar{\theta} + \mu(\lambda_t r_t - r^*), \tag{26}$$

where r_t is the return on savings and $\lambda_t < 1$ is the signal discount (women perceive lower returns).

(a) Starting from $\theta_0^F = 0.5$ and $\theta^M = 2$ (fixed), simulate for 20 generations with $\lambda = 0.7$, $\mu = 0.1$, $\bar{\theta} = 2$, $r^* = 0.1$.

(b) Plot the trajectory of θ_t^F and the gender gap $\theta^M - \theta_t^F$.

(c) At what generation does the gap halve?

(d) Now set $\lambda = 1$ (signal distortion removed) at $t = 10$. How many additional generations until the gap halves from its $t = 10$ value? This is the hysteresis result from Chapter 10.

Mathematical Appendices

Appendix 3A: Block Matrices and Their Eigenvalues

Block matrices. A **block matrix** is a matrix partitioned into rectangular submatrices. A 2×2 block matrix:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \tag{27}$$

where A is $n_1 \times n_1$, D is $n_2 \times n_2$, B is $n_1 \times n_2$, and C is $n_2 \times n_1$. The total matrix M is $(n_1 + n_2) \times (n_1 + n_2)$.

Block-diagonal matrices. When $B = 0$ and $C = 0$, the matrix is **block-diagonal**: $M = \text{diag}(A, D)$. The eigenvalues of a block-diagonal matrix are simply the union of the eigenvalues of each block. If A has eigenvalues $\{\alpha_1, \ldots, \alpha_{n_1}\}$ and D has eigenvalues $\{\delta_1, \ldots, \delta_{n_2}\}$, then M has eigenvalues $\{\alpha_1, \ldots, \alpha_{n_1}, \delta_1, \ldots, \delta_{n_2}\}$.

Why this matters. A block-diagonal W^P means the economy's sectors don't interact across blocks. Each block has its own dynamics, its own eigenvalues, its own growth rate. The spectral gap of the whole matrix is $|\rho(A) - \rho(D)|$ (approximately): the distance between the dominant eigenvalues of the two blocks. If both blocks have similar spectral radii, $\Gamma \approx 0$: two competing "modes" of growth, neither dominating.

Near-block-diagonal matrices. When B and C are small but nonzero (weak cross-block linkages), the eigenvalues are perturbed from the block-diagonal case. The Bauer-Fike theorem states: if λ is an eigenvalue of M and λ_0 is the nearest eigenvalue of $\text{diag}(A, D)$, then $|\lambda - \lambda_0| \leq \kappa(V)\|(B, C)\|$, where $\kappa(V)$ is the condition number of the eigenvector matrix and $\|(B, C)\|$ is the norm of the off-diagonal perturbation.

In plain language: small cross-block linkages produce small changes in eigenvalues. The qualitative structure (two competing modes, small spectral gap) is preserved under small perturbation. This is why structural heterogeneity is robust: adding a few weak linkages between the gas sector and the quinoa sector does not fundamentally change the spectral structure. The economy remains approximately block-diagonal until the cross-block linkages are comparable in strength to the within-block linkages.

Example. For Bolivia's stylized W^P from Example 4.2:

$$W^P = \begin{pmatrix} 0.15 & 0.03 \\ 0.02 & 0.20 \end{pmatrix}. \tag{28}$$

Block-diagonal part: $\text{diag}(0.15, 0.20)$ with eigenvalues 0.15 and 0.20. Spectral gap of the block-diagonal part: $|0.20 - 0.15| = 0.05$.

Full matrix eigenvalues (from the characteristic equation $\lambda^2 - 0.35\lambda + 0.0294 = 0$): $\lambda_1 \approx 0.204$, $\lambda_2 \approx 0.144$. Spectral gap: 0.060. Close to the block-diagonal approximation (0.05), confirming that the perturbation is small.

Compare with the connected case ($W^P_{\text{connected}}$ from Example 4.2):

$$W^P_{\text{connected}} = \begin{pmatrix} 0.15 & 0.15 \\ 0.15 & 0.20 \end{pmatrix}. \tag{29}$$

Eigenvalues: $\lambda_1 \approx 0.325$, $\lambda_2 \approx 0.025$. Spectral gap: 0.30. Five times larger than the disconnected case. Growth propagates faster, convergence is quicker, and the Perron eigenvector assigns weight to both sectors rather than concentrating on one.

Appendix 3B: Convergence in Multi-Sector Growth Models

The convergence question. In the one-sector Solow model, the economy converges to a unique steady state at rate $\beta = (1-\alpha)(\delta + g + n)$. In the multi-sector model, convergence depends on the eigenvalues of the *Jacobian of the growth mapping* evaluated at the steady state.

The growth mapping. Define $G(\mathbf{K}) = S(I - \gamma W^P)^{-1}\mathbf{f}(\mathbf{K}) + (I - \Delta)\mathbf{K}$ from equation (10). The Jacobian at steady state $\mathbf{K}^*$ is:

$$J = DG(\mathbf{K}^*) = S(I - \gamma W^P)^{-1} F'(\mathbf{K}^*) + (I - \Delta), \tag{30}$$

where $F'(\mathbf{K}^*) = \text{diag}(\alpha_1 f_1/K_1^*, \ldots, \alpha_N f_N/K_N^*)$ is the diagonal matrix of marginal products.

Convergence condition. The economy converges to $\mathbf{K}^*$ from nearby initial conditions if and only if all eigenvalues of J have modulus less than 1: $\rho(J) < 1$. The *speed* of convergence is $-\ln \rho(J)$: faster when $\rho(J)$ is further below 1.

How W^P affects convergence speed. The Leontief inverse $(I - \gamma W^P)^{-1}$ amplifies the marginal products $F'(\mathbf{K}^*)$, which makes the eigenvalues of J larger (closer to 1). Stronger inter-sectoral linkages (larger $\gamma\rho(W^P)$) mean stronger amplification, which means *slower* convergence in levels but faster propagation of shocks across sectors.

The tradeoff: a well-connected economy ($\rho(W^P)$ close to $1/\gamma$) propagates growth broadly but converges slowly to the steady state. A disconnected economy (small $\rho(W^P)$) converges fast *within* each block but does not propagate *between* blocks.

The two-speed result. In a block-diagonal economy, the Jacobian J is approximately block-diagonal, and convergence proceeds at two speeds:

- **Within-block convergence**: governed by the eigenvalues of the individual blocks. Fast if each block has moderate linkages.
- **Between-block convergence**: governed by the cross-block entries of J, which are small (proportional to ε). Very slow. The productivity gap between sectors persists long after each sector individually reaches its own approximate steady state.

This is the "two-speed" convergence that Pinto observed: the modern sector grows rapidly to its (high) steady state; the traditional sector grows slowly to its (low) steady state; the gap between them persists for decades or centuries because the cross-block convergence rate is proportional to the cross-block linkage strength, which is small.

In the formation model (Chapter 10), the same two-speed pattern appears for preferences: enforcement effects converge fast (within-generation timescale), formation effects converge slow (between-generation timescale, governed by ρ). The mathematical structure is identical: block-diagonal dynamics with weak cross-block coupling.

Structuralist Case Study: Pinto's Structural Heterogeneity and the Spectral Gap

Aníbal Pinto Santa Cruz (1919–1996) was a Chilean economist who spent most of his career at CEPAL. In 1970, he published a short essay in *El Trimestre Económico* — "Naturaleza e implicaciones de la 'heterogeneidad estructural' de la América Latina" — that reframed the structuralist research program. His argument was that the center-periphery framework, while correct about international asymmetries, had missed an equally important asymmetry *within* peripheral economies.

The problem was not only that the periphery was exploited by the center but that the periphery was exploiting itself.

Pinto's thesis has three parts, each of which maps to a result in this chapter.

Part 1: The peripheral economy is not one economy but three. Pinto identified a tripartite structure within Latin American countries: a "primitive" stratum (with productivity levels comparable to colonial times), a "modern pole" (with productivity levels comparable to the industrialized countries), and an "intermediate" stratum (near the national average). The productivity ratio between the modern pole and the primitive stratum could reach 20:1. Crucially, this was not a simple urban-rural divide — the modern pole included urban enclaves (export platforms, MNC subsidiaries) while the intermediate stratum included both urban industry and commercial agriculture.

In this chapter's notation: the domestic production network W^P has at least two blocks (we simplified to two for tractability), with *weak cross-block entries*. The tripartite structure is a block-diagonal matrix with three blocks of very different productivity. The spectral gap $\Gamma(W^P)$ is small because the blocks' spectral radii are similar — each block has its own internal dynamics, and no single mode dominates.

Part 2: Import-substitution industrialization added new layers of heterogeneity without eliminating old ones. The ISI strategy, which CEPAL had championed, successfully built a modern industrial sector in several Latin American countries (Brazil, Mexico, Argentina). But Pinto observed that this modern sector coexisted with the pre-existing traditional sector rather than absorbing it. The economy became more complex — more blocks in the matrix — without becoming more *connected*. The off-diagonal entries of W^P remained small. He called this "modernización excluyente" — exclusionary modernization.

This is the dynamic result from Section 5: new blocks can be added to a block-diagonal matrix (new sectors created through ISI) without increasing the spectral gap, if the new blocks connect only to themselves and to the global economy rather than to the pre-existing traditional sector. The Leontief inverse gains new rows and columns, but the off-diagonal entries between the new modern block and the old traditional block remain dark. The heatmap gets bigger without getting brighter.

Part 3: The structural policy implication. Pinto's prescription was not more ISI (adding more blocks) but structural integration: building the cross-block linkages that would allow productivity gains in the modern sector to propagate to the traditional sector. In the language of this chapter: the policy goal is to increase the off-diagonal entries of W^P — to make the heatmap brighter off the diagonal —

which increases the spectral gap Γ, which reduces the between-group welfare loss (Proposition 3.3 in Chapter 2), which makes growth broadly shared rather than enclave-concentrated.

Pinto's argument	**Formal object**	**This chapter**
Tripartite structure	Block-diagonal W^P with 2–3 blocks	Sec. 4
Productivity ratio 20:1	$\bar{\omega}_{\text{modern}}/\bar{\omega}_{\text{traditional}} \gg 1$	Example 4.2
ISI adds blocks, not links	New blocks with dark cross-entries	Eq. (20)
Exclusionary modernization	Growth stays in bright blocks	$\ell_{ij} \approx 0$ for $i \neq j$
Small spectral gap	$\Gamma(W^P) \approx \lvert\rho_1 - \rho_2\rvert$ small	Prop. 4.3
Structural integration	Increase off-diagonal entries	Example 4.2: 0.03 $\rightarrow$ 0.15

Pinto's essay is 18 pages. The mathematical content of those 18 pages is a 2×2 block-diagonal matrix, its Leontief inverse, and its spectral gap. He did not have the notation. He had the diagnosis.

Further Reading

Growth theory: Acemoglu (2009), *Introduction to Modern Economic Growth*, chapters on Solow, OLG, multi-sector models. Barro and Sala-i-Martin (2004), *Economic Growth*, for the empirical convergence literature.

Multi-sector growth with networks: Acemoglu et al. (2012), "Network Origins of Aggregate Fluctuations." Baqaee and Farhi (2019), "The Macroeconomic Impact of Microeconomic Shocks." Liu and Tsyvinski (2020), "Dynamical Structure and Spectral Properties of Input-Output Networks."

Structural heterogeneity: Pinto (1970), "Naturaleza e implicaciones de la 'heterogeneidad estructural'" *El Trimestre Económico*. This is the source. Read it. CEPAL (2010), *Time for Equality: Closing Gaps, Opening Trails*, for the modern institutional statement.

The CEPAL tradition: Prebisch (1950), start here. Furtado (1961), *Development and Underdevelopment*. Ocampo, Rada, and Taylor (2009), *Growth and Policy in Developing Countries: A Structuralist Approach*. Bielschowsky (2009), "Sixty Years of ECLAC."

Bolivia: Morales and Sachs (1990), "Bolivia's Economic Crisis" for the structural adjustment context. Weisbrot, Ray, and Johnston (2009), "Bolivia: The Economy Under Morales" for the resource-transfer period. The National Statistics Institute (INE) input-output tables for the data.

Chapter 4

What the Standard Model Assumes

Six Properties of One Matrix

> *"El subdesarrollo no es un 'momento' ni una 'etapa' en la evolución de una sociedad aislada y autónoma, sino parte del proceso histórico global de desarrollo del capitalismo."*
>
> ["Underdevelopment is not a 'moment' or a 'stage' in the evolution of an isolated and autonomous society, but part of the global historical process of capitalist development."]
>
> — Osvaldo Sunkel and Pedro Paz, *El subdesarrollo latinoamericano y la teoría del desarrollo* (1970)

The first three chapters rewrote the standard model in matrix notation and showed that it assumes a specific interaction matrix: $W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$. Chapter 1 named it. Chapter 2 showed that the welfare theorems require it. Chapter 3 showed that growth propagation depends on it.

This chapter pulls the threads together. It catalogs, in one place, every assumption embedded in W^{CE} — six properties of one matrix — and for each assumption states: (a) the formal condition, (b) when it approximately holds, (c) when it fails, (d) what the world looks like when it does, and (e) which chapter in this book develops the general case.

This is the hinge. Part I ends here. Part II begins with W as an object to be studied rather than assumed.

1 The Six Assumptions

Recall from Chapter 1 that the competitive interaction matrix $W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$ has five properties: doubly stochastic, symmetric, rank 1, equal centrality, maximal spectral gap. To these we add a sixth assumption that is not a property of W^{CE} itself but of the model that uses it: preferences are exogenous.

Definition 1.1 (The Six Assumptions of the Standard Model)**.** *The standard competitive model assumes:*

A1. No utility externalities. $W^U = 0$ *(or equivalently* $\gamma_U = 0$*). Agent i's utility depends only on* x_i*.*

A2. Exogenous production structure. $W^P = A$ *is taken as given from input-output data. It is a technological fact, not a social choice.*

A3. No information network. W^I *does not exist. Beliefs are rational, information is symmetric, and preferences are common knowledge.*

A4. Anonymous market interaction. *The price mechanism aggregates demands symmetrically:* $W^{market} = \frac{1}{N}\mathbf{1}\mathbf{1}'$*.*

A5. Equal power. *The left Perron eigenvector of the composite W is* $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$*. No agent has more structural influence than any other.*

A6. Exogenous preferences. θ_i *(the preference parameter governing patience, risk aversion, time discount, labor supply) is fixed. It does not evolve in response to the network.*

We now examine each assumption in detail.

2 A1: No Utility Externalities ($W^U = 0$)

2.1 The Formal Condition

Agent i maximizes $U_i(x_i)$, not $V_i(\mathbf{x}) = U_i(x_i) + \gamma_U \sum_j W^U_{ij} U_j(x_j)$. Nobody's consumption enters anyone else's utility function. No altruism, no envy, no status competition, no compadrazgo bonds, no peer effects.

2.2 When It Approximately Holds

Large anonymous markets for standardized goods. The buyer of a barrel of oil on the NYMEX does not care who produced it. The seller of wheat at the Chicago Board of Trade does not know who will eat it. In these settings, the externality weights W^U_{ij} are effectively zero because the transacting parties have no social relationship.

2.3 When It Fails

Small communities. In a village or a tight-knit neighborhood, people know each other. The api vendor cares about her comadre's income. The shopkeeper cares about the schoolteacher's welfare. Utility externalities are large, local, and asymmetric.

Status competition. Veblen (1899), Duesenberry (1949), Frank (1985): consumption generates externalities through social comparison. A neighbor's new car reduces your satisfaction with your old one. The externality is negative and proportional to the visibility of consumption.

Network effects. Adoption of a technology by agent j raises the value of the technology for agent i (telephones, social media, payment systems). $W^U_{ij} > 0$ for complementary users.

Proposition 2.1 (Welfare Cost of Ignoring W^U)**.** *When $W^U \neq 0$ and $\gamma_U > 0$, the competitive equilibrium (which ignores W^U) generates a welfare loss of:*

$$L_{A1} = \gamma_U^2 \cdot \text{Var}(c_j^U) \cdot \Psi, \tag{1}$$

where $c_j^U = \sum_{i \neq j} W^U_{ij}$ is agent j's externality centrality in the utility network (Chapter 2, Proposition 2.4).

Relaxed in: Chapters 5 (the utility network defined), 7 (bargaining equilibrium with externalities), 12 (welfare under endogenous preferences).

3 A2: Exogenous Production Structure (W^P Given)

3.1 The Formal Condition

The technical coefficients matrix A (= W^P) is treated as a technological parameter: determined by engineering, not by economic choices. The Leontief system takes A as data and solves for output given final demand.

3.2 When It Approximately Holds

Short-run analysis. In a given year, a steel plant requires a specific amount of iron ore, coal, and electricity per ton of steel. These technical coefficients change slowly. For short-run comparative statics, treating A as fixed is reasonable.

3.3 When It Fails

ISI as structural policy. The entire history of import-substitution industrialization is an attempt to *change* A — to build domestic production of goods previously imported. Tavares (1972) documented how each external bottleneck provoked a structural response that altered the domestic input-output matrix. A is a policy variable, not a constant.

Endogenous specialization. Countries choose what to produce. The product space (Hidalgo et al., 2007) shows that diversification follows path-dependent trajectories determined by the existing A: a country that produces wine can more easily diversify into grapes, bottles, and corks than into semiconductors. The production network evolves.

FDI and global value chains. When a multinational opens an assembly plant, it inserts new rows and columns into the domestic A, connecting the domestic economy to the global production network. The entries of A reflect corporate decisions, not just technology.

Proposition 3.1 (Endogenous W^P Changes the Growth Trajectory)**.** *In the multi-sector growth model of Chapter 3, a change ΔA in the technical coefficients shifts the steady-state capital vector by:*

$$\Delta \mathbf{K}^* \approx S(I - \gamma W^P)^{-1} \cdot \gamma \Delta A \cdot (I - \gamma W^P)^{-1} \mathbf{f}(\mathbf{K}^*) \cdot (\textit{Jacobian terms}). \tag{2}$$

The effect is amplified by the Leontief inverse twice — once for the direct impact, once for the propagation through the existing network. A small change in A can have a large effect on $\mathbf{K}^*$ if the Leontief inverse has large off-diagonal entries.

Relaxed in: Chapters 3 (production network in growth), 13 (how networks evolve), 15 (operator design — policy as deliberate change of W^P).

4 A3: No Information Network (W^I Absent)

4.1 The Formal Condition

The standard model assumes: agents have rational expectations, information is symmetric (everyone observes the same signals), and preferences are common knowledge. There is no mechanism through which one agent's beliefs influence another's. The information network W^I is not merely unknown — it is assumed not to exist.

4.2 When It Approximately Holds

Financial markets with mandatory disclosure, standardized goods with transparent pricing, and repeated interactions where agents learn the true model. In these settings, heterogeneous beliefs converge to the truth through Bayesian updating, making W^I irrelevant.

4.3 When It Fails

DeGroot learning dominates Bayesian learning in practice. Agents in most real-world settings do not observe the data-generating process. They observe their neighbors' behavior, their social media feeds, their peers' outcomes. They update naïvely (weighted average of neighbors' beliefs) rather than optimally (Bayesian). Golub and Jackson (2010) showed that DeGroot convergence is governed by the Perron eigenvector of W^I — the agents with the most epistemic power determine the long-run consensus, regardless of whether their initial beliefs are correct.

Media concentration. When a few outlets control information flow, W^I is star-shaped: everyone listens to the hub. The consensus converges to the hub's beliefs. State media, monopoly platforms, and algorithmic feeds all produce this topology.

Aspiration formation. The girl in Potosí does not observe the "true" return to education. She observes the returns *in her network* — her mother, her school friends, her cousin in Buenos Aires, her TikTok feed. Her perceived returns are W^I-weighted averages of the returns experienced by her reference group. This shapes her aspirations, her investment in education, and ultimately her economic outcomes.

Proposition 4.1 (Epistemic Power and Belief Convergence)**.** *Under DeGroot learning on an information network W^I (row-stochastic, irreducible, aperiodic):*

$$\lim_{t\to\infty} \mathbf{b}^{(t)} = (\boldsymbol{\pi}^I)'\mathbf{b}^{(0)} \cdot \mathbf{1}, \tag{3}$$

where $\boldsymbol{\pi}^I$ is the left Perron eigenvector of W^I. The long-run consensus is a weighted average of initial beliefs, with weights given by epistemic power π_j^I. Agents with $\pi_j^I \gg 1/N$ dominate the consensus.

Relaxed in: Chapters 5 (the information network defined), 8 (belief manipulation and epistemic power), 9–12 (the formation model, where W^I shapes preferences).

5 A4: Anonymous Market Interaction ($W^{\text{market}} = \frac{1}{N}\mathbf{1}\mathbf{1}'$)

5.1 The Formal Condition

The competitive price mechanism aggregates demands anonymously. Each agent contributes $1/N$ to the determination of the equilibrium price (with equal budgets). Nobody has market power. The interaction is $W^{\text{market}} = \frac{1}{N}\mathbf{1}\mathbf{1}'$.

5.2 When It Approximately Holds

Markets with many small agents, homogeneous goods, free entry and exit, and complete information. The textbook examples: wheat markets, foreign exchange (at the retail level), standardized commodity markets with many participants.

5.3 When It Fails

Monopsony and monopoly. A single buyer (monopsony employer in a company town) or single seller (YPFB for Bolivian gas) sets prices rather than taking them. The interaction matrix has a star topology: $W_{ij}^{\text{market}} \approx 0$ for all j except the hub.

Platform economies. Amazon, Uber, and Airbnb are hubs through which millions of small agents must transact. The platform sets the terms of trade. W^{market} is star-shaped with the platform at the center.

Bargaining. In thin markets (few participants, differentiated goods, relationship-specific investments), prices are determined by bilateral bargaining rather than anonymous market clearing. The allocation depends on relative bargaining power, which is a function of the network position (Chapter 7).

Proposition 5.1 (Welfare Loss from Market Power)**.** *When $W^{market} \neq \frac{1}{N}\mathbf{1}\mathbf{1}'$, the competitive equilibrium does not obtain. The bargaining equilibrium (Chapter 7) generates allocations:*

$$x_i^* = \frac{\Omega}{N} + \frac{1}{\kappa}\left(\pi_i(W) - \frac{1}{N}\right), \tag{4}$$

with welfare loss $L_{A4} = \frac{\kappa N}{2}\,\mathrm{Var}(\pi_i)\Phi^2$ (Chapter 2, Proposition 2.3).

Relaxed in: Chapters 6 (centrality and power from W), 7 (the bargaining equilibrium).

6 A5: Equal Power ($\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$)

6.1 The Formal Condition

The left Perron eigenvector of the composite weight matrix $W = \alpha_U W^U + \alpha_P W^P + \alpha_I W^I$ is uniform: $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$. No agent has more structural influence than any other. This requires W to be doubly stochastic (Chapter 2, Proposition 2.2).

6.2 When It Approximately Holds

Economies where interactions are dense, symmetric, and relatively uniform. Small cooperatives, well-functioning democracies with distributed power, competitive markets with many participants of similar size.

6.3 When It Fails

Everywhere that A1–A4 fail. Unequal power is not a separate assumption — it is the *consequence* of any departure from A1–A4. When W^U has clustered externalities, W^P is block-diagonal, W^I is star-shaped, or W^{market} is concentrated, the composite W is not doubly stochastic, and $\boldsymbol{\pi} \neq \frac{1}{N}\mathbf{1}$.

Proposition 6.1 (Power Decomposition)**.** *The Perron eigenvector of the composite $W = \alpha_U W^U + \alpha_P W^P + \alpha_I W^I$ satisfies:*

$$\pi_i = \alpha_U \underbrace{\sum_j \pi_j W^U_{ji}}_{\text{social power}} + \alpha_P \underbrace{\sum_j \pi_j W^P_{ji}}_{\text{production power}} + \alpha_I \underbrace{\sum_j \pi_j W^I_{ji}}_{\text{epistemic power}} . \tag{5}$$

An agent's total power is the weighted sum of power directed toward her through all three channels: how much powerful agents care about her (W^U), depend on her output (W^P), and listen to her signals (W^I).

Intuition: Power is recursive

The equation is self-referential: π_i depends on π_j for all j who point toward i, and π_j depends on π_k for all k who point toward j, and so on. Power comes from being valued, depended upon, and listened to *by other powerful agents*. This recursion is the defining feature of the Perron eigenvector and the mathematical content of "privilege reproduces itself."

Relaxed in: Chapters 5 (three networks), 6 (Perron-Frobenius), 7 (bargaining allocation).

7 A6: Exogenous Preferences (θ_i Fixed)

7.1 The Formal Condition

The preference parameter θ_i — patience, risk aversion, time discount, aspiration level, labor supply elasticity — is a primitive of the model. It does not change over time, does not respond to the network, and is not influenced by the allocations that the model produces.

This is the Becker-Stigler (1977) position: "de gustibus non est disputandum." Tastes are given. The economist's job is to derive behavior from fixed preferences, not to explain the preferences themselves.

7.2 When It Approximately Holds

Short-run analysis with stable social environments. If nothing in the agent's information network changes, and no significant shocks alter her reference group or her perceived returns, then treating θ_i as fixed over a budget period or a business cycle is harmless.

7.3 When It Fails

Intergenerational change. Preferences visibly evolve across generations: attitudes toward gender roles, savings behavior, educational aspirations, labor force participation norms. These are not random mutations — they correlate with changes in the information environment. The girl in Potosí with a phone has different aspirations than her mother, not because of a random preference shock but because W^I changed.

Peer effects. Sacerdote (2001), Duflo and Saez (2003): randomly assigned roommates, randomly placed coworkers influence each other's savings, retirement decisions, and educational investments. Preferences are contagious — they travel along W^I.

Advertising and media. W^I is deliberately shaped by actors who benefit from specific preference configurations. Beverage companies don't advertise to inform; they advertise to change θ.

Proposition 7.1 (The Formation Equation (Preview))**.** *When θ_i evolves according to the information network:*

$$\theta_{i,t+1} = \rho\theta_{i,t} + (1-\rho)\bar{\theta} + \mu\left(\sum_j W^I_{ij} r_{j,t} - r^*\right), \tag{6}$$

where $\rho \in (0,1)$ is persistence, $\bar{\theta}$ is a long-run anchor, $r_{j,t}$ is the return observed from agent j, and r^* is the reference return. The term $\sum_j W^I_{ij} r_{j,t}$ is the W^I-weighted average of observed returns: what you see depends on who you watch.

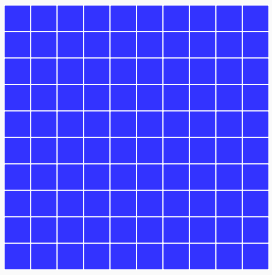
This equation is developed in full in Chapter 10. Its key implication: *the structure produces the preferences that justify the structure.* If W^I is block-diagonal (the girl in Potosí observes only traditional-sector outcomes), she forms low aspirations, which lead to low investment, which reproduces the traditional-sector allocation, which keeps W^I block-diagonal. The fixed point is Furtado's "autonomous process of underdevelopment."

Relaxed in: Chapters 9 (where preferences come from), 10 (the formation model), 11 (the Ψ ratio), 12 (welfare under endogenous preferences).

8 The Assumptions Together

Visualization:

Six assumptions, one heatmap



$W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$

A1: $W^U = 0$ (no off-diagonal in utility)
A2: W^P exogenous (technology, not choice)
A3: W^I absent (no belief channel)
A4: Uniform blue (anonymous aggregation)
A5: $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$ (equal power)
A6: θ_i fixed (preferences exogenous)

The uniform-blue heatmap is the visual signature of all six assumptions holding simultaneously. Every departure from uniform blue — a bright column, a dark off-diagonal region, a block structure — is a violation of at least one assumption.

The six assumptions are not independent. They form a logical hierarchy:

Proposition 8.1 (The Hierarchy of Assumptions). 1. *A1–A3 are about the **existence** of the three networks. The standard model denies that W^U matters, takes W^P as exogenous, and ignores W^I entirely.*

2. *A4 is about the **symmetry** of the market mechanism. Given that W^U and W^I are absent, the remaining interaction (through prices) is assumed to be anonymous and equal.*

3. *A5 is a **consequence** of A1–A4: if the only interaction is through the symmetric market, then the composite W is doubly stochastic and* $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$.

4. *A6 is the **deepest** assumption: it says the structure cannot change preferences. Without A6, even a correct diagnosis of the other five becomes dynamically unstable — the preferences used to evaluate welfare are themselves produced by the network.*

Key Structural Insight

The six assumptions can be summarized in one sentence: *the standard model assumes that social structure is irrelevant for economic outcomes.*
$W^U = 0$: social bonds don't affect utility. W^P exogenous: production structure is engineering, not social choice. W^I absent: information doesn't flow through social channels. $W^{\text{market}} = \frac{1}{N}\mathbf{1}\mathbf{1}'$: the market treats everyone the same. $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$: nobody has structural power. θ fixed: the structure doesn't shape who you become.
Structural economics is the systematic relaxation of this sentence.

9 The Structural Program

Part II of this book introduces the three networks (W^U, W^P, W^I) as distinct objects, derives power from the Perron eigenvector, and computes the bargaining equilibrium — relaxing A1, A3, A4, and A5.

Part III relaxes A6 — the formation model, where W^I shapes preferences.

Part IV studies dynamics — how the networks evolve and reproduce, relaxing A2 (production structure becomes endogenous).

Parts V–VII apply the framework to three scales (global, national, individual), develop measurement tools, and analyze structural change.

Assumption	Standard model	Structural economics	Chapters
A1 (W^U)	$= 0$	General W^U	5, 7, 12
A2 (W^P)	Exogenous	Endogenous, policy variable	3, 13, 15
A3 (W^I)	Absent	Drives beliefs and aspirations	5, 8, 9–12
A4 (Market)	$\frac{1}{N}\mathbf{1}\mathbf{1}'$	Asymmetric, concentrated	6, 7
A5 (Power)	$\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$	$\boldsymbol{\pi}(W)$, computed	6, 7, 16–19
A6 (Preferences)	Fixed θ	$\theta_{t+1} = f(W^I, \theta_t)$	9–12

Each row of this table is a chapter (or group of chapters). Each cell is a research program. The table is the book.

10 How Far Is the Real World from W^{CE}?

A natural question: are we tilting at windmills? How badly do the assumptions fail in practice?

Proposition 10.1 (Empirical Departures)**.** *For each assumption, we can define a scalar measure of departure and estimate it from data:*

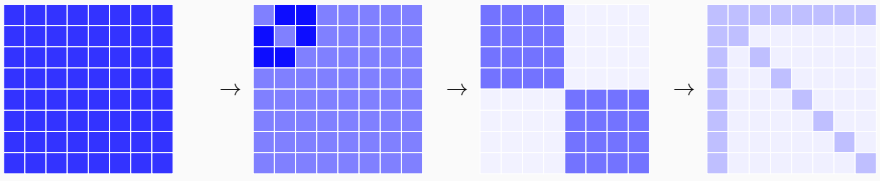
	Departure measure	***Data source***	***Typical value***
A1	$\gamma_U \|W^U\|_F$ *(externality intensity)*	*Social surveys*	0.1–0.5
A2	$\Delta A / A$ *(I-O change per decade)*	*I-O time series*	5–20%
A3	$\mathrm{Var}(\pi_i^I)$ *(epistemic power concentration)*	*Media data*	0.01–0.10
A4	$\Delta(W^{market})$ *(column-sum asymmetry)*	*Market shares*	0.1–0.8
A5	$\mathrm{Var}(\pi_i)$ *(power asymmetry)*	*Composite W*	0.001–0.05
A6	Ψ *ratio (formation vs. enforcement)*	*Cohort-year data*	0.5–4.0

The departures are not small. In Bolivia's domestic production network, the column-sum asymmetry $\Delta(W^P)$ is approximately 0.4 (Chapter 3). The Ψ ratio for female labor force participation in the US is approximately 2.1 (Chapter 11). The epistemic power concentration of global financial media gives $\mathrm{Var}(\pi_i^I) \approx 0.03$ for a 20-country sample.

The standard model is a useful benchmark — like Euclidean geometry in a curved universe. But the curvature is measurable, and in development economics it is large.

Visualization:

From W^{CE} to the real world: five departures



Progressive departure from the competitive benchmark. Each step adds a structural feature that the standard model assumes away. The real world is on the right — and further right than the last panel shows.

11 Summary

1. The standard competitive model makes six assumptions, all expressible as properties of the weight matrix: $W^U = 0$, W^P exogenous, W^I absent, $W^{\text{market}} = \frac{1}{N}\mathbf{1}\mathbf{1}'$, $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$, and θ fixed.

2. The assumptions form a hierarchy: A1–A3 deny the existence of three networks; A4 asserts symmetry of the remaining interaction; A5 follows as a consequence; A6 freezes the structure by fixing preferences.

3. Each assumption has a measurable departure: externality intensity, I-O change rates, epistemic power concentration, column-sum asymmetry, power variance, and the Ψ ratio. In developing economies, the departures are large.

4. Structural economics is the systematic relaxation of these six assumptions: Parts II–VII develop general W^U, endogenous W^P, general W^I, asymmetric power, and endogenous preferences.

5. The standard model is not wrong. It is the special case $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$ with θ fixed. This book is the general case.

12 Exercises

4.1 Classifying failures. For each scenario, identify which of A1–A6 is violated and which is the most important:

(a) A landlord and three tenant farmers negotiate sharecropping rates in a village with no outside labor market.

(b) A social media platform shows users algorithmically selected content that shifts purchasing behavior.

(c) Bolivia's gas revenues are redistributed through transfers, but the gas sector remains an enclave.

(d) A girl in Potosí with a smartphone develops aspirations her mother never had.

(e) Two ethnic groups in the same city trade goods but do not intermarry, share schools, or attend the same churches.

4.2 Measuring departure. For a 3×3 row-stochastic matrix $W = \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.1 & 0.7 & 0.2 \\ 0.2 & 0.1 & 0.7 \end{pmatrix}$:

(a) Compute column sums. Is W doubly stochastic?

(b) Compute $\Delta(W) = \|\mathbf{1}'W - \mathbf{1}'\|_2$.

(c) Compute the Perron eigenvector $\boldsymbol{\pi}$ and $\mathrm{Var}(\pi_i)$.

(d) If this were Bolivia's composite W (with $N = 3$ sectors), which assumption is most severely violated?

4.3 The hierarchy in action. Construct a 4×4 economy where:

(a) A1 fails but A4 holds (utility externalities exist, but the market interaction is still symmetric). Is $\boldsymbol{\pi}$ uniform? (Hint: the composite W includes W^U.)

(b) A4 fails but A1 holds (market power exists, no utility externalities). Is $\boldsymbol{\pi}$ uniform?

(c) Both A1 and A4 fail simultaneously. Compute $\boldsymbol{\pi}$. Is the welfare loss larger than the sum of the individual losses? (This tests whether the interaction term L_{int} is positive.)

4.4 Bolivia's six assumptions. Using the two-sector Bolivia model from Chapter 3 (W^P with Gas and Traditional):

(a) Assess each of A1–A6 for Bolivia. Which hold approximately? Which fail clearly?

(b) For each failing assumption, estimate the departure measure from publicly available data (INE, CEPAL).

(c) Rank the six departures from largest to smallest. Which structural feature of Bolivia is the most severe deviation from the competitive benchmark?

4.5 Sunkel's five processes. Sunkel (1971) argued that development, underdevelopment, dependency, marginalization, and spatial inequality are "diverse manifestations of a single process." Formalize this: define each of the five processes as a property of the weight matrix W or its components (W^U, W^P, W^I). What is the "single process" in matrix terms?

4.6 A6 is the deepest. Consider two economists:

(a) Economist A relaxes A1–A5 but keeps A6 (preferences are fixed). She finds the optimal policy: a transfer T^* that maximizes welfare given the structural power $\boldsymbol{\pi}(W)$.

(b) Economist B relaxes all six, including A6. She finds that the policy changes W^I, which changes θ, which changes the welfare function itself.

(c) Show that Economist A's policy can be welfare-*reducing* under Economist B's model. (Hint: the transfer may reinforce the information structure that produces low aspirations.)

(d) What does this imply about the "growth first, redistribute later" consensus?

4.7 The spectral gap of the assumptions. For $N = 8$, construct a sequence of matrices $W^{(k)}$ for $k = 0, 1, \ldots, 5$ where $W^{(0)} = \frac{1}{8}\mathbf{1}\mathbf{1}'$ and each successive matrix adds one structural feature (a cluster in W^U, a block in W^P, a hub in W^I, etc.). Compute $\Gamma^{(k)}$ and $L^{(k)}_{\text{power}}$ for each. Plot both as functions of k. Does the welfare loss grow linearly, convexly, or concavely in the number of structural features?

Mathematical Appendix

Appendix 4A: Composite Matrices and Their Perron Eigenvectors

The composite. Given three nonnegative matrices W^U, W^P, W^I (each $N \times N$) and weights $\alpha_U + \alpha_P + \alpha_I = 1$ with $\alpha_k \geq 0$:

$$W = \alpha_U W^U + \alpha_P W^P + \alpha_I W^I. \tag{7}$$

If each component is row-stochastic, so is W: $W\mathbf{1} = \alpha_U W^U\mathbf{1} + \alpha_P W^P\mathbf{1} + \alpha_I W^I\mathbf{1} = \alpha_U\mathbf{1} + \alpha_P\mathbf{1} + \alpha_I\mathbf{1} = \mathbf{1}$.

Perron eigenvector of the composite. In general, $\boldsymbol{\pi}(W)$ is NOT a convex combination of $\boldsymbol{\pi}(W^U)$, $\boldsymbol{\pi}(W^P)$, $\boldsymbol{\pi}(W^I)$. The eigenvectors of a sum of matrices are not the sums of the eigenvectors. This is a nonlinearity that makes the composite power vector genuinely different from any single-network power vector.

Special cases. If all three components have the same eigenvector ($\boldsymbol{\pi}^U = \boldsymbol{\pi}^P = \boldsymbol{\pi}^I = \boldsymbol{\pi}$), then $\boldsymbol{\pi}(W) = \boldsymbol{\pi}$ — the composite inherits the common power structure. This is the "aligned networks" case from Chapter 5: all three networks agree on who is powerful. In this case, power is reinforced across channels.

If the components have different eigenvectors, the composite $\boldsymbol{\pi}(W)$ is a nonlinear blend that depends on the α_k weights. Changing α_P (the importance of production linkages) relative to α_I (the importance of information) can shift $\boldsymbol{\pi}$ discontinuously when the leading eigenvectors of W^P and W^I point in very different directions. This is a structural phase transition: the economy's power structure flips when one network channel comes to dominate.

Computing $\boldsymbol{\pi}(W)$. Form $W = \alpha_U W^U + \alpha_P W^P + \alpha_I W^I$, then apply power iteration (Appendix 6A).

Structuralist Case Study: Sunkel and Paz's Systematic Critique

In 1970, Osvaldo Sunkel and Pedro Paz published *El subdesarrollo latinoamericano y la teoría del desarrollo* through ILPES/Siglo XXI — a 385-page treatise that remains the most comprehensive systematic critique of orthodox development theory from the structuralist perspective. Where Prebisch (1949) was a manifesto and Furtado (1961) was a theoretical essay, Sunkel and Paz was a textbook: methodical, exhaustive, and organized as a point-by-point examination of what mainstream economics assumes and why those assumptions fail in the periphery.

Their argument maps directly onto the six assumptions of this chapter.

Part 1: Mainstream theory assumes a closed, self-contained economy. Sunkel and Paz began by noting that the neoclassical model analyzes an economy in isolation — abstracting from its insertion in the global system. They argued that this is not an innocuous simplification but a fundamental distortion: the peripheral economy's structure is shaped by its relationship to the center. In our notation: the standard model treats W^P as a domestic object, ignoring the global production network in which the domestic matrix is embedded. The domestic W^P is not independent of the global W^P — the off-diagonal entries linking the domestic economy to the world are among the most important entries in the matrix.

Part 2: Five processes, one process. The book's most original contribution was the claim that development, underdevelopment, dependency, marginalization, and spatial inequality are not five separate phenomena but "diverse manifestations of a single process" — the simultaneous transnational integration and national disintegration produced by global capitalism. In matrix terms: the global weight matrix W_{global} is being rewired in a way that increases cross-country connectivity among elite nodes (transnational integration) while decreasing within-country connectivity between elite and non-elite nodes (national disintegration). The five "processes" are five observables of one matrix transformation.

Part 3: The theory is part of the structure. Sunkel and Paz argued — anticipating Quijano by three decades — that the economic theories taught at the center and exported to the periphery are not neutral analytical tools but products of the center's structural position. The theory of comparative advantage, the Heckscher-Ohlin model, the welfare theorems — these are theorems about $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$, which is the center's idealization of a world that serves the center's interests. Teaching these theorems in La Paz without noting that they assume a specific W is an act of epistemic power: it naturalizes the assumption that makes the welfare theorems hold, thereby legitimating the structure that the theorems describe.

Sunkel & Paz's argument	**This chapter's assumption**	**Formal object**
Economy not self-contained	A2: W^P treated as domestic	Global embedding of W^P
Five processes, one process	A1–A5 fail simultaneously	$W \neq \frac{1}{N}\mathbf{1}\mathbf{1}'$, all channels
National disintegration	A4: market not symmetric	Block-diagonal domestic W
Transnational integration	A2: W^P changes endogenously	Elite cross-border linkages
Theory is part of structure	A6: preferences exogenous	W^I shapes beliefs about theory

Sunkel and Paz's book is the structuralist critique organized as a textbook. This

chapter is the same critique organized as a table of matrix properties. The mathematics adds precision; the diagnosis was always theirs.

Further Reading

The welfare theorems and their assumptions: MWG (1995), Chapter 16; Debreu (1959); Arrow (1951). For the assumptions' failure in developing economies: Stiglitz (1989), "Markets, Market Failures, and Development," *AER P&P*.

Endogenous production networks: Acemoglu et al. (2012); Baqaee and Farhi (2019); Hidalgo et al. (2007), "The Product Space Conditions the Development of Nations," *Science*.

Information networks and belief dynamics: DeGroot (1974); Golub and Jackson (2010); Acemoglu et al. (2011), "Bayesian Learning in Social Networks," *ReStud*.

Endogenous preferences: Bowles (1998), "Endogenous Preferences: The Cultural Consequences of Markets and Other Economic Institutions," *JEL*; Bisin and Verdier (2001), "The Economics of Cultural Transmission and the Dynamics of Preferences," *JET*.

The structuralist critique: Sunkel and Paz (1970), *El subdesarrollo latinoamericano y la teoría del desarrollo*, México: Siglo XXI / ILPES (the definitive systematic treatment; the full text in Spanish is available through CEPAL's digital repository). Sunkel (1971), "Capitalismo transnacional y desintegración nacional en América Latina," *Estudios Internacionales* 4(16), pp. 3–61.

Part II

The Three Networks

Chapter 5

The Three Networks

Where the Structure Enters the Model

> *"Por dependencia entendemos una situación en la cual la economía de ciertos países está condicionada por el desarrollo y la expansión de otra economía a la cual la primera está sometida."*
>
> ["By dependence we mean a situation in which the economy of certain countries is conditioned by the development and expansion of another economy to which the former is subjected."]
> — Theotônio dos Santos, *La estructura de la dependencia* (1970)

> *"Tell me who you walk with, and I'll tell you who you are."*
> — Proverb, every language that has one

> *"Dime con quién andas y te diré tu vector de Perron."*
> — This book

Part I showed that the standard competitive model assumes a specific interaction matrix — $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$ — and that this assumption is what makes the welfare theorems hold. Part II replaces that matrix with the one the world actually has.

This chapter introduces the three weight matrices that organize economic life:

1. W^U: the **utility network**. Who affects your wellbeing directly?
2. W^P: the **production network**. Who depends on whom for inputs and outputs?
3. W^I: the **information network**. Whose signals shape your beliefs and desires?

Every economic interaction passes through at least one of these networks. Many pass through all three simultaneously. The salteña vendor in La Paz is embedded in a production network (she buys flour from the miller, sells to the office worker), a utility network (her comadre's success makes her happy, her competitor's success makes her anxious), and an information network (she watches what other vendors charge, listens to the radio about the dollar exchange rate, hears from her cousin that there is work in El Alto). Her economic behavior — what she produces, how she prices, whether she invests, what she wants for her daughter — is determined not by her individual utility function in isolation but by her position in all three networks simultaneously.

The standard model collapses all three to $\frac{1}{N}\mathbf{1}\mathbf{1}'$ or ignores them entirely. This chapter makes them visible, distinct, and formal.

1 Before the Math: Three Stories

1.1 The Utility Network: Keeping Up and Falling Behind

María sells api on Calle Comercio. Her stall is between Doña Rosa's stall (empanadas) and Don Carlos's stall (newspapers). María has known Rosa for twenty years. They share a comadrazgo bond: Rosa is godmother to María's youngest daughter. When Rosa's sales are good, María feels genuinely happy for her. When Rosa's sales collapse — say, because a new empanada chain opens nearby — María worries, helps Rosa with a small loan, and feels the loss almost as her own.

Don Carlos, on the other hand, is just a neighbor. María doesn't dislike him, but his success or failure does not move her emotionally.

Now: how much of Rosa's consumption enters María's utility function? Quite a lot. How much of Carlos's? Almost nothing. How much of a stranger's in Santa Cruz? Zero.

This is the utility network W^U. Entry W^U_{ij} measures how much agent j's consumption or wellbeing directly affects agent i's utility. For María: $W^U_{\text{María, Rosa}} \approx 0.3$ (strong positive externality), $W^U_{\text{María, Carlos}} \approx 0.02$ (negligible), $W^U_{\text{María, stranger}} = 0$.

In the standard model, $W^U = I$: your utility depends only on your own consumption. No externalities, no altruism, no envy, no keeping-up-with-the-Joneses. The salteña vendor is an island.

1.2 The Production Network: The Chain from Gas Well to Quinoa Bowl

The gas well in Tarija produces natural gas. The gas goes to the processing plant (run by YPFB), which separates it into components. One component goes to the thermoelectric plant in Santa Cruz, which generates electricity. The electricity powers the flour mill in Cochabamba. The flour goes to the bakery. The bakery sells bread to the miner in Potosí. The miner extracts tin. The tin is exported. The export revenue enters the treasury. The treasury funds the school where María's daughter studies.

Each link in this chain is an entry in W^P. The gas well's output is an input to the processing plant: $W^P_{\text{plant, well}} > 0$. The processing plant's output is an input to the thermoelectric plant: $W^P_{\text{electric, plant}} > 0$. And so on. The chain from gas to quinoa bowl passes through six intermediate steps.

Now: how many of these links actually exist in Bolivia? The gas processing plant imports most of its equipment from Brazil. The thermoelectric plant was built by a Spanish firm. The flour mill buys Argentine wheat because Bolivian wheat production collapsed in the 1990s. At each step, the domestic chain breaks, and the production link points outward — to the global economy rather than inward to the domestic economy.

This is Pinto's structural heterogeneity as a property of W^P: the off-diagonal entries between the gas block and the traditional block are tiny. The chain from gas well to quinoa bowl is mostly imaginary.

1.3 The Information Network: What the Girl in Potosí Sees

María's daughter, Luisa, is fifteen. She lives in Potosí and has a phone. On her phone she sees: TikTok videos from La Paz showing university students, Instagram posts from her cousin in Buenos Aires who is studying engineering, WhatsApp messages from her school friends who are all planning to finish secondary school and then — what? Work in the market like their mothers? Go to the normal school in Sucre? Try for the public university in Oruro?

Twenty years ago, Luisa would have seen only what Potosí showed her: market vendors, miners, teachers. Her information network W^I would have been local, dense within the traditional sector, disconnected from the modern sector. The signals she received about what life could look like — the perceived returns to different life paths — would have been calibrated entirely by her immediate environment.

The phone changes W^I. It adds long-range connections. Luisa now receives signals from Buenos Aires (her cousin's engineering career), from La Paz (the uni-

versity), from TikTok (a globally curated feed of aspirational content). These new entries in $W^I_{\text{Luisa},j}$ — nonzero weights on agents j who are outside her geographic and social cluster — change the information she uses to form beliefs about returns. They change her λ (the signal discount from the formation model in Chapter 10). They change what she wants.

This is W^I: the network through which signals about the world reach you, filtered by who you pay attention to, who you trust, who you can observe. In the standard model, W^I does not exist — preferences are given, beliefs are rational, information is symmetric. In the structural model, W^I is the mechanism through which the structure reproduces itself or, occasionally, breaks.

2 The Utility Network W^U

2.1 Utility with Externalities

In Chapter 1, agent i's utility depended only on her own consumption: $U_i(x_i)$. We now let it depend on everyone else's consumption too, weighted by the utility network.

Definition 2.1 (Utility with Social Externalities)**.** *Agent i's utility is:*

$$V_i(x_1, \ldots, x_N) = U_i(x_i) + \gamma_U \sum_{j \neq i} W^U_{ij}\, U_j(x_j), \tag{1}$$

where:

- $U_i(x_i)$ *is agent i's private utility from her own consumption.*
- $W^U_{ij} \geq 0$ *is the weight that agent i places on agent j's utility. This is the* ***social externality****: how much j's wellbeing directly affects i.*
- $\gamma_U \geq 0$ *is the overall strength of social externalities.*

In matrix form, stacking the N agents:

$$\mathbf{V} = \mathbf{U} + \gamma_U W^U \mathbf{U} = (I + \gamma_U W^U)\mathbf{U}, \tag{2}$$

where $\mathbf{U} = (U_1(x_1), \ldots, U_N(x_N))'$ is the vector of private utilities and $\mathbf{V}$ is the vector of social utilities.

Intuition: What W^U does to utility

Without W^U (the standard model), your happiness depends only on your own consumption. With W^U, other people's happiness enters directly. If W^U_{ij} is large and positive, you are *altruistic* toward j: their consumption makes you happier. If we allowed $W^U_{ij} < 0$ (which some models do), you would be *envious*: their consumption makes you worse off.

The parameter γ_U controls how much you care about others overall. When $\gamma_U = 0$, we are back to the standard model regardless of W^U. When $\gamma_U > 0$, the structure of W^U matters: *who* you care about — not just *how much* — shapes the equilibrium.

Example 2.2 (Three Vendors on Calle Comercio)**.** *María ($i = 1$), Rosa ($j = 2$), Carlos ($j = 3$). Quadratic private utility: $U_i = x_i - \frac{1}{2}x_i^2$.*

$$W^U = \begin{pmatrix} 0 & 0.30 & 0.02 \\ 0.25 & 0 & 0.05 \\ 0.02 & 0.05 & 0 \end{pmatrix}. \tag{3}$$

María places weight 0.30 on Rosa (comadre bond) and only 0.02 on Carlos. Rosa places 0.25 on María and 0.05 on Carlos. Carlos barely cares about either.

With $\gamma_U = 0.5$ and allocations $x_1 = 0.6$, $x_2 = 0.4$, $x_3 = 0.8$:

$$\begin{aligned} V_1 &= U_1(0.6) + 0.5[0.30 \cdot U_2(0.4) + 0.02 \cdot U_3(0.8)] && (4)\\ &= 0.42 + 0.5[0.30 \times 0.32 + 0.02 \times 0.48] && (5)\\ &= 0.42 + 0.5[0.096 + 0.0096] = 0.42 + 0.053 = 0.473. && (6) \end{aligned}$$

María's social utility (0.473) is higher than her private utility (0.42) because her comadre Rosa exists and has positive consumption. The externality adds 12.6% to her well-being. This is small in levels but enormous in marginal terms: María's willingness to help Rosa (e.g., with a loan during a bad week) is proportional to $\gamma_U W^U_{12} = 0.15$, which is large enough to generate substantial mutual aid.

2.2 What W^U Is Not

Three common confusions:

W^U is not the market. The market is the price mechanism, which aggregates demands anonymously. W^U is the direct externality channel: j's consumption enters i's utility *without* passing through the price. Both channels operate simultaneously. Chapter 7 embeds both in general equilibrium.

W^U is not altruism alone. Altruism is $W^U_{ij} > 0$: you benefit from j's consumption. But W^U also captures status competition (j's consumption raises the bar for i), reference-dependent preferences (i evaluates her consumption relative to j's), and network externalities (j's adoption of a technology raises i's benefit from the same technology). All of these are direct externalities mediated by a weight matrix.

W^U is not W^I. The utility network says whose consumption affects your happiness. The information network says whose signals affect your beliefs. María cares about Rosa's wellbeing (W^U) AND learns about prices from Rosa's behavior (W^I). These are different channels with different matrices. A dictator might have high W^I centrality (everyone watches what the dictator does) but low W^U weight (nobody cares about the dictator's personal happiness). Chapter 8 develops W^I in detail.

2.3 Properties of W^U

Proposition 2.3 (Basic Properties of the Utility Network)**.** *For the utility externality model (1):*

1. ***Diagonal is zero**: $W^U_{ii} = 0$ by convention (own utility is captured by U_i, not by the externality term).*

2. ***Row sums need not equal 1**: the total externality weight $\sum_j W^U_{ij}$ varies across agents. A "social" agent (María) has high row sum; an "isolated" agent has low row sum.*

3. ***Not necessarily symmetric**: $W^U_{ij} \neq W^U_{ji}$ in general. María may care more about Rosa than Rosa cares about María.*

4. ***The standard model is** $W^U = 0$: no externalities. Equivalently, $\gamma_U = 0$.*

Key Structural Insight

The distinction between $W^U = 0$ (standard model) and $W^U = \frac{1}{N}\mathbf{1}\mathbf{1}' - I$ (anonymous equal externalities) matters. The first says nobody's consumption affects anyone else. The second says *everyone's* consumption affects everyone else *equally*. Both are extreme; neither is realistic. The real W^U is sparse (most pairs have zero or near-zero weight), asymmetric (compadrazgo bonds are not symmetric), and clustered (your utility network is local: family, friends, neighbors, compadres).

3 The Production Network W^P

3.1 From Chapter 1 to Chapter 5

We have already met W^P — it is the input-output matrix A from Chapters 1 and 3, now given its structural name. Chapter 1 introduced it as a technological object: how much of good j is needed to produce one unit of good i. Chapter 3 embedded it in a growth model and showed that the block-diagonal structure of W^P determines whether growth propagates.

Here we develop W^P as one of three networks, clarify what it shares with W^U and W^I and how it differs, and show how all three interact in the same economy.

Definition 3.1 (Production Network)**.** *The **production network** W^P is the $N \times N$ matrix where $W^P_{ij} \geq 0$ is the production externality of agent (or sector) j on agent i: how much j's output contributes to i's production.*

In the inter-sectoral interpretation, W^P_{ij} is the technical coefficient: units of good j required per unit of good i's output.

In the inter-agent interpretation, W^P_{ij} captures production spillovers: how much j's activity increases i's productivity. This includes knowledge spillovers, shared infrastructure, supply chain dependencies, and agglomeration effects.

The output equation from Chapter 3:

$$\mathbf{Y} = \mathbf{f}(\mathbf{K}, \mathbf{L}) + \gamma_P W^P \mathbf{Y} \qquad \Rightarrow \qquad \mathbf{Y} = (I - \gamma_P W^P)^{-1} \mathbf{f}(\mathbf{K}, \mathbf{L}), \tag{7}$$

where $(I - \gamma_P W^P)^{-1}$ is the Leontief inverse and $\gamma_P > 0$ scales the strength of inter-sectoral linkages.

3.2 W^P vs. W^U: Different Channels, Different Matrices

Example 3.2 (Why They Differ: The Gas Engineer)**.** *Consider Patricia, a petroleum engineer working for YPFB in Tarija. She interacts with two groups: her coworkers at the gas plant (production network) and her family and friends in Cochabamba (utility network).*

*Patricia's **production network** W^P: high weights on the gas plant operators, the pipeline maintenance crew, the Brazilian equipment supplier, the Argentine gas buyer. Low or zero weights on the quinoa farmer in Oruro, the api vendor in La Paz, her mother in Cochabamba. The production network is organized by the technology of gas extraction, not by social bonds.*

*Patricia's **utility network** W^U: high weights on her mother, her partner, her children, her close friends. Moderate weights on her coworkers (she likes some of them). Zero*

weights on the Argentine gas buyer (she has never met him). The utility network is organized by affection and social proximity, not by technological linkages.

The two matrices have almost disjoint support: the people who matter for Patricia's production barely overlap with the people who matter for Patricia's happiness. This is typical. W^P *and* W^U *are not the same matrix, and confusing them leads to analytical errors.*

The most common error: assuming that strengthening production linkages (W^P*) automatically strengthens social bonds (*W^U*) or changes aspirations (*W^I*). It does not. Bolivia can build a gas-to-fertilizer plant that links the gas sector to agriculture (*W^P *changes) without affecting whether the quinoa farmer's daughter aspires to be an engineer (*W^I *unchanged).*

Property	W^U (Utility)	W^P (Production)
What it captures	Who affects your happiness	Who affects your output
Channel	Direct externality in U	Intermediate inputs / spillovers
Typical structure	Sparse, clustered, local	Sparser, sectoral, sometimes global
Symmetry	Often asymmetric	Often asymmetric
Diagonal	Zero by convention	Positive (own-sector recycling)
Structural amplifier	$(I + \gamma_U W^U)$	$(I - \gamma_P W^P)^{-1}$ (Leontief)
Standard model	$W^U = 0$	Varies (I-O tables exist)

4 The Information Network W^I

4.1 The Missing Network

W^U has been studied (under the name "social preferences" or "other-regarding preferences") since at least Becker (1974). W^P has been studied (under the name "input-output analysis") since Leontief (1941). But W^I — the network through which information, signals, and norms flow, shaping beliefs and eventually preferences — is the network that standard economics has most systematically ignored.

It exists. It is not W^U and it is not W^P. And it is arguably the most important of the three.

Definition 4.1 (Information Network)**.** *The **information network** W^I is the $N \times N$ row-stochastic matrix where $W^I_{ij} \geq 0$ is the weight that agent i places on agent j's signals when forming beliefs about the world. Row-stochasticity ($\sum_j W^I_{ij} = 1$ for all i) means that each agent's attention is a probability distribution over other agents: you allocate 100% of your attention, and W^I describes how.*

Intuition: What W^I does

Think of W^I as your *attention allocation*. Each day, you observe the world — but not all of it. You observe a weighted sample of other people's behaviors, outcomes, and signals. The weights are W^I_{ij}. Your neighbor gets high weight (you see her every day). The president gets moderate weight (you see him on TV). A farmer in Beni gets zero weight (you don't know he exists).

The information you receive determines your beliefs. Your beliefs determine your choices. So W^I — who you pay attention to — shapes your economic behavior even though it never enters your utility function directly.

In the DeGroot learning model (developed in Chapter 8), repeated application of W^I drives beliefs toward a consensus determined by the left Perron eigenvector of W^I. The agents with the highest Perron weight are the ones whose initial beliefs dominate the long-run consensus. This is **epistemic power**: the ability to shape what others believe, not through coercion but through structural position in the information network.

4.2 DeGroot Learning: A Preview

Suppose each agent starts with an initial belief $b_i^{(0)}$ about some quantity (the return on education, the value of saving, the probability of success in the modern sector). At each period, agents update their beliefs by taking a weighted average of their neighbors' beliefs:

$$\mathbf{b}^{(t+1)} = W^I \mathbf{b}^{(t)}. \tag{8}$$

This is the DeGroot learning model. It is extremely simple: just multiply the belief vector by W^I at each step. Yet its implications are profound.

Proposition 4.2 (DeGroot Convergence)**.** *If W^I is row-stochastic, aperiodic, and strongly connected (every agent can reach every other agent through a chain of positive weights),*

then:

$$\lim_{t\to\infty}(W^I)^t = \mathbf{1}\boldsymbol{\pi}', \tag{9}$$

where $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_N)'$ *is the* ***left Perron eigenvector*** *of* W^I*: the unique vector satisfying* $\boldsymbol{\pi}' W^I = \boldsymbol{\pi}'$ *with* $\sum_i \pi_i = 1$.

All agents converge to the same belief:

$$b_i^{(\infty)} = \boldsymbol{\pi}'\mathbf{b}^{(0)} = \sum_{j=1}^{N} \pi_j b_j^{(0)} \quad \textit{for all } i. \tag{10}$$

The long-run belief is a weighted average of the initial beliefs, with weights given by $\boldsymbol{\pi}$.

Intuition: What this says in words

Everyone eventually agrees — but they agree on a *weighted average* of initial beliefs, not the simple average. The weights π_j are the Perron eigenvector entries: agents with high π_j have more influence on the long-run consensus. This is not because they shout louder or coerce others. It is because the network structure channels information flow through them. They are listened to more, directly or indirectly, by more people.

The Perron eigenvector $\boldsymbol{\pi}$ is the **structural power** of each agent in the information network. It is not a choice or an attribute of the agent. It is a property of the network topology — a property of W^I.

Example 4.3 (Luisa's Information Network)**.** *Luisa in Potosí,* $N = 5$ *reference agents: Mother (*$j = 1$*), School friend (*$j = 2$*), Cousin in Buenos Aires (*$j = 3$*), TikTok feed (*$j = 4$*), Teacher (*$j = 5$*).*

$$W^I_{Luisa} = \begin{pmatrix} 0.35 & 0.25 & 0.15 & 0.15 & 0.10 \end{pmatrix}. \tag{11}$$

Luisa allocates 35% of her attention to her mother (she sees her every day, trusts her judgment), 25% to her school friend (constant contact), 15% to her cousin in Buenos Aires (WhatsApp, intermittent but aspirational), 15% to TikTok (algorithmically curated, diverse signals), and 10% to her teacher.

Twenty years ago, before the phone: $W^I_{Luisa,\,1990} = (0.50, 0.30, 0, 0, 0.20)$*. Mother and friend dominated. The cousin in Buenos Aires had zero weight (no WhatsApp). TikTok did not exist. The information network was local: Potosí signals only.*

The shift from W^I_{1990} *to* W^I_{2020} *is the single most important structural change in Luisa's life. It exposes her to signals about returns (her cousin's engineering career) that were previously invisible. Whether those signals change her* θ *(patience, investment in education)*

depends on the formation model in Chapter 10. But the precondition — the change in W^I — is what makes formation change possible.

Prebisch Connection: Information as structural power

In the global economy, W^I has a clear center-periphery structure. The center produces the signals that the periphery consumes: financial news from New York, technology standards from Silicon Valley, fashion from Paris, economic policy from Washington. The periphery does not produce signals that the center consumes. Entry $W^I_{\text{Bolivia, USA}}$ is large (Bolivian policymakers watch the Fed). Entry $W^I_{\text{USA, Bolivia}}$ is approximately zero (the Fed does not watch the Banco Central de Bolivia).

The information asymmetry compounds the production asymmetry. Not only does the periphery export primary commodities and import manufactures (W^P); it also exports attention and imports beliefs (W^I). The Perron eigenvector of the global W^I assigns almost all weight to center countries. The long-run consensus about "how economies should be run" converges to the initial beliefs of the center — which is how the Washington Consensus becomes consensus even in countries where its prescriptions are structurally inappropriate.

Prebisch saw this as "ideological dependency." We can now measure it: it is the Perron eigenvector of W^I_{global}.

5 The Three Networks Together

5.1 The Full Model

We now write the economy with all three networks active simultaneously.

Definition 5.1 (The Three-Network Economy)**.** *An economy with N agents, L goods, and three networks (W^U, W^P, W^I) is defined by:*

***Preferences** (shaped by W^U and, indirectly, by W^I):*

$$V_i(\mathbf{x}) = U_i(x_i; \theta_i) + \gamma_U \sum_{j \neq i} W^U_{ij} U_j(x_j; \theta_j), \tag{12}$$

where θ_i is agent i's preference parameter, which in the formation model (Chapter 10) evolves according to W^I.

***Production** (shaped by W^P):*

$$\mathbf{Y} = (I - \gamma_P W^P)^{-1} \mathbf{f}(\mathbf{K}, \mathbf{L}). \tag{13}$$

Beliefs *(shaped by W^I):*

$$\mathbf{b}^{(t+1)} = W^I \mathbf{b}^{(t)}. \tag{14}$$

Market clearing: $\sum_i x_{i\ell}^* = \bar{\omega}_\ell$ *for each good* ℓ.

Key Structural Insight

The three networks enter through three different channels:

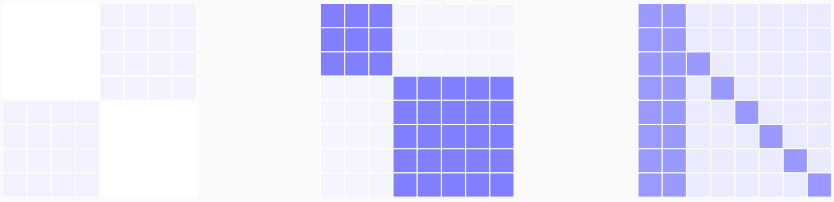
Network	Channel	Object modified	Structural amplifier
W^U	Utility	What you want	$(I + \gamma_U W^U)$
W^P	Production	What you can make	$(I - \gamma_P W^P)^{-1}$
W^I	Information	What you believe	$(W^I)^t \to \mathbf{1}\boldsymbol{\pi}'$

The standard competitive model sets $W^U = 0$, takes W^P from input-output data (the one network economists have always measured), and ignores W^I entirely. This book studies general (W^U, W^P, W^I).

Note the three amplifiers. The utility amplifier $(I + \gamma_U W^U)$ is a small perturbation of the identity: externalities add a correction to private utility. The production amplifier $(I - \gamma_P W^P)^{-1}$ is the Leontief inverse: potentially large, summing infinite rounds of indirect effects. The information amplifier $(W^I)^t$ converges to a rank-1 matrix: in the long run, everyone agrees, and the agreement reflects the Perron eigenvector. The three amplifiers operate on different timescales: utility effects are instantaneous, production effects play out over business cycles, and information effects play out over generations.

Visualization:

Three networks, one economy, three heatmaps



W^U **(utility)**
Who you care about

W^P **(production)**
Who depends on your output

W^I **(information)**
Who you listen to

Same eight agents, three different networks. W^U is clustered (two social groups, caring mostly within). W^P is block-diagonal (enclave: three agents in the modern sector, five in the traditional). W^I is star-shaped (two media sources dominate everyone's attention). The three heatmaps have different topologies — the agent who is central in one may be peripheral in another. The composite $W = \alpha_U W^U + \alpha_P W^P + \alpha_I W^I$ blends all three.

5.2 When the Networks Align

In the simplest case, all three networks are the same: $W^U = W^P = W^I = W$. Your friends are your business partners are your information sources. This is the world of a small village: the same 50 people appear in all three matrices.

When the networks align, the analysis simplifies dramatically: there is one weight matrix, one Perron eigenvector, one spectral gap. Power is power across all channels. The salteña vendor who is central in the utility network is also central in the production network and the information network. Life is simple and oppressive.

5.3 When the Networks Diverge

In a complex economy, the three networks diverge. Your production network (the global supply chain) has nothing to do with your utility network (your family in Cochabamba) or your information network (TikTok, the Banco Central, your teacher).

This divergence creates both opportunities and tensions:

Opportunity: You can be peripheral in W^P (a quinoa farmer in Oruro) but central in W^I (a TikTok influencer with 200,000 followers). Information power can compensate for production powerlessness — Luisa's phone gives her access to signals about the modern sector even though her production network is entirely traditional.

Tension: You can be central in W^P (the YPFB gas engineer) but peripheral in W^U (she moved to Tarija for work, leaving her family in Cochabamba). Economic success coexists with social isolation. The production network rewards one location; the utility network rewards another.

Structural trap: When W^P is block-diagonal (Pinto) AND W^I is block-diagonal (traditional-sector people attend only to traditional-sector signals), the two networks reinforce each other. Growth in the modern sector does not propagate through production (W^P disconnected) AND does not propagate through information (W^I disconnected). The trap is double-locked. Breaking it requires changing both matrices — which is why structural policy is harder than it looks.

6 Canonical Topologies

Any of the three networks can have any topology. But certain topologies appear again and again across economic settings. We catalog the most important ones and develop intuition for reading their structural properties.

(Readers should consult the interactive *Seeing the Structure* companion tool, which visualizes all topologies below for $N = 3$ to 100.)

6.1 Complete (Anonymous Symmetric)

$W = \frac{1}{N}\mathbf{1}\mathbf{1}'$. Every entry is $1/N$. Doubly stochastic, symmetric, rank 1, maximal spectral gap $\Gamma = 1$, equal Perron eigenvector $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$.

As W^U: universal, symmetric altruism. You care about everyone equally.

As W^P: every sector uses every other sector's output in equal proportion. No specialization, no comparative advantage, no structural heterogeneity. Unrealistic but useful as a benchmark.

As W^I: you listen to everyone equally. No opinion leaders, no media concentration, no filter bubbles. Information aggregates to the simple average of initial beliefs.

Economics: The welfare theorems hold. The First Welfare Theorem says: if $W^U = 0$ and $W^P = \frac{1}{N}\mathbf{1}\mathbf{1}'$, the competitive equilibrium is Pareto efficient. This is the standard result. It is the structural content of "perfect competition."

6.2 Star (Hub-Spoke)

One hub (h) connected to all spokes; spokes connected only to the hub.

$$W_{ih} \text{ large for all } i \neq h; \qquad W_{ij} \approx 0 \text{ for } i, j \neq h. \tag{15}$$

Spectral gap $\Gamma \approx 0$. Perron eigenvector concentrates on the hub: $\pi_h \gg \pi_i$ for $i \neq h$.

As W^U: everyone cares about the hub (celebrity, patron, chief), nobody cares about each other. Patron-client structures, populist politics.

As W^P: all production passes through a single firm or sector (the platform, the middleman, the colonial trading company). Amazon, the East India Company.

As W^I: one source broadcasts, all others listen. State media, monopoly news, papal authority. The long-run consensus equals the hub's initial belief. Epistemic dictatorship.

6.3 Block-Diagonal (Dual Economy)

Two or more blocks with strong internal links and weak cross-block links.

$$W \approx \begin{pmatrix} W_{11} & \varepsilon \\ \varepsilon' & W_{22} \end{pmatrix}, \quad \|\varepsilon\| \ll \|W_{11}\|, \|W_{22}\|. \tag{16}$$

Small spectral gap $\Gamma \approx |\rho(W_{11}) - \rho(W_{22})|$. Perron eigenvector concentrates on the block with higher spectral radius.

As W^U: two social groups that care about their own members but not about the other group. Ethnic enclaves, caste, class segregation.

As W^P: Pinto's structural heterogeneity. The modern sector and the traditional sector, barely connected. Bolivia's gas and quinoa economies. Growth does not propagate (Chapter 3).

As W^I: two information ecosystems. Rural and urban. Spanish-language and Quechua-language. Fox News and MSNBC. Each group converges to its own consensus internally, but the two groups' consensuses may diverge. This is the formation-level mechanism of structural reproduction: if traditional-sector people attend only to traditional-sector signals, their beliefs about returns are calibrated to the traditional sector, which reproduces traditional-sector behavior, which keeps them in the traditional sector.

6.4 Core-Periphery (Prebisch)

A dense core and a sparse periphery, with asymmetric links: periphery depends on core, core barely depends on periphery.

$$W_{ij} \text{ large for } i \in \text{periphery}, j \in \text{core}; \qquad W_{ij} \approx 0 \text{ for } i \in \text{core}, j \in \text{periphery}. \tag{17}$$

Perron eigenvector assigns high weight to the core and low weight to the periphery: $\pi_{\text{core}} \gg \pi_{\text{periphery}}$.

As W^U: the periphery admires and envies the center; the center is indifferent to the periphery. Colonial relations, celebrity culture, academic prestige hierarchies.

As W^P: Prebisch's center-periphery. The periphery exports raw materials to the center and imports manufactures. The Leontief multiplier is high for the center (column sums of $(I - \gamma W^P)^{-1}$ are large) and low for the periphery. Terms of trade deterioration is a spectral property of this asymmetric W^P (Chapter 16).

As W^I: the periphery absorbs signals from the center (economic theory, cultural norms, consumption patterns, policy advice) but not vice versa. The long-run consensus converges to center beliefs. The Washington Consensus as a DeGroot equilibrium.

6.5 Other Topologies

Several additional topologies appear in applications:

Ring (supply chain): Each agent connects to neighbors. Sequential production. Shocks propagate slowly along the chain. Manufacturing assembly lines, agricultural value chains.

Hierarchy (tree): Top-down connections. Firms, bureaucracies, military, caste. Power decreases monotonically with depth in the tree. Information flows down; surplus flows up.

Small-world: Local clusters plus random long-range shortcuts. Most social networks exhibit this structure (Watts and Strogatz, 1998). High clustering with short path lengths. Ideas diffuse faster than in a ring but less uniformly than in a complete graph.

Scale-free: Few super-connected hubs, many weakly connected nodes. Power-law degree distribution. Wealth, internet, citation networks. Extreme power concentration in the Perron eigenvector.

Random (Erdős-Rényi): Each link exists with probability p. No structure by design. Useful as a null model: any deviation from random IS structure.

7 The Composite Weight Matrix

In many applications, the three networks operate simultaneously and interact. We define the composite.

Definition 7.1 (Composite Weight Matrix)**.** *The **composite weight matrix** is:*

$$W = \alpha_U W^U + \alpha_P W^P + \alpha_I W^I, \quad \alpha_U + \alpha_P + \alpha_I = 1, \tag{18}$$

where $\alpha_U, \alpha_P, \alpha_I \geq 0$ are the relative importance of each channel.

The composite W is the single matrix that summarizes the overall structure of economic interaction. Its Perron eigenvector $\boldsymbol{\pi}(W)$ is the **power vector**: agent i's structural power is π_i, determined jointly by her position in all three networks.

Proposition 7.2 (Power from the Composite)**.** *If W is irreducible and aperiodic, the left Perron eigenvector $\boldsymbol{\pi}$ of the composite W satisfies:*

$$\pi_i = \alpha_U \sum_j \pi_j W^U_{ji} + \alpha_P \sum_j \pi_j W^P_{ji} + \alpha_I \sum_j \pi_j W^I_{ji}. \tag{19}$$

An agent's power is the weighted sum of power directed toward her through all three channels: how much powerful agents care about her (utility channel), how much powerful agents depend on her output (production channel), and how much powerful agents listen to her signals (information channel).

Intuition: What determines power

Your structural power π_i is high if:

1. People who care about you (W^U_{ji} large) are themselves powerful.
2. People who depend on your output (W^P_{ji} large) are themselves powerful.
3. People who listen to your signals (W^I_{ji} large) are themselves powerful.

This is recursive: power begets power, through the Perron eigenvector's self-referential definition. This is the mathematical content of "privilege reproduces itself." It is not a moral judgment. It is a theorem about nonnegative matrices.

Example 7.3 (Power in La Paz)**.** *Consider three agents: a Cabinet Minister ($i = 1$), a Gas Executive ($i = 2$), and a Market Vendor ($i = 3$).*

Utility network (who cares about whom):

$$W^U = \begin{pmatrix} 0 & 0.3 & 0.0 \\ 0.2 & 0 & 0.0 \\ 0.0 & 0.0 & 0 \end{pmatrix}. \tag{20}$$

The Minister and the Executive care about each other. Nobody cares about the Vendor.

Production network (who depends on whose output):

$$W^P = \begin{pmatrix} 0.1 & 0.4 & 0.1 \\ 0.3 & 0.2 & 0.0 \\ 0.0 & 0.0 & 0.3 \end{pmatrix}. \tag{21}$$

The Minister depends on gas revenue (0.4 weight on Executive). The Executive depends on regulatory approval (0.3 weight on Minister). The Vendor is self-contained.

Information network (who listens to whom):

$$W^I = \begin{pmatrix} 0.3 & 0.5 & 0.2 \\ 0.4 & 0.3 & 0.3 \\ 0.2 & 0.1 & 0.7 \end{pmatrix}. \tag{22}$$

The Minister listens mostly to the Executive. The Executive listens to the Minister and somewhat to the Vendor (market prices as signal). The Vendor listens mostly to herself (local knowledge).

Composite ($\alpha_U = 0.2, \alpha_P = 0.5, \alpha_I = 0.3$):

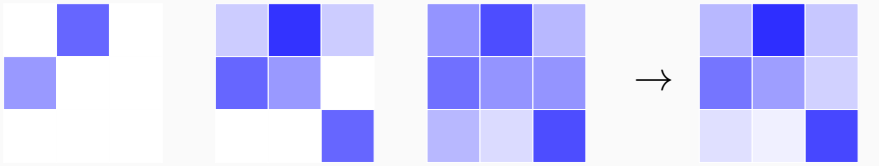
$$W = 0.2W^U + 0.5W^P + 0.3W^I = \begin{pmatrix} 0.14 & 0.41 & 0.11 \\ 0.27 & 0.19 & 0.09 \\ 0.06 & 0.03 & 0.36 \end{pmatrix}. \tag{23}$$

The left Perron eigenvector (computed numerically): $\boldsymbol{\pi} \approx (0.40, 0.38, 0.22)$. The Minister has the most power (0.40), followed closely by the Executive (0.38). The Vendor has some power (0.22) — mostly from her information self-reliance — but substantially less than either.

This is the power structure of La Paz in three numbers.

Visualization:

La Paz: three networks and their composite



W^U W^P W^I W **(composite)**

$\boldsymbol{\pi} = (0.40, 0.38, 0.22)$

The La Paz economy: Minister, Gas Executive, Market Vendor. Three component networks with different topologies blend into the composite. Column 2 of W^P is brightest (everyone depends on the Executive's output). Column 3 of W^I is dimmer (nobody much listens to the Vendor). The composite $\boldsymbol{\pi}$: Minister 40%, Executive 38%, Vendor 22%. The Vendor's self-reliance in W^I saves her from zero power, but she is structurally marginal.

8 What the Standard Model Assumes

We can now state precisely what the standard competitive model assumes about the three networks.

Proposition 8.1 (The Standard Model as a Network Assumption)**.** *The standard competitive model assumes:*

1. $W^U = 0$ *(no utility externalities).*
2. W^P *is given by technology (the input-output matrix, taken as data).*
3. W^I *does not exist (preferences and beliefs are exogenous and common knowledge).*
4. *The market interaction is* $W^{market} = \frac{1}{N}\mathbf{1}\mathbf{1}'$ *(prices aggregate demands anonymously and symmetrically).*

Under these assumptions, the welfare theorems hold, the competitive equilibrium is Pareto efficient, and social structure is irrelevant.

Structural economics relaxes each assumption:

1. *General* W^U *(utility externalities matter for welfare, efficiency, and policy).*
2. *General* W^P *(the topology of production determines growth propagation and structural change).*
3. *General* W^I *(information flows shape beliefs and preferences, reproducing or disrupting the structure).*

4. *The market interaction is mediated by all three networks, not just by anonymous prices.*

This is the complete structural program. The rest of the book develops each relaxation: Chapter 6 derives power from W, Chapter 7 derives the bargaining equilibrium, Chapter 8 develops belief manipulation through W^I, and Chapters 9–12 develop the formation model where W^I shapes preferences.

9 Summary

1. The economy is organized by three networks: W^U (utility externalities), W^P (production spillovers), and W^I (information flows). The standard model ignores the first, measures the second, and denies the existence of the third.

2. W^U captures who affects your happiness. It enters utility directly: $V_i = U_i + \gamma_U \sum_j W^U_{ij} U_j$. The standard model assumes $W^U = 0$, which means nobody's consumption affects anyone else.

3. W^P captures who depends on whose output. It determines the Leontief inverse $(I - \gamma_P W^P)^{-1}$, which amplifies shocks through inter-sectoral linkages. This is the matrix from Chapters 1 and 3, now recognized as one of three structural objects.

4. W^I captures who listens to whom. It drives belief dynamics through DeGroot learning: $\mathbf{b}^{(t+1)} = W^I \mathbf{b}^{(t)}$, converging to a consensus weighted by the Perron eigenvector $\boldsymbol{\pi}(W^I)$. This is epistemic power.

5. The composite $W = \alpha_U W^U + \alpha_P W^P + \alpha_I W^I$ summarizes the full structure. Its Perron eigenvector is the power vector. Power is recursive: it comes from being valued, depended upon, and listened to by other powerful agents.

6. Canonical topologies — complete, star, block-diagonal, core-periphery, ring, hierarchy, small-world, scale-free, random — characterize the qualitative features of each network. The topology determines the spectral properties, which determine propagation, convergence, power concentration, and structural persistence.

10 Exercises

5.1 Utility externalities. For the three-vendor economy (Example 2.2), with $\gamma_U = 0.5$:

(a) Compute V_2 and V_3 (Rosa's and Carlos's social utilities).

(b) A social planner maximizes $\sum_i V_i$. Show that the planner's optimal allocation differs from the competitive equilibrium (which maximizes $\sum_i U_i$). Who gets more under the planner?

(c) What happens as $\gamma_U \to 1$? As $\gamma_U \to 0$?

5.2 Three networks for a factory town. A town has $N = 4$ agents: Factory Owner (F), Worker (W), Shopkeeper (S), Mayor (M).

(a) Write plausible W^U, W^P, and W^I matrices (each 4×4).

(b) Compute the composite W with equal weights $\alpha = 1/3$.

(c) Find the Perron eigenvector. Who has the most power? Explain why.

(d) Now suppose the factory closes. Which entries of W^P go to zero? How does the Perron eigenvector change?

5.3 DeGroot learning. For $N = 3$ with:

$$W^I = \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{pmatrix}. \tag{24}$$

(a) Verify W^I is row-stochastic.

(b) Compute $(W^I)^2$ and $(W^I)^5$ (use Python). Are the rows becoming more similar?

(c) Find the left Perron eigenvector $\boldsymbol{\pi}$. Which agent has the most epistemic power?

(d) Starting from $\mathbf{b}^{(0)} = (0, 0.5, 1)'$, compute $\mathbf{b}^{(t)}$ for $t = 1, 2, 5, 10, 50$. Verify convergence to $\boldsymbol{\pi}'\mathbf{b}^{(0)}$.

5.4 The phone as structural change. Luisa's pre-phone W^I and post-phone W^I from Example 4.3. Extend to $N = 5$ agents (Mother, Friend, Cousin, TikTok, Teacher) where each agent also has a W^I row (Mother listens to: herself 0.6, friend 0.2, teacher 0.2; etc.).

(a) Construct the full 5×5 pre-phone and post-phone W^I.

(b) Compute the Perron eigenvector for both. How does epistemic power shift?

(c) If initial beliefs about "returns to education" are $\mathbf{b}^{(0)} = (0.3, 0.3, 0.8, 0.6, 0.7)$ (Mother and Friend think returns are low; Cousin, TikTok, Teacher think returns are high), what is the long-run consensus pre-phone? Post-phone?

(d) Interpret: how much does the phone change Luisa's long-run belief about education?

5.5 When networks align. Construct a 4×4 economy where $W^U = W^P = W^I$ (a village where everyone is simultaneously your friend, your supplier, and your information source).

(a) Compute the composite W. Note: it equals each component matrix.

(b) Now perturb: add one external production link (agent 1 begins importing from outside). How does W^P change? How does the composite change? What happens to the Perron eigenvector?

(c) Interpret: what does it mean for a village to enter the global supply chain?

5.6 Prebisch in W^I. Construct a 6×6 global W^I: agents 1–2 are "center" countries, agents 3–6 are "periphery" countries. Center listens mostly to itself; periphery listens mostly to center.

(a) Compute the Perron eigenvector. How much epistemic power do center countries have?

(b) Initial beliefs about "the optimal tariff policy": center believes $b = 0$ (free trade), periphery believes $b = 1$ (protection). What is the long-run consensus?

(c) What fraction of the consensus is determined by the center? Is this consistent with the Washington Consensus?

(d) What would it take (in terms of W^I entries) for the periphery's belief to survive?

5.7 Bolivia: composite power. Using Bolivia's three-agent model (Example 7.3):

(a) Vary α_P from 0 to 1 (holding α_U / α_I constant). Plot π_3 (Vendor's power) as a function of α_P. When does the Vendor have the most power? The least?

(b) Interpret: in what kind of economy does the market vendor matter most? When does she become invisible?

(c) Add a fourth agent: "Luisa (Student)." Construct plausible W^U, W^P, W^I rows for Luisa. How does her inclusion change the power structure?

Mathematical Appendices

Appendix 5A: Graph Theory for Economists

A **directed graph** (digraph) $G = (V, E)$ consists of a set of *vertices* (nodes, agents) $V = \{1, \ldots, N\}$ and a set of *edges* (links, connections) $E \subseteq V \times V$. An edge (i, j) means agent i is connected to agent j.

Every $N \times N$ weight matrix W defines a weighted directed graph: the vertices are agents $1, \ldots, N$, and the edge from j to i has weight W_{ij}. (Convention: $W_{ij} > 0$ means there is an edge from j to i, i.e., j affects i.)

Key definitions. *Path*: a sequence of edges connecting i to j. The *length* of a path is the number of edges. A path of length k from j to i exists if and only if $(W^k)_{ij} > 0$: the (i, j) entry of the k-th power of W.

Connected: i can reach j through a path of some finite length. *Strongly connected*: every agent can reach every other agent. Equivalently, W is *irreducible*: $(I + W)^{N-1}$ has all positive entries.

Aperiodic: the greatest common divisor of the lengths of all cycles through any agent is 1. Sufficient condition: some diagonal entry $W_{ii} > 0$ (some agent has a self-loop, i.e., some of their influence stays with themselves).

Why these matter: The Perron-Frobenius theorem (Chapter 6) requires W to be nonnegative. Strong connectivity ensures a unique Perron eigenvector. Aperiodicity ensures convergence of (W^t) to a limit (the consensus matrix $\mathbf{1}\boldsymbol{\pi}'$). Without strong connectivity, the network fragments into components, each with its own Perron eigenvector. Without aperiodicity, the system cycles rather than converging.

Degree and centrality. *In-degree*: $d_i^{\text{in}} = \sum_j W_{ij}$ (row sum). How much total influence agent i receives. For row-stochastic W^I: always 1.

Out-degree: $d_i^{\text{out}} = \sum_j W_{ji}$ (column sum). How much total influence agent i exerts. Not necessarily 1 even for row-stochastic matrices.

Eigenvector centrality: the Perron eigenvector $\boldsymbol{\pi}$ assigns each agent a centrality proportional to the centrality of agents who point toward her. This is the recursive "power" we have been developing. It is not the same as degree (a node can have high degree but low eigenvector centrality if its neighbors are peripheral). Chapter 6 develops this fully.

Adjacency matrix vs. weight matrix. The *adjacency matrix* A is binary: $A_{ij} = 1$ if there is an edge from j to i, and 0 otherwise. The *weight matrix* W generalizes this:

$W_{ij} \geq 0$ is the strength of the edge. In this book, we always use weight matrices. The adjacency matrix is the special case where all edges have weight 1.

Appendix 5B: Stochastic Matrices and the Perron Eigenvector

Row-stochastic matrices. A nonnegative matrix W is **row-stochastic** if $W\mathbf{1} = \mathbf{1}$: every row sums to 1. This means each row is a probability distribution. In the context of W^I, row-stochasticity means each agent allocates 100% of her attention across all agents (including possibly herself).

Key property. If W is row-stochastic, then $\rho(W) = 1$ and $\mathbf{1}$ is a right eigenvector: $W\mathbf{1} = 1 \cdot \mathbf{1}$.

The left Perron eigenvector. The left eigenvector corresponding to eigenvalue 1 is $\boldsymbol{\pi}$: the vector satisfying $\boldsymbol{\pi}'W = \boldsymbol{\pi}'$ with $\boldsymbol{\pi} \geq 0$ and $\mathbf{1}'\boldsymbol{\pi} = 1$. For a row-stochastic matrix, $\boldsymbol{\pi}$ is also the *stationary distribution* of the Markov chain defined by W: if you start at a random node drawn from $\boldsymbol{\pi}$ and take a random step according to W, you remain distributed according to $\boldsymbol{\pi}$.

Why left, not right? The right eigenvector of a row-stochastic matrix is always $\mathbf{1}$ — uninformative. The left eigenvector is the interesting object: it tells you which agents have the most influence in the long run. "Left Perron" is the power vector throughout this book.

Computing $\boldsymbol{\pi}$ by iteration. Start with any positive vector $\mathbf{v}^{(0)}$. Iterate: $\mathbf{v}^{(t+1)\prime} = \mathbf{v}^{(t)\prime}W$. Normalize: $\boldsymbol{\pi}^{(t)} = \mathbf{v}^{(t)}/\|\mathbf{v}^{(t)}\|_1$. If W is irreducible and aperiodic, $\boldsymbol{\pi}^{(t)} \to \boldsymbol{\pi}$ as $t \to \infty$. Convergence is geometric at rate $|\lambda_2(W)|$, the second-largest eigenvalue in magnitude. Faster convergence means larger spectral gap $\Gamma = 1 - |\lambda_2|$.

Example. For the 3×3 matrix from Exercise 5.3:

$$W^I = \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{pmatrix}. \tag{25}$$

Starting from $\mathbf{v}^{(0)} = (1, 1, 1)$:

t	π_1	π_2	π_3	Max change
0	0.333	0.333	0.333	—
1	0.267	0.367	0.367	0.067
2	0.244	0.374	0.382	0.023
5	0.232	0.375	0.394	0.002
∞	0.231	0.375	0.395	0

Agent 3 has the most epistemic power (0.395), despite not having the highest self-weight (that's agent 2 with $W^I_{22} = 0.6$). Why? Because agents 1 and 2 both listen to agent 3 more than agent 3 listens to them. Structural position, not self-confidence, determines power.

Doubly stochastic matrices. If W is both row-stochastic AND column-stochastic ($\mathbf{1}'W = \mathbf{1}'$), then $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$: all agents have equal power. This is the competitive benchmark from Chapter 1 ($W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$ is doubly stochastic). Any deviation from doubly stochastic generates power asymmetry.

Structuralist Case Study: Dos Santos's Three Regimes of the Weight Matrix

In 1970, Theotônio dos Santos published "The Structure of Dependence" in the *American Economic Review* — a six-page paper presented at an AEA panel on "Economics of Imperialism" chaired by Paul Sweezy. The paper's core contribution was a **taxonomy of three historical forms of dependence**, each corresponding to a specific phase of world capitalist development. Read through this chapter's framework, the three forms are three historical regimes of the weight matrix W.

Regime 1: Colonial dependence (trade-export). The metropole extracts raw materials from the colony through monopoly control of land, mines, and labor. The weight matrix is a **star graph**: all edges point to the colonial power. W^P has one bright column (the metropole) and $N-1$ dim columns (the colonies). The colonies do not trade with each other — the off-diagonal entries among periphery nodes are zero. W^I is similarly star-shaped: information (prices, technology, legal frameworks) flows from the metropole to the colonies, not between colonies. Power is maximally concentrated: $\boldsymbol{\pi} \approx (1, 0, \ldots, 0)$.

Regime 2: Financial-industrial dependence (late 19th century). Big capital invests in raw material and agricultural production for export to hegemonic centers. The star becomes a **hub-spoke with multiple hubs**: several industrial powers (Britain, France, the US) each have their own peripheral suppliers. W^P now has a few bright columns instead of one. The periphery begins to develop limited internal linkages (railways for commodity transport, ports, basic processing), adding

small off-diagonal entries among peripheral nodes. But the dominant flows still point toward the center. W^I remains center-dominated: economic policy knowledge flows from London and New York.

Regime 3: Technological-industrial dependence (post-WWII). Multinational corporations invest in industries oriented toward the internal market of the underdeveloped countries. The topology becomes more complex: MNCs create **bridging nodes** that connect the domestic W^P to the global W^P through intra-firm trade. The domestic matrix acquires new blocks (the MNC-owned modern sector), but these blocks connect outward to headquarters rather than inward to the traditional domestic sector. This is Sunkel's "transnational integration and national disintegration" as a matrix rewiring: cross-country edges among elite nodes strengthen while within-country edges between modern and traditional blocks weaken. The information network W^I becomes layered: the MNC subsidiary receives technical knowledge from headquarters ($W^I_{\text{subsidiary, HQ}}$ large) but does not transmit local knowledge back ($W^I_{\text{HQ, subsidiary}} \approx 0$).

Dos Santos's regime	**Topology of W**	**Power structure**	Γ
Colonial	Star	$\pi_{\text{metropole}} \approx 1$	≈ 0
Financial-industrial	Multi-hub	$\pi_{\text{centers}} \gg \pi_{\text{periphery}}$	Small
Technological-industrial	Complex asymmetric	$\pi_{\text{MNC}} > \pi_{\text{domestic elite}} \gg \pi_{\text{traditional}}$	Small

The taxonomy's power is its specificity: each regime generates a *different* heatmap, a different Perron eigenvector, and a different spectral gap, even though all three are "dependent." The colonial heatmap has one blazing column. The financial-industrial heatmap has several bright columns and dim periphery. The technological-industrial heatmap has a complex block structure with MNC bridging nodes creating cross-border links while domestic cross-block links remain dark.

Dos Santos's definition — "a situation in which the economy of certain countries is conditioned by the development and expansion of another economy" — is the statement that the Perron eigenvector of the global W assigns dominant weight to the conditioning economy. The taxonomy refines this: *how* the conditioning occurs (through which channel of W and with which topology) determines the specific developmental consequences. The colonial star produces extraction. The financial hub-spoke produces commodity dependence. The technological-industrial complex produces what Pinto called "modernización excluyente" — a modern sector that connects to the world but not to its own country.

The three networks of this chapter — W^U, W^P, W^I — are the analytical tools

that Dos Santos needed but did not have. His taxonomy describes what the matrices look like. The Perron eigenvector tells you who benefits.

Further Reading

Social preferences and externalities: Fehr and Schmidt (1999), "A Theory of Fairness, Competition, and Cooperation," *QJE*. Sobel (2005), "Interdependent Preferences and Reciprocity," *JEL*. Bowles and Gintis (2011), *A Cooperative Species*.

Production networks: See readings from Chapters 1 and 3. The comprehensive reference is now Carvalho and Tahbaz-Salehi (2019), "Production Networks: A Primer," *Annual Review of Economics*.

DeGroot learning: DeGroot (1974), "Reaching a Consensus." Golub and Jackson (2010), "Naïve Learning in Social Networks and the Wisdom of Crowds," *AEJ: Microeconomics*. This is the key paper on convergence, influence, and the Perron eigenvector in learning models.

Network economics textbooks: Jackson (2008), *Social and Economic Networks*. Goyal (2023), *Networks: An Economics Approach*. Sargent and Stachurski (2024), *Economic Networks*.

The interaction of multiple networks: Kivelä et al. (2014), "Multilayer Networks," *Journal of Complex Networks*. For the economic application: the composite W is related to the "multiplex" framework in network science.

Prebisch and information dependency: Prebisch (1950) on ideological dependency. Cardoso and Faletto (1969), *Dependency and Development in Latin America*. For the modern formalization: Acemoglu et al. (2010), "Spread of (Mis)information in Social Networks," *Games and Economic Behavior*.

Dependency theory: Dos Santos (1970), "The Structure of Dependence," *AER* 60(2), pp. 231–236 (the six pages that define the taxonomy formalized in the case study above). Dos Santos (1978), *Imperialismo y Dependencia*, México: Era. Sunkel (1971), "Capitalismo transnacional y desintegración nacional," *Estudios Internacionales* 4(16).

Chapter 6

Centrality, Power, and the Perron Vector

Where Power Comes From

> *"En el capitalismo periférico, la apropiación del excedente económico está íntimamente ligada a las relaciones de poder entre los distintos estratos sociales."*
>
> ["In peripheral capitalism, the appropriation of the economic surplus is intimately linked to the power relations between different social strata."]
>
> — Raúl Prebisch, *Capitalismo Periférico: Crisis y Transformación* (1981)

> *"Power is not an institution, and not a structure; neither is it a certain strength we are endowed with; it is the name that one attributes to a complex strategical situation in a particular society."*
>
> — Michel Foucault, *The History of Sexuality* (1976)

> *"Power is the left Perron eigenvector of the weight matrix."*
>
> — This book

Foucault was right that power is not an attribute of individuals. He was wrong that it cannot be measured. Power is a property of the network — specifically, of the weight matrix W that encodes who affects whom. It is computed by the Perron-Frobenius theorem, the single most important result in this book.

This chapter does three things. First, it develops the concept of centrality — the idea that some positions in a network are more "important" than others — through examples, building from simple degree counting to the recursive logic of eigenvector centrality. Second, it states and proves the Perron-Frobenius theorem, which guarantees that the centrality measure we want actually exists and is unique. Third, it derives the properties of the Perron eigenvector — the power vector π — and shows what the spectral gap Γ tells us about the concentration and persistence of power.

By the end, the student will know where power comes from (the network), how to compute it (the Perron eigenvector), when it concentrates (small spectral gap), and when the welfare theorems hold (power is equal, meaning W is doubly stochastic).

1 Who Matters? Four Attempts at an Answer

1.1 A Small Economy

Consider five agents in a production network:

$$W = \begin{pmatrix} 0.2 & 0.4 & 0.1 & 0.0 & 0.0 \\ 0.3 & 0.1 & 0.2 & 0.1 & 0.0 \\ 0.1 & 0.3 & 0.2 & 0.2 & 0.1 \\ 0.0 & 0.1 & 0.3 & 0.2 & 0.3 \\ 0.0 & 0.0 & 0.1 & 0.3 & 0.4 \end{pmatrix}. \tag{1}$$

Read row i: these are the weights that agent i places on others. Agent 1 depends heavily on agent 2 ($W_{12} = 0.4$). Agent 5 depends heavily on herself ($W_{55} = 0.4$) and on agent 4 ($W_{54} = 0.3$). Agent 3 is in the middle — connected to everyone.

Question: which agent is the most powerful?

Four different answers emerge from four different ways of measuring importance. All four are "correct" in their own domain. Only one is the right measure of structural power.

1.2 Attempt 1: Count the Connections (Degree Centrality)

The simplest measure: how many connections does each agent have?

Definition 1.1 (Degree Centrality)**.** *For a weight matrix W, agent i's **in-degree** is the column sum $d_i^{in} = \sum_j W_{ji}$ (total weight that others place on i). The **out-degree** is the row sum $d_i^{out} = \sum_j W_{ij}$ (total weight that i places on others).*

For our W: the column sums (in-degree) are:

Agent	1	2	3	4	5
In-degree d_i^{in}	0.60	0.90	0.90	0.80	0.80

Agents 2 and 3 are tied at the top: both receive the most total weight from others. Agent 1 is least important by this measure.

Intuition: What degree gets right and wrong

Degree counts *direct* connections: how many people depend on you (in-degree) or how much you depend on others (out-degree). It is easy to compute and often correlates with importance. But it misses something critical: *who* depends on you matters. If you are depended upon by powerful agents, that should count more than being depended upon by peripheral agents. Degree treats all connections equally. A farmer who sells to the president and a farmer who sells to another farmer have the same in-degree, but they do not have the same power.

1.3 Attempt 2: Count the Paths (Katz Centrality)

Degree counts direct connections. A more sophisticated measure counts *all* paths — direct and indirect.

Definition 1.2 (Katz Centrality)**.** *Agent i's **Katz centrality** is:*

$$c_i^{Katz} = \sum_{k=1}^{\infty} \delta^k \sum_{j=1}^{N} (W^k)_{ji} = \left[(I - \delta W')^{-1} - I \right]_i \cdot \mathbf{1}, \tag{2}$$

where $\delta \in (0, 1/\rho(W))$ is a decay parameter that downweights longer paths, and $(W^k)_{ji}$ is the total weight of all paths of length k from i to j.

Katz centrality says: agent i is important if many paths lead to her, directly or indirectly, weighted by length. The parameter δ ensures convergence (same condition as the Leontief inverse from Chapter 1).

Intuition: The Leontief connection

Katz centrality is the column sum of the Leontief inverse $(I - \delta W')^{-1}$. This is the output multiplier from Chapter 1 reinterpreted as a centrality measure. An agent with a high output multiplier generates a lot of total activity when her output increases — she is "important" in the production network.

Katz improves on degree by counting indirect connections. But it still has a flaw: it adds a fixed bonus δ^k for each path of length k, regardless of whether the agents along that path are themselves important. The weight of a path through the president should count more than a path through another farmer.

1.4 Attempt 3: Ask the Neighbors (Bonacich Centrality)

Definition 1.3 (Bonacich Centrality)**.** *Agent i's **Bonacich centrality** is the i-th component of:*

$$\mathbf{c}^{Bon} = (I - \beta W')^{-1} W' \mathbf{1} = (I - \beta W')^{-1} \mathbf{d}^{in}, \tag{3}$$

where $\beta \in (-1/\rho(W), 1/\rho(W))$ is a parameter and $\mathbf{d}^{in} = W'\mathbf{1}$ is the in-degree vector.

Bonacich centrality starts from degree (the $\mathbf{d}^{\text{in}}$ term) and then amplifies it through the Leontief-type multiplier $(I - \beta W')^{-1}$, which accounts for the centrality of your neighbors.

The parameter β has a remarkable interpretation. When $\beta > 0$, an agent is more central if her neighbors are also central — this captures *cooperation* (your friends' power helps you). When $\beta < 0$, an agent is more central if her neighbors are *less* central — this captures *bargaining* (your power comes from your partners' weakness, because they have no outside options).

Ballester, Calvó-Armengol, and Zenou (2006) proved the foundational result: in linear-quadratic games on networks, the Nash equilibrium action of each agent is proportional to her Bonacich centrality. This is the "Bonacich-Nash linkage" — the first clean mapping from network position to economic outcome.

Intuition: Getting closer

Bonacich centrality is a major advance: it makes the centrality of your neighbors matter for your own centrality. But it still depends on a parameter β that must be chosen by the analyst. Different values of β give different rankings. There is no "canonical" Bonacich centrality.

What we want is a centrality measure that is *parameter-free* — determined entirely by the network, with no analyst degrees of freedom. That is the eigenvector.

1.5 Attempt 4: The Recursive Definition (Eigenvector Centrality)

Here is the idea that resolves everything.

An agent is powerful if powerful agents depend on her.

This is circular. The power of agent i depends on the power of agents j who point to i, which depends on the power of agents k who point to j, and so on. We seem to need to know the answer before we can compute it.

But this circularity is not a bug. It is a feature. The circularity defines a *fixed-point equation*: we seek a vector $\boldsymbol{\pi}$ such that the power of each agent equals the total power directed toward her.

Definition 1.4 (Eigenvector Centrality — Left Perron Eigenvector)**.** *Agent i's **eigenvector centrality** (or **structural power**) is π_i, the i-th component of the vector $\boldsymbol{\pi}$ satisfying:*

$$\boldsymbol{\pi}' W = \rho(W) \cdot \boldsymbol{\pi}', \tag{4}$$

with $\boldsymbol{\pi} \geq 0$ and $\|\boldsymbol{\pi}\|_1 = \sum_i \pi_i = 1$.

Equivalently, for each agent:

$$\pi_i = \frac{1}{\rho(W)} \sum_{j=1}^{N} \pi_j W_{ji}. \tag{5}$$

Intuition: Reading the equation

Equation (5) says: agent i's power is proportional to the sum, over all agents j, of (a) j's power π_j times (b) how much j depends on i (that is, W_{ji}). You are powerful if powerful agents point toward you.

The factor $1/\rho(W)$ is just normalization — it ensures $\sum_i \pi_i = 1$.

This is the recursive definition we wanted: power depends on the power of those who depend on you. The Perron-Frobenius theorem (next section) guarantees that this circular definition has a unique solution.

Example 1.5 (Computing Power for the Five-Agent Economy)**.** *For the matrix W in (1), the left Perron eigenvector (computed iteratively — see Appendix 6A) is:*

$$\boldsymbol{\pi} \approx (0.122,\ 0.226,\ 0.257,\ 0.214,\ 0.181). \tag{6}$$

Agent 3 is the most powerful *($\pi_3 = 0.257$), even though she tied with Agent 2 on in-degree. Why? Because Agent 3 is depended upon by Agent 4 ($W_{43} = 0.3$), who is himself fairly powerful ($\pi_4 = 0.214$). Agent 2 is depended upon by Agent 1 ($W_{12} = 0.4$), who is the least powerful ($\pi_1 = 0.122$). Being depended upon by a weak agent counts for less.*

Agent 1 is the least powerful *($\pi_1 = 0.122$), despite having a nonzero connection to the most powerful agent. She is peripheral: few agents depend on her, and those who do are not particularly powerful themselves.*

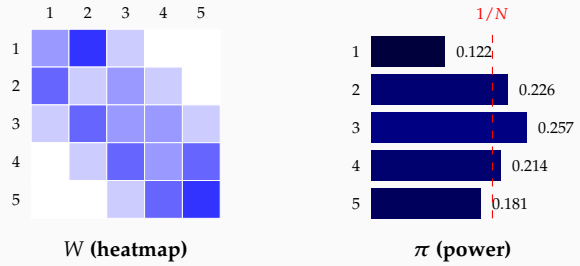
Compare the four centrality measures:

	Agent 1	*Agent 2*	*Agent 3*	*Agent 4*	*Agent 5*
In-degree	*0.60*	***0.90***	***0.90***	*0.80*	*0.80*
Katz ($\delta = 0.5$)	*0.87*	*1.32*	***1.41***	*1.24*	*1.16*
Bonacich ($\beta = 0.3$)	*0.71*	*1.15*	***1.28***	*1.12*	*1.04*
Eigenvector π_i	*0.122*	*0.226*	***0.257***	*0.214*	*0.181*

All four agree that Agent 3 is the most important, but they disagree on the rest. The eigenvector is the only measure that correctly discounts connections to weak agents and amplifies connections to strong ones, recursively and without free parameters.

Visualization:

The five-agent economy: heatmap and power distribution



Left: the 5×5 weight matrix as a heatmap. Agent 3's column (column 3) receives weight from many rows — she is depended upon broadly. Right: the Perron eigenvector. Agent 3 has the most power (0.257). The dashed red line marks $1/N = 0.20$ (equal power). Agents above the line are structurally advantaged; agents below it are disadvantaged. The distance from the red line is the power correction that enters the bargaining allocation (Chapter 7).

2 The Perron-Frobenius Theorem

The recursive definition of eigenvector centrality (4) raises an obvious question: does it have a solution? Is the solution unique? Is it positive (do all agents have nonnegative power)?

The answer to all three is yes, and the proof is the Perron-Frobenius theorem — the single most important result in matrix theory for economics.

2.1 Statement

Theorem 2.1 (Perron-Frobenius)**.** *Let W be an $N \times N$ nonnegative matrix ($W_{ij} \geq 0$ for all i, j). Then:*

Part I (Perron, 1907): *If $W > 0$ (all entries strictly positive):*

1. *W has a real eigenvalue $\rho = \rho(W) > 0$ that is strictly larger in magnitude than all other eigenvalues: $|\lambda_k| < \rho$ for all $k \neq 1$.*

2. *The corresponding right eigenvector $\mathbf{v}$ can be chosen strictly positive: $v_i > 0$ for all i.*

3. *The corresponding left eigenvector $\boldsymbol{\pi}$ can be chosen strictly positive: $\pi_i > 0$ for all i.*

4. *Both $\mathbf{v}$ and $\boldsymbol{\pi}$ are unique up to normalization.*

Part II (Frobenius, 1912): *If $W \geq 0$ (nonnegative, some entries may be zero) and W is **irreducible** (the network is strongly connected — every agent can reach every other agent through some chain of positive weights):*

5. *All of (1)–(4) still hold.*

6. *If additionally W is **aperiodic** (the greatest common divisor of cycle lengths through any node is 1), then $\frac{1}{\rho^t} W^t \to \mathbf{v}\boldsymbol{\pi}'$ as $t \to \infty$.*

Intuition: What each part says in words

(1) Dominant eigenvalue: Among all the eigenvalues of W, there is one — the spectral radius ρ — that is real, positive, and strictly larger than all others. This means the long-run behavior of the system is dominated by a single "mode."

(2)–(3) Positive eigenvectors: The right eigenvector $\mathbf{v}$ and left eigenvector $\boldsymbol{\pi}$ have all positive entries. No agent has zero power. No agent has negative power. Everyone matters, at least a little.

(4) Uniqueness: There is only one such $\boldsymbol{\pi}$ (up to scaling). Power is determined uniquely by the network. There is no ambiguity, no analyst choice, no free parameter.

(5) Extension to sparse matrices: Even if some entries of W are zero (not everyone is directly connected to everyone), the theorem still holds as long as the network is *connected*: there exists an indirect path between every pair of agents.

(6) Convergence: If you repeatedly apply W to any initial vector — iterate beliefs through DeGroot learning, cascade shocks through the Leontief system, propagate influence through the network — the system converges to a state described by the Perron eigenvectors. In the long run, only $\boldsymbol{\pi}$ matters.

Key Structural Insight

The Perron-Frobenius theorem is why structural power is well-defined. Without it, the recursive definition $\pi_i = \frac{1}{\rho}\sum_j \pi_j W_{ji}$ might have no solution, multiple solutions, or solutions with negative entries (meaningless as a power measure). With it, the solution exists, is unique, and is positive. Power is a theorem, not an assumption.

The theorem requires two conditions: nonnegativity ($W \geq 0$, which is natural — dependencies are nonnegative) and irreducibility (the network is connected — which is an empirical question). When the network is reducible (disconnected), it fragments into irreducible components, each with its own Perron eigenvector. This is the mathematical content of structural heterogeneity: a block-diagonal W has multiple Perron vectors, one per block.

2.2 Proof Sketch

The full proof is in Appendix 7. Here we give the key ideas for the positive case ($W > 0$).

Step 1: The map T. Define $T(\mathbf{u}) = W\mathbf{x}/\|W\mathbf{x}\|_1$ for $\mathbf{x} \geq 0$, $\mathbf{x} \neq 0$. This maps the positive simplex $\Delta = \{\mathbf{x} \geq 0 : \|\mathbf{x}\|_1 = 1\}$ to itself (because $W > 0$ ensures $W\mathbf{x} > 0$).

Step 2: T is a contraction. On the positive simplex with the Hilbert projective metric $d_H(\mathbf{u}, \mathbf{w}) = \ln \frac{\max_i x_i/y_i}{\min_i x_i/y_i}$, the map T satisfies $d_H(T(\mathbf{u}), T(\mathbf{w})) \leq \tau \cdot d_H(\mathbf{u}, \mathbf{w})$ where $\tau < 1$ depends on the ratio of the smallest to largest entry of W. This is Birkhoff's contraction theorem (1957).

Step 3: Fixed point. By the Banach fixed-point theorem, T has a unique fixed point $\mathbf{v}^*$. This satisfies $W\mathbf{v}^* = \rho\mathbf{v}^*$ where $\rho = \|W\mathbf{v}^*\|_1$. So $\mathbf{v}^*$ is the right Perron eigenvector.

Step 4: Left eigenvector. Apply the same argument to W' (the transpose). The unique fixed point of $T'(\boldsymbol{\pi}) = W'\boldsymbol{\pi}/\|W'\boldsymbol{\pi}\|_1$ is $\boldsymbol{\pi}$, the left Perron eigenvector.

Step 5: Dominance. Any other eigenvalue λ satisfies $|\lambda| < \rho$ because the contraction property forces all initial conditions to converge to the same fixed point — there is no room for competing eigenvectors at the same magnitude.

3 Properties of the Power Vector

3.1 Six Propositions About Power

Proposition 3.1 (Power Sums to One)**.** $\sum_{i=1}^{N} \pi_i = 1$. *Power is a distribution: it sums to 100%. An agent's power π_i can be interpreted as her "share of total structural influence."*

Proposition 3.2 (Equal Power iff Doubly Stochastic)**.** $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$ *(equal power for all agents) if and only if W is* ***doubly stochastic*** *(every row sum and every column sum equals 1).*

Proof. ($\Leftarrow$): If W is doubly stochastic, $\mathbf{1}'W = \mathbf{1}'$, so $\boldsymbol{\pi}' = \frac{1}{N}\mathbf{1}'$ satisfies $\boldsymbol{\pi}'W = \frac{1}{N}\mathbf{1}'W = \frac{1}{N}\mathbf{1}'$. By uniqueness of the Perron eigenvector, $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$.

($\Rightarrow$): If $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$, then $\frac{1}{N}\mathbf{1}'W = \frac{1}{N}\mathbf{1}'$, so $\mathbf{1}'W = \mathbf{1}'$, meaning column sums equal 1. Combined with row-stochasticity (if W is row-stochastic), this gives doubly stochastic. □

Intuition: The deepest result in this chapter

Proposition 3.2 is the mathematical foundation of the entire book. It says: *power is equal if and only if the interaction matrix is doubly stochastic.* The competitive model assumes $W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$ (doubly stochastic), so power is equal, so the welfare theorems hold. Any deviation from doubly stochastic creates power asymmetry. The welfare theorems fail because someone has more power than others.

The question "why don't the welfare theorems hold in the real world?" has a precise answer: because the real W is not doubly stochastic. The column sums are not equal. Some agents receive more total weight than others. That asymmetry is power.

Proposition 3.3 (Power is Monotone in Connections)**.** *If W_{ji} increases (agent j places more weight on agent i) while all other entries are adjusted to maintain row-stochasticity, then π_i weakly increases. Being depended upon by more people, or more heavily, raises your power.*

Proposition 3.4 (Power Concentrates in Stars)**.** *In a star network with hub h: $\pi_h = 1 - O(1/N)$, i.e., the hub captures almost all the power as N grows. The spectral gap $\Gamma \to 0$.*

Proposition 3.5 (Power Fragments in Block-Diagonal Networks)**.** *If W is approximately block-diagonal with k blocks, $\boldsymbol{\pi}$ is approximately concentrated on the block with the largest spectral radius, with weights proportional to the internal Perron eigenvector of that block.*

Proposition 3.6 (The Welfare Theorem Revisited)**.** *In the economy from Chapters 1–5, the competitive equilibrium is Pareto efficient if and only if the composite weight matrix W is doubly stochastic (equivalently, if and only if $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$). When power is unequal, the equilibrium is inefficient, and the direction of inefficiency is determined by $\boldsymbol{\pi}$: powerful agents consume too much relative to the efficient allocation, and powerless agents consume too little.*

4 The Spectral Gap

4.1 Definition and Meaning

Definition 4.1 (Spectral Gap)**.** *The **spectral gap** of W is:*

$$\Gamma = \rho(W) - |\lambda_2(W)|, \tag{7}$$

where $\rho(W)$ is the largest eigenvalue (the Perron root) and $|\lambda_2(W)|$ is the second-largest eigenvalue in magnitude.

The spectral gap is the single most informative structural diagnostic in this book. It measures four things simultaneously:

1. **Speed of convergence.** In DeGroot learning, beliefs converge at rate $|\lambda_2/\rho|^t$. Large Γ means fast convergence to consensus. Small Γ means slow convergence and persistent disagreement.

2. **Strength of propagation.** In the Leontief system, shocks propagate through the network. The off-diagonal entries of $(I - \gamma W)^{-1}$ are bounded by terms involving Γ. Large Γ means strong cross-agent propagation. Small Γ means shocks stay localized.

3. **Symmetry of power.** Large Γ implies that $\boldsymbol{\pi}$ is close to uniform ($\frac{1}{N}\mathbf{1}$): power is relatively equal. Small Γ means the Perron eigenvector can be highly concentrated: power is unequal.

4. **Structural integrity.** Large Γ means the network is well-connected: there is one dominant "mode" and the system behaves as a unified whole. Small Γ means competing modes: the network is approximately reducible (block-diagonal), and different parts of the system can evolve independently.

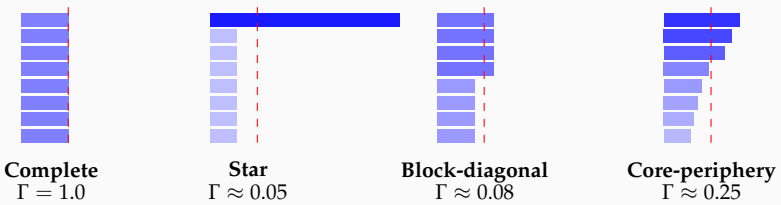
Example 4.2 (Spectral Gap Across Topologies)**.** *For $N = 10$:*

Topology	Γ	*Max* π_i	*Interpretation*
Complete	*1.000*	*0.100*	*Maximum symmetry, fast convergence*
Ring	≈ 0.19	*0.100*	*Slow propagation along chain*
Small-world	≈ 0.35	≈ 0.12	*Moderate: shortcuts help*
Star	≈ 0.05	≈ 0.50	*Near-zero: hub dominates*
Block-diagonal	≈ 0.08	≈ 0.13	*Near-zero: disconnected sectors*
Core-periphery	≈ 0.25	≈ 0.20	*Moderate asymmetry*
Scale-free	≈ 0.10	≈ 0.35	*Hubs concentrate power*

The complete graph has $\Gamma = 1$ *(maximum). The star and block-diagonal have* $\Gamma \approx 0$ *(minimum among connected graphs). The spectral gap collapses whenever there are competing power centers (block-diagonal) or a dominant hub (star).*

Visualization:

Four topologies, four power distributions



Power distributions ($\boldsymbol{\pi}$) for four topologies at $N = 8$. Dashed red line: $1/N = 0.125$ (equal power). Complete: perfectly flat. Star: one bar dominates. Block-diagonal: two tiers. Core-periphery: smooth gradient. The spectral gap Γ predicts the shape: large Γ means flat; small Γ means concentrated or tiered.

4.2 The Spectral Gap as Development Diagnostic

Prebisch Connection: The spectral gap of the global economy

Combining the insights from Chapters 3 and 5: the spectral gap of the global production network W^P_{global} measures how well-integrated the global economy is. If Γ is large, productivity shocks propagate from center to periphery and back — growth is shared. If Γ is small, the global economy is approximately block-diagonal: center and periphery evolve independently, and gains from trade accrue primarily to the center.

Prebisch's argument is that Γ_{global} is small — not because the global economy is disconnected (trade volumes are enormous) but because the linkages are *asymmetric*: the periphery depends on the center ($W^P_{\text{periphery, center}}$ is large) but the center barely depends on the periphery ($W^P_{\text{center, periphery}} \approx 0$). The matrix is not block-diagonal; it is core-periphery. The spectral gap is reduced not by disconnection but by asymmetry.

The spectral gap of Bolivia's domestic W^P measures internal structural heterogeneity (Chapter 3). The spectral gap of the global W^I measures epistemic dependence (Chapter 5). Both are small for the periphery. This is the double trap: weak economic propagation AND weak informational autonomy. The eigenvalues tell the same story at both scales.

5 Power in Practice

5.1 Where Does Power Come From?

The Perron eigenvector gives us a precise answer. From equation (5):

$$\pi_i = \frac{1}{\rho} \sum_{j=1}^{N} \pi_j W_{ji}. \tag{8}$$

Your power is the weighted sum of the power of agents who depend on you, where the weights are how much they depend on you. This generates three *sources* of structural power:

1. Being needed (production power). If many agents use your output as an input (W^P_{ji} large for many j), you have production power. Saudi Arabia has production power in the global oil network. TSMC has production power in the semiconductor network.

2. Being cared about (social power). If powerful agents care about your well-

being (W^U_{ji} large for powerful j), you have social power. The president's spouse has social power. A patron's favored client has social power.

3. Being listened to (epistemic power). If powerful agents pay attention to your signals (W^I_{ji} large for powerful j), you have epistemic power. The Federal Reserve chair has epistemic power. A TikTok influencer has epistemic power.

5.2 Power vs. Wealth

Power ($\boldsymbol{\pi}$) and wealth ($\mathbf{w}$) are not the same thing.

Wealth is what you have — your endowment ω_i, your capital K_i, your income Y_i. Wealth can be redistributed by transfers: tax the rich, give to the poor, and $\mathbf{w}$ changes.

Power is where you sit — your position in W. Power cannot be redistributed by transfers, because transfers change $\boldsymbol{\omega}$ but not W. The Perron eigenvector is a property of the matrix, not of the endowment vector.

Example 5.1 (A Rich Farmer and a Poor Minister)**.** *Suppose agent 1 is a wealthy farmer ($w_1 = 100$) and agent 2 is a poor government minister ($w_2 = 10$). In a 2×2 economy with:*

$$W = \begin{pmatrix} 0.8 & 0.2 \\ 0.9 & 0.1 \end{pmatrix}, \tag{9}$$

the farmer depends on himself ($W_{11} = 0.8$) and barely on the minister ($W_{12} = 0.2$). But the minister depends heavily on the farmer ($W_{21} = 0.9$) — because agricultural output feeds the government's tax base.

The left Perron eigenvector: $\boldsymbol{\pi}'W = \rho\boldsymbol{\pi}'$. From the first column: $0.8\pi_1 + 0.9\pi_2 = \rho\pi_1$, so $\pi_2/\pi_1 = (\rho - 0.8)/0.9$. The spectral radius is $\rho = (0.8+0.1)/2 + \sqrt{((0.8-0.1)/2)^2 + 0.18} = 0.45 + \sqrt{0.1225 + 0.18} = 0.45 + 0.55 = 1.0$. Wait — check: for a row-stochastic matrix, $\rho = 1$. Let's verify: row sums are $0.8 + 0.2 = 1$ and $0.9 + 0.1 = 1$. Yes, $\rho = 1$.

So $\pi_2/\pi_1 = (1 - 0.8)/0.9 = 0.2/0.9 = 2/9$. With $\pi_1 + \pi_2 = 1$: $\pi_1 = 9/11 \approx 0.818$, $\pi_2 = 2/11 \approx 0.182$.

The farmer has 82% of the structural power despite the minister having the political title. Why? Because the minister depends on the farmer (for tax revenue) more than the farmer depends on the minister (for governance). The dependency is asymmetric, and asymmetry is power.

The farmer is rich AND powerful. The minister is poor AND less powerful. This is a structurally stable configuration: power and wealth reinforce each other. Chapter 13 will show that such configurations are fixed points of the structural reproduction process.

6 Summary

1. **Four centrality measures** capture different aspects of network importance: degree (direct connections), Katz (all paths), Bonacich (neighbor-weighted paths), and eigenvector (recursive power). Only eigenvector centrality is parameter-free and self-consistent.

2. **The Perron-Frobenius theorem** guarantees that the eigenvector centrality $\boldsymbol{\pi}$ exists, is unique, and is positive for any connected nonnegative matrix. Power is a theorem, not an assumption.

3. **Power is equal if and only if W is doubly stochastic.** This is the structural content of the welfare theorems: efficiency requires symmetric interaction. Any deviation from doubly stochastic creates power asymmetry and inefficiency.

4. **The spectral gap** $\Gamma = \rho - |\lambda_2|$ measures convergence speed, propagation strength, power symmetry, and structural integrity — four dimensions of the same underlying property.

5. **Power comes from position**, not endowment. Transfers change $\boldsymbol{\omega}$ but not $\boldsymbol{\pi}$. Structural policy changes W and therefore $\boldsymbol{\pi}$.

7 Exercises

6.1 Computing centralities. For the five-agent economy (1):

(a) Compute all four centrality measures (degree, Katz with $\delta = 0.5$, Bonacich with $\beta = 0.3$, eigenvector). Use Python.

(b) Do they agree on the ranking? Where do they disagree?

(c) Now double W_{31} (Agent 3 depends more on Agent 1). Recompute the eigenvector centrality. Does Agent 1's power rise? By how much?

6.2 Doubly stochastic or not. Classify each matrix as doubly stochastic (equal power) or not (asymmetric power), and for those that are not, find the Perron eigenvector:

(a) $W = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(b) $W = \begin{pmatrix} 0.5 & 0.5 \\ 0.8 & 0.2 \end{pmatrix}$

(c) $W = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$

(d) $W = \begin{pmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.4 \end{pmatrix}$

6.3 The spectral gap and convergence. For each topology (complete, star, ring, block-diagonal) with $N = 8$:

(a) Compute Γ.

(b) Starting from $\mathbf{b}^{(0)} = (1, 0, 0, 0, 0, 0, 0, 0)'$, iterate $\mathbf{b}^{(t+1)} = W\mathbf{b}^{(t)}$ for $t = 1, \ldots, 50$. Plot $\|\mathbf{b}^{(t)} - \boldsymbol{\pi}'\mathbf{b}^{(0)}\mathbf{1}\|$ (distance from consensus) against t.

(c) Verify that the convergence rate matches $|\lambda_2|^t$.

6.4 Bonacich with negative β. For a buyer-seller network:

$$W = \begin{pmatrix} 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \end{pmatrix}. \tag{10}$$

(a) Compute Bonacich centrality with $\beta = 0.3$ (cooperation) and $\beta = -0.3$ (bargaining).

(b) Under bargaining ($\beta < 0$), which agents have higher centrality? Why?

(c) Relate to Manea's (2011) bargaining result: in symmetric bipartite networks, all agents have equal bargaining power. Does the eigenvector centrality confirm this?

6.5 Power iteration by hand. For $W = \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix}$:

(a) Start with $\mathbf{v}^{(0)} = (1, 1)$. Compute $\mathbf{v}^{(1)} = W'\mathbf{v}^{(0)}$, normalize. Repeat for $t = 2, 3, 4, 5$.

(b) At what iteration does $\mathbf{v}^{(t)}$ converge to two decimal places?

(c) Compute the exact Perron eigenvector algebraically (solve $\boldsymbol{\pi}'W = \boldsymbol{\pi}'$).

(d) Compare with the iterative result. Compute Γ and verify the convergence rate.

6.6 Structural power vs. formal power. An economy has 4 agents: President, General, Businesswoman, Farmer. Construct a plausible 4×4 weight matrix where the Farmer has the highest eigenvector centrality despite having the lowest formal rank. (Hint: make everyone's food supply depend on the Farmer, directly or indirectly.) Compute $\boldsymbol{\pi}$ and verify.

6.7 Bolivia's power structure. Using the Bolivia three-sector model from Chapter 3 (W^P with Gas, Manufacturing, Traditional):

(a) Compute the left Perron eigenvector. Which sector has the most structural power?

(b) Now suppose gas revenues are redistributed to the traditional sector (a transfer). Does $\boldsymbol{\pi}$ change? Why or why not?

(c) Now suppose a gas-to-fertilizer plant is built, adding a link from Gas to Traditional ($W^P_{\text{Trad, Gas}}$ increases from 0.02 to 0.15). Recompute $\boldsymbol{\pi}$. How does the traditional sector's power change?

(d) Which policy — the transfer or the plant — changes the power structure?

Mathematical Appendices

Appendix 6A: Computing the Perron Eigenvector

Power iteration. The simplest algorithm:

1. Start with $\mathbf{v}^{(0)} = \frac{1}{N}\mathbf{1}$ (or any positive vector).
2. Iterate: $\tilde{\mathbf{v}}^{(t+1)} = W'\mathbf{v}^{(t)}$ (multiply by the transpose).
3. Normalize: $\mathbf{v}^{(t+1)} = \tilde{\mathbf{v}}^{(t+1)} / \|\tilde{\mathbf{v}}^{(t+1)}\|_1$.
4. Repeat until $\|\mathbf{v}^{(t+1)} - \mathbf{v}^{(t)}\|_\infty < \varepsilon$ (convergence tolerance).

The spectral radius is estimated as $\rho \approx \|\tilde{\mathbf{v}}^{(t+1)}\|_1 / \|\mathbf{v}^{(t)}\|_1$ (the growth factor before normalization).

Convergence rate. The error after t iterations is $\|\mathbf{v}^{(t)} - \boldsymbol{\pi}\| = O(|\lambda_2/\rho|^t)$. This is geometric with rate $|\lambda_2/\rho| = 1 - \Gamma/\rho$. For the complete graph ($\Gamma = 1$, $\rho = 1$): convergence in one step. For a near-block-diagonal graph ($\Gamma \approx 0.01$): convergence requires ~ 100 iterations for each digit of accuracy.

Python code.

```
import numpy as np

def perron_eigenvector(W, tol=1e-10, maxiter=1000):
    """Left Perron eigenvector of W."""
    n = W.shape[0]
    v = np.ones(n) / n
    for _ in range(maxiter):
        v_new = W.T @ v
        v_new = v_new / v_new.sum()
        if np.max(np.abs(v_new - v)) < tol:
            return v_new
        v = v_new
    return v

def spectral_gap(W):
    """Spectral gap of W."""
    eigenvalues = np.sort(np.abs(np.linalg.eigvals(W)))[::-1]
    return eigenvalues[0] - eigenvalues[1]
```

Computational complexity. Each iteration of power iteration requires one matrix-vector multiplication: $O(N^2)$ for a dense matrix, $O(\text{nnz})$ for a sparse matrix (where nnz is the number of nonzero entries). For most economic networks (sparse, $\text{nnz} \ll N^2$), this is efficient even for N in the millions.

Appendix 6B: Proof of Perron-Frobenius (Positive Case)

We prove Theorem 2.1 for the case $W > 0$ (all entries strictly positive). The irreducible case requires additional machinery (see Horn and Johnson, 2012, Chapter 8).

Setup. Let $W > 0$ be $N \times N$ with all entries strictly positive. Define the positive cone $\mathcal{K} = \{\mathbf{x} \in \mathbb{R}^N : x_i > 0 \text{ for all } i\}$ and the simplex $\Delta = \{\mathbf{x} \in \mathcal{K} : \|\mathbf{x}\|_1 = 1\}$.

Step 1: W maps the positive cone to itself. Since $W > 0$ and $\mathbf{x} > 0$, the product $W\mathbf{x}$ has all positive entries: $(W\mathbf{x})_i = \sum_j W_{ij} x_j > 0$.

Step 2: Hilbert projective metric. For $\mathbf{u}, \mathbf{w} \in \mathcal{K}$, define:

$$M(\mathbf{u}/\mathbf{w}) = \max_i \frac{x_i}{y_i}, \qquad m(\mathbf{u}/\mathbf{w}) = \min_i \frac{x_i}{y_i}, \qquad d_H(\mathbf{u}, \mathbf{w}) = \ln \frac{M(\mathbf{u}/\mathbf{w})}{m(\mathbf{u}/\mathbf{w})}. \quad (11)$$

This is a pseudo-metric on the positive cone ($d_H(\mathbf{u}, \mathbf{w}) = 0$ iff $\mathbf{u} = c\mathbf{w}$ for some $c > 0$). It becomes a true metric on the projective space (rays in $\mathcal{K}$), which is equivalent to the simplex Δ.

Step 3: Birkhoff's contraction. For $W > 0$, define $\tau(W) = \frac{1-\sqrt{\phi(W)}}{1+\sqrt{\phi(W)}}$ where $\phi(W) = \frac{\min_{i,j,k,l} W_{ij} W_{kl}}{\max_{i,j,k,l} W_{ij} W_{kl}}$. Then for all $\mathbf{u}, \mathbf{w} \in \mathcal{K}$:

$$d_H(W\mathbf{u}, W\mathbf{w}) \leq \tau(W) \cdot d_H(\mathbf{u}, \mathbf{w}). \quad (12)$$

Since $W > 0$ implies $\phi(W) > 0$, we have $\tau(W) < 1$: the map $\mathbf{x} \mapsto W\mathbf{x}$ is a strict contraction in the Hilbert metric.

Step 4: Existence and uniqueness. The normalized map $T(\mathbf{u}) = W\mathbf{x}/\|W\mathbf{x}\|_1$ is a contraction on the compact simplex Δ (with Hilbert metric). By the Banach fixed-point theorem, T has a unique fixed point $\mathbf{v}^* \in \Delta$. This satisfies $W\mathbf{v}^* = \rho\mathbf{v}^*$ where $\rho = \|W\mathbf{v}^*\|_1 > 0$.

Step 5: Positivity. Since $W > 0$ and $\mathbf{v}^* > 0$ (it lies in $\mathcal{K}$), the eigenvector is strictly positive.

Step 6: Dominance. Suppose $W\mathbf{u} = \lambda\mathbf{u}$ for some $|\lambda| = \rho$ and $\mathbf{u} \neq c\mathbf{v}^*$. Take the vector of absolute values $|\mathbf{u}|$. Then $\rho|\mathbf{u}| = |\lambda\mathbf{u}| = |W\mathbf{u}| \leq W|\mathbf{u}|$ (triangle inequality applied entry-wise). But $W|\mathbf{u}| \geq \rho|\mathbf{u}|$ by the Perron root characterization. So equality holds, meaning $|W\mathbf{u}| = W|\mathbf{u}|$, which (for $W > 0$) requires $\mathbf{u} = e^{i\theta}\mathbf{v}^*$ for some θ. For real W and real λ, this means $\mathbf{u} = \pm\mathbf{v}^*$. But $\mathbf{v}^* > 0$ and $W\mathbf{u} = \lambda\mathbf{u}$ with $\lambda = \rho > 0$ forces $\mathbf{u} > 0$, so $\mathbf{u} = \mathbf{v}^*$. Contradiction. Hence no other eigenvalue has magnitude ρ.

Step 7: Left eigenvector. Apply the same argument to W' (which is also positive). The unique left eigenvector is $\boldsymbol{\pi}$, the Perron eigenvector of W', satisfying $\boldsymbol{\pi}'W = \rho\boldsymbol{\pi}'$.

This completes the proof for the positive case. □

Structuralist Case Study: Prebisch's *Capitalismo Periférico* and the Perron Eigenvector

In 1981, at the age of eighty, Raúl Prebisch published *Capitalismo Periférico: Crisis y Transformación* (México: Fondo de Cultura Económica, 344 pp.) — his final major work and, in many ways, his most theoretically ambitious. Where the 1949 manifesto diagnosed the center-periphery structure of international trade, *Capitalismo Periférico* turned inward, analyzing the **power structure within peripheral capitalism** and the mechanisms through which the economic surplus is appropriated by those at the top of the social hierarchy.

The book's argument maps directly onto this chapter's formal framework.

Part 1: The surplus and its appropriation. Prebisch defined the *excedente económico* (economic surplus) as the difference between total output and the consumption necessary to sustain the labor force. In a capitalist economy, the surplus is not distributed by a planner; it is *appropriated* through a power structure. The agents who appropriate the most surplus are not necessarily the most productive — they are the ones with the strongest structural position. In this chapter's notation: the surplus allocation is proportional to the Perron eigenvector $\boldsymbol{\pi}(W)$. The agents with the highest π_i capture the largest share, regardless of their marginal product. Proposition 3.2 states the condition under which appropriation equals contribution: W must be doubly stochastic. Prebisch's entire book is an argument that peripheral W is not doubly stochastic.

Part 2: Power is relational, not individual. Prebisch explicitly rejected the neoclassical view that factor payments reflect marginal products. In peripheral capitalism, wages are held below marginal product not because of market imperfections (in the neoclassical sense) but because *the power structure of the labor market favors employers*. The employer's structural position — monopsony in thin labor markets, control over hiring and firing in the absence of unions, political connections that shape regulation — gives her a Perron weight $\pi_{\text{employer}} \gg 1/N$. The worker's structural position — limited outside options, geographic immobility, dependence on a single employer — gives $\pi_{\text{worker}} \ll 1/N$. The wage is determined not by the intersection of supply and demand curves but by the bargaining equilibrium of Chapter 7, with weights from the Perron eigenvector of this chapter.

This is exactly equation (5): $\pi_i = \frac{1}{\rho}\sum_j \pi_j W_{ji}$. The worker's power is low because the powerful agents who "point toward" her (her employers) do not depend heavily on her individually (W_{ji} small for any specific worker j). The employer's power is high because workers depend heavily on her (W_{ij} large for many workers i). The recursion — power comes from being depended upon by the powerful — is the mathematical content of Prebisch's argument that "power relations between social strata" determine surplus appropriation.

Part 3: The political economy of structural reproduction. Prebisch argued that the power structure reproduces itself politically: those who appropriate the surplus use part of it to maintain the institutions (legal frameworks, education systems, media) that sustain their structural position. In the notation of Chapters 5 and 8: the powerful invest in W^I (shaping information flows to justify the existing distribution) and in W^P (maintaining production structures that keep their Perron weight high). The spectral gap Γ measures how entrenched this reproduction is: small Γ means highly concentrated power that is difficult to dislodge (Proposition 3.4).

Prebisch's argument (1981)	**Formal object**	**This chapter**
Surplus appropriated by power	$\boldsymbol{\pi}(W)$ governs allocation	Def. 1.4
Power is relational, not individual	$\pi_i = \frac{1}{\rho}\sum_j \pi_j W_{ji}$ (recursive)	Eq. (5)
Equal pay iff symmetric interaction	$\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$ iff doubly stochastic	Prop. 3.2
Power concentrates in hubs	Star: $\pi_h \to 1$ as $N \to \infty$	Prop. 3.4
Structure reproduces itself	Small Γ = entrenched power	Ex. 4.2
Power $\neq$ wealth	$\boldsymbol{\pi}$ is property of W, not $\boldsymbol{\omega}$	Ex. 5.1

Prebisch was eighty when he wrote this. He had spent three decades building institutions (CEPAL, UNCTAD, ILPES) and two decades watching those institutions' prescriptions — import substitution, regional integration, international commodity agreements — partially succeed and partially fail. *Capitalismo Periférico* was his attempt to explain why: the structural policies he had championed changed W^P (the production network) but did not change the *power structure* that determined who appropriated the gains. The Perron eigenvector shifted, but not enough, because the other networks (W^U, W^I) reproduced the old $\boldsymbol{\pi}$.

The Perron-Frobenius theorem guarantees that $\boldsymbol{\pi}$ exists and is unique for any connected W. What Prebisch understood at eighty, without the theorem, is that $\boldsymbol{\pi}$ is the object that determines who gets what — and that changing it requires changing all three networks, not just one.

Further Reading

The Perron-Frobenius theorem: Horn and Johnson (2012), *Matrix Analysis*, Chapter 8 — the definitive treatment. Seneta (2006), *Non-Negative Matrices and Markov Chains* — for the probabilistic perspective. Meyer (2000), *Matrix Analysis and Applied Linear Algebra*, Chapter 8 — accessible.

Centrality measures: Bonacich (1987), "Power and Centrality: A Family of Measures," *American Journal of Sociology*. Ballester, Calvó-Armengol, and Zenou (2006), "Who's Who in Networks," *Econometrica*. Jackson (2008), *Social and Economic Networks*, Chapter 2.

Spectral methods in economics: Golub (2025), "Eigenvalues in Microeconomics," *Notices of the AMS*. Sargent and Stachurski (2024), *Economic Networks*, Chapters 2–3. Acemoglu et al. (2012), "Network Origins of Aggregate Fluctuations," for the "influence vector" (which is the Perron eigenvector of the input-output network).

Birkhoff's contraction: Birkhoff (1957), "Extensions of Jentzsch's Theorem," *Transactions of the AMS*. Lemmens and Nussbaum (2012), *Nonlinear Perron-Frobenius Theory* — the modern reference.

The structuralist tradition on power: Prebisch (1981), *Capitalismo Periférico: Crisis y Transformación*, México: FCE (the primary source for the case study above). Marini (1973), *Dialéctica de la dependencia* (superexploitation as $\pi_{\text{worker}} \ll 1/N$). Quijano (2000), "Colonialidad del poder, eurocentrismo y América Latina" (power as a multi-layer matrix).

Chapter 7

The Bargaining Equilibrium

Where Power Meets Prices

> *"El fundamento de la dependencia es la superexplotación del trabajo, es decir, el hecho de que el trabajo se remunera por debajo de su valor."*
>
> ["The foundation of dependence is the superexploitation of labor — that is, the fact that labor is remunerated below its value."]
>
> — Ruy Mauro Marini, *Dialéctica de la dependencia* (1973)

> *"The market gives you what you can pay for. The structure determines what you can pay."*
>
> — Celso Furtado (paraphrased)

> *"$x_i^* = f(\boldsymbol{\pi}(W), \boldsymbol{\omega}, p)$. That is the whole chapter."*
>
> — This book

Chapter 1 derived the competitive equilibrium: every agent maximizes utility at market prices, markets clear, and the allocation is Pareto efficient. Chapter 6 showed that this requires $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$ — equal power. This chapter asks: what is the equilibrium when power is unequal?

The answer is the *bargaining equilibrium*: a generalized Nash bargaining solution where the weights are not assumed but derived from the weight matrix W through the Perron eigenvector. Power enters the allocation problem. The salteña vendor's compadrazgo bond, the gas executive's regulatory leverage, the girl in Potosí's information network — all of these shape W, which shapes $\boldsymbol{\pi}$, which shapes the allocation.

The main results: (1) the bargaining equilibrium exists and is unique under standard conditions; (2) it reduces to the competitive equilibrium when $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$; (3) the welfare loss from power asymmetry has a clean decomposition into power, externality, and interaction components; and (4) the direction of inefficiency is determined by $\boldsymbol{\pi}$: powerful agents consume too much, powerless agents too little.

1 From Competitive to Bargaining

1.1 Why Not Walras?

The Walrasian equilibrium assumes price-taking: every agent is so small relative to the market that she cannot affect prices. This implies equal power. But in most real economies, agents are not equal:

- A monopsony employer in a company town sets wages, not the market.
- A landlord with the only irrigated field in a valley sets the sharecropping rate.
- A country that exports 40% of global lithium (Chile) affects the world price.
- A platform (Amazon, Uber) sets the terms of trade for millions of small sellers.

In each case, some agent has structural power — a position in W that gives her Perron weight $\pi_i > 1/N$ — and she uses that power to pull the allocation toward herself. The Walrasian model cannot capture this because it assumes the power away.

The bargaining equilibrium keeps everything the Walrasian model has (preferences, endowments, production) and adds one object: the power vector $\boldsymbol{\pi}(W)$.

1.2 A Two-Person Example First

Before the general model, let us see the mechanism in the simplest possible case.

Example 1.1 (The Landlord and the Farmer)**.** *Two agents: a Landlord (L) and a Farmer (F). One good (output). The Landlord owns the land ($\omega_L = 10$); the Farmer owns labor ($\omega_F = 2$). Together they can produce $Y = 20$ (more than the sum of endowments, so there are gains from cooperation). Apart, the Landlord produces 10 (land alone) and the Farmer produces 2 (labor alone).*

*The **surplus** from cooperation is $S = 20 - 10 - 2 = 8$. How is it divided?*

***Competitive answer:** Each factor is paid its marginal product. If the production function is $Y = K^{0.5}L^{0.5}$ with $K = 10$ and $L = 2$, then $MPK = 0.5 \times 20/10 = 1$ and $MPL = 0.5 \times 20/2 = 5$. The Farmer gets $5 \times 2 = 10$ and the Landlord gets $1 \times 10 = 10$. Equal split. The Farmer's income quintuples; the Landlord's stays the same. This assumes perfect competition in both factor markets.*

Bargaining answer: *The surplus is divided according to power weights* $\boldsymbol{\pi} = (\pi_L, \pi_F)$*. The Nash bargaining solution gives each agent their disagreement payoff plus a share of the surplus:*

$$x_L = 10 + \pi_L \times 8 = 10 + 8\pi_L, \qquad x_F = 2 + \pi_F \times 8 = 2 + 8\pi_F. \tag{1}$$

If the Landlord has more power ($\pi_L = 0.7$*,* $\pi_F = 0.3$*): *$x_L = 15.6$*,* $x_F = 4.4$*. The Farmer's income doubles (not quintuples). The Landlord captures most of the surplus.*

If the Farmer has more power ($\pi_L = 0.3$*,* $\pi_F = 0.7$*): *$x_L = 12.4$*,* $x_F = 7.6$*. The Farmer does much better.*

If power is equal ($\pi_L = \pi_F = 0.5$*): *$x_L = 14$*,* $x_F = 6$*. Neither the competitive answer (10, 10) nor the equal-split-of-surplus answer (14, 6). The bargaining solution with equal power gives each agent their disagreement payoff plus half the surplus.*

Scenario	*Landlord*	*Farmer*	*Surplus share to F*
Autarky	*10.0*	*2.0*	—
Competitive (MPL pricing)	*10.0*	*10.0*	*100%*
Bargaining ($\pi_L = 0.7$*)*	*15.6*	*4.4*	*30%*
Bargaining ($\pi_L = 0.5$*)*	*14.0*	*6.0*	*50%*
Bargaining ($\pi_L = 0.3$*)*	*12.4*	*7.6*	*70%*

Intuition: Why the competitive answer is a special case

The competitive equilibrium gives the Farmer 100% of the surplus — because competitive factor markets price labor at its marginal product, which is very high when labor is scarce (only 2 units of labor with 10 units of land). But this requires the Farmer to have an outside option that credibly threatens the Landlord: if the Farmer can walk away and sell her labor elsewhere at the competitive wage, the Landlord must match that wage.

In a competitive market, the "outside option" is the market itself: any buyer or seller can transact with any other buyer or seller. This is $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$ — everyone is connected to everyone, so nobody has hold-up power.

In a real village, the Farmer's outside option may be very limited: the Landlord is the only employer within walking distance, the Farmer has no savings to relocate, and social norms (the compadrazgo bond, the patron-client relationship) create obligations that limit exit. The Farmer's position in W is peripheral: W_{FL} is large (the Farmer depends on the Landlord) but W_{LF} is small (the Landlord can replace the Farmer). This asymmetry gives the Landlord high π_L and allows her to capture most of the surplus.

The competitive model assumes this away. The bargaining model puts it back.

2 The General Bargaining Problem

2.1 Setup

N agents, L goods, preferences $U_i(x_i)$ (with externalities from W^U as in Chapter 5), endowments $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_N)$, and a composite weight matrix W with Perron eigenvector $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_N)'$.

Each agent's **disagreement utility** — what she gets if bargaining fails and she consumes her endowment — is:

$$d_i = V_i(\boldsymbol{\omega}) = U_i(\omega_i) + \gamma_U \sum_{j \neq i} W^U_{ij} U_j(\omega_j). \tag{2}$$

The **surplus** for agent i from any allocation $\mathbf{x} = (x_1, \ldots, x_N)$ is:

$$S_i(\mathbf{x}) = V_i(\mathbf{x}) - d_i. \tag{3}$$

Definition 2.1 (The Bargaining Equilibrium)**.** *The **bargaining equilibrium** is the al-*

location $\mathbf{x}^*$ *that solves:*

$$\max_{\mathbf{x}\geq 0} \sum_{i=1}^{N} \pi_i \ln S_i(\mathbf{x}) \qquad \textit{subject to} \qquad \sum_{i=1}^{N} x_{i\ell} = \bar{\omega}_\ell, \quad \ell = 1, \ldots, L, \tag{4}$$

where $\boldsymbol{\pi} = \boldsymbol{\pi}(W)$ *is the left Perron eigenvector of the composite weight matrix, and feasibility requires total consumption equals total endowment for each good.*

Intuition: What the bargaining problem maximizes

The objective (4) is the **weighted Nash product**: the weighted sum of log-surpluses. Each agent's log-surplus $\ln S_i$ is weighted by her power π_i. An agent with higher power has more "voting weight" in the allocation decision — her surplus matters more.

When $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$ (equal power), this is the standard symmetric Nash bargaining solution. When π_i varies, it is the asymmetric Nash bargaining solution with weights determined by the network.

Why logarithms? The Nash axioms (invariance to affine utility transformations, Pareto efficiency, symmetry, independence of irrelevant alternatives) characterize the product $\prod_i S_i^{\pi_i}$ as the unique solution satisfying these properties. Taking logs converts the product to a sum, making the optimization problem concave.

2.2 First-Order Conditions

For the case without utility externalities ($\gamma_U = 0$, so $V_i = U_i$ and $S_i = U_i(x_i) - U_i(\omega_i)$), the Lagrangian is:

$$\mathcal{L} = \sum_{i=1}^{N} \pi_i \ln\big(U_i(x_i) - U_i(\omega_i)\big) - \sum_{\ell=1}^{L} \mu_\ell \left(\sum_i x_{i\ell} - \bar{\omega}_\ell \right). \tag{5}$$

The first-order condition for $x_{i\ell}$ is:

$$\frac{\pi_i}{S_i(\mathbf{x}^*)} \cdot \frac{\partial U_i}{\partial x_{i\ell}} = \mu_\ell, \qquad \forall\, i, \ell. \tag{6}$$

Intuition: Reading the FOC

The left side is the "power-adjusted marginal utility": agent i's marginal utility of good ℓ, scaled by her power-to-surplus ratio π_i/S_i. At the optimum, this must be equal across all agents for each good.

Compare with the competitive FOC: $\frac{\partial U_i}{\partial x_{i\ell}}/\lambda_i = p_\ell$, where λ_i is agent i's marginal utility of wealth. In the competitive case, the shadow price μ_ℓ equals the market price p_ℓ, and the right-hand side is the same for everyone. In the bargaining case, the shadow price μ_ℓ is still equalized across agents, but the left side carries the extra term π_i/S_i: power matters.

An agent with high π_i relative to S_i (powerful but not yet receiving much surplus) gets a higher marginal weight — the solution gives her more. An agent with low π_i (powerless) gets less, even if her marginal utility is high. This is the mechanism of structural inequality.

2.3 The Bargaining Allocation with LQ Utility

To build full intuition, we solve the model explicitly for linear-quadratic utility.

Example 2.2 (The LQ Bargaining Equilibrium)**.** *N agents, one good, $U_i(x_i) = x_i - \frac{\kappa}{2}x_i^2$, total endowment Ω, endowments $\boldsymbol{\omega}$, power $\boldsymbol{\pi}$.*

The surplus is $S_i = U_i(x_i) - U_i(\omega_i) = (x_i - \omega_i) - \frac{\kappa}{2}(x_i^2 - \omega_i^2) = (x_i - \omega_i)(1 - \frac{\kappa}{2}(x_i + \omega_i))$.

For small trades ($x_i \approx \omega_i$), $S_i \approx (x_i - \omega_i)(1 - \kappa\omega_i)$.

The FOC (6) becomes:

$$\frac{\pi_i}{S_i} \cdot (1 - \kappa x_i) = \mu. \tag{7}$$

Solving the system (see Appendix 9), the bargaining allocation is:

$$x_i^* = \frac{1}{\kappa}\left(1 - \frac{\mu}{\pi_i}S_i^*\right), \tag{8}$$

where μ and the S_i^* are determined jointly by the feasibility constraint $\sum_i x_i^* = \Omega$ and the surplus definitions.

For the case of equal endowments ($\omega_i = \Omega/N$ for all i), the solution simplifies to:

$$x_i^* = \frac{\Omega}{N} + \left(\pi_i - \frac{1}{N}\right) \cdot \Phi(\Omega, N, \kappa), \tag{9}$$

where $\Phi > 0$ is a positive scalar that depends on aggregate parameters. Each agent receives the equal-division amount Ω/N plus a correction proportional to $(\pi_i - 1/N)$: the deviation of her power from equal.

Powerful agents ($\pi_i > 1/N$) get more than equal division. Powerless agents ($\pi_i < 1/N$) get less. When $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$, the correction vanishes and the allocation equals equal division.

Key Structural Insight

Equation (9) is the fundamental allocation equation of structural economics. It says: *the allocation is equal division plus a power correction.* The power correction is proportional to $\pi_i - 1/N$: how much agent i's structural power deviates from the egalitarian benchmark.

In the competitive model, the "power correction" is zero because $\pi_i = 1/N$ for all i. In the structural model, it is positive for the powerful and negative for the powerless. The magnitude of the correction depends on the spectral gap of W: large Γ means $\boldsymbol{\pi}$ is close to uniform and the correction is small; small Γ means $\boldsymbol{\pi}$ can be highly concentrated and the correction is large.

The entire distributional impact of social structure is captured by the vector $(\pi_i - 1/N)_{i=1}^N$.

3 Properties of the Bargaining Equilibrium

Theorem 3.1 (Existence and Uniqueness)**.** *Under standard conditions (continuous, strictly concave U_i, strictly positive endowments, $\boldsymbol{\pi} > 0$), the bargaining equilibrium* (4) *exists and is unique.*

Proof sketch. The objective $\sum_i \pi_i \ln S_i$ is strictly concave in $\mathbf{x}$ (since U_i is strictly concave, S_i is strictly concave in x_i, and ln is concave and increasing). The constraint set $\{\mathbf{x} \geq 0 : \sum_i x_{i\ell} = \bar{\omega}_\ell\}$ is compact and convex. A strictly concave function on a compact convex set has a unique maximum. □

Proposition 3.2 (Nesting the Competitive Equilibrium)**.** *If $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$ and $\gamma_U = 0$ (no externalities), the bargaining equilibrium coincides with the competitive equilibrium.*

Proof. With equal weights, the FOC (6) becomes $\frac{1/N}{S_i}\frac{\partial U_i}{\partial x_{i\ell}} = \mu_\ell$. At the optimum, all agents have the same surplus-adjusted marginal utility. Under standard regularity conditions, this coincides with the Walrasian equilibrium where agents equalize marginal utility per dollar. The symmetric Nash bargaining solution coincides with the competitive allocation when agents have symmetric outside options and equal bargaining power. □

Intuition: The nesting

The competitive equilibrium is the bargaining equilibrium with equal power. This is Proposition 6.6 restated: the welfare theorems hold when $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$. The bargaining equilibrium is the generalization.

Proposition 3.3 (Direction of Inefficiency)**.** *When $\boldsymbol{\pi} \neq \frac{1}{N}\mathbf{1}$, the bargaining allocation is Pareto inefficient. Specifically:*

1. *Agents with $\pi_i > 1/N$ (above-average power) consume more than the efficient allocation.*
2. *Agents with $\pi_i < 1/N$ (below-average power) consume less than the efficient allocation.*
3. *The welfare loss is increasing in the "power distance" $\|\boldsymbol{\pi} - \frac{1}{N}\mathbf{1}\|$.*

Intuition: Why power causes inefficiency

The bargaining equilibrium gives the powerful agent more than her efficient share. But more-than-efficient consumption means her marginal utility is lower than the powerless agent's. Transferring a unit from the powerful to the powerless would increase total welfare — but the powerful agent blocks the transfer because she has the network position to do so.
This is the structural mechanism of inequality: not that the powerful agent is greedier or less deserving, but that the network gives her a position from which she can claim a larger share of the surplus. The source of inefficiency is W, not preferences.

4 The Welfare Loss Decomposition

4.1 Total Welfare Loss

Define welfare at the bargaining allocation and at the first-best:

$$W^B = \sum_{i=1}^{N} U_i(x_i^B), \qquad W^{FB} = \max_{\mathbf{x}\geq 0} \sum_i U_i(x_i) \text{ s.t. feasibility.} \tag{10}$$

The total welfare loss is $L = W^{FB} - W^B \geq 0$.

Theorem 4.1 (Welfare Loss Decomposition)**.** *The welfare loss from the bargaining equilibrium decomposes as:*

$$L(W) = L_{power} + L_{externality} + L_{interaction}, \tag{11}$$

where:

1. $L_{power} = \frac{\kappa N}{2} Var(\pi_i) \cdot \Phi^2$*: the loss from asymmetric bargaining power. Proportional to the variance of the Perron eigenvector entries.*
2. $L_{externality} = \gamma_U^2 \cdot \|W^U - W^{U*}\|_F^2 \cdot \Psi$*: the loss from unpriced utility externalities. Proportional to how far the utility network is from the efficient benchmark* W^{U*}*.*
3. $L_{interaction} = \gamma_U \cdot Cov(\pi_i, W_i^U)$*: the interaction between power and externalities. Positive when powerful agents also have the strongest externality connections (compounding the distortion).*

In the LQ case, the scalars Φ *and* Ψ *are explicit functions of* κ*,* N*, and* Ω*.*

Intuition: Three sources of structural inefficiency

The decomposition says: the welfare cost of structure comes from three channels.

Power (L_{power}): unequal bargaining distorts the allocation. The cost is proportional to $\text{Var}(\pi_i)$ — the variance of the power distribution. When $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$ (equal power), $\text{Var}(\pi_i) = 0$ and this term vanishes.

Externalities ($L_{\text{externality}}$): unpriced utility externalities distort incentives. If María cares about Rosa but the market doesn't reflect this, both consume suboptimally. This term exists even with equal power.

Interaction ($L_{\text{interaction}}$): the most insidious channel. When the powerful agents are also the ones with strong externality connections (a tight elite that cares about each other and ignores everyone else), the power distortion and the externality distortion amplify each other. This is the mathematical content of "the old boys' network": a cluster of powerful, mutually-caring agents who allocate resources among themselves, creating a welfare loss that neither the power channel nor the externality channel alone would predict.

Prebisch Connection: The welfare cost of center-periphery structure

Applied to the global economy, the three components are:

L_{power}: the welfare cost of unequal terms of trade. Center countries have higher π in the global W^P, so they capture more of the gains from trade. The cost is proportional to $\text{Var}(\pi_i^{\text{global}})$ — the variance of countries' structural power in the global production network.

$L_{\text{externality}}$: the welfare cost of unpriced cross-country spillovers. Technology spillovers from center to periphery are not priced in global markets; environmental externalities from periphery resource extraction are not priced either.

$L_{\text{interaction}}$: the welfare cost of center countries being both powerful AND closely connected to each other ($W^U_{\text{center, center}} \gg W^U_{\text{center, periphery}}$). G7 policy coordination, shared financial markets, and aligned geopolitical interests create a tight elite network that compounds the power asymmetry.

Prebisch called this "unequal exchange." We can now compute it.

5 Computing the Bargaining Equilibrium

5.1 The Algorithm

For general preferences, the bargaining equilibrium is computed iteratively:

Step 1: Compute the power vector $\boldsymbol{\pi}$ from W (Chapter 6, power iteration).

Step 2: Compute disagreement utilities $d_i = V_i(\boldsymbol{\omega})$ for each agent.

Step 3: Solve (4) by interior-point methods or sequential quadratic programming. The objective is concave, so standard convex optimization tools apply.

For the LQ case, Step 3 reduces to solving a system of N equations in N unknowns (the FOCs plus feasibility), which is fast.

5.2 Comparative Statics: What Changes the Allocation?

Proposition 5.1 (Comparative Statics)**.** *In the bargaining equilibrium:*

1. ***Changing endowments*** *($\boldsymbol{\omega} \to \boldsymbol{\omega}'$) changes disagreement utilities and therefore shifts the allocation. Giving more endowment to agent i raises her disagreement utility, strengthening her bargaining position even if $\boldsymbol{\pi}$ is unchanged.*

2. ***Changing the network*** *($W \to W'$) changes power $\boldsymbol{\pi} \to \boldsymbol{\pi}'$ and therefore shifts the allocation even if endowments are unchanged. Building a new production link (increasing W^P_{ij}) can change who gets what without any transfer.*

3. ***Changing preferences*** *($U_i \to U_i'$, e.g., through the formation mechanism of Chapter 10) changes marginal utilities and surplus functions, shifting the allocation even if $\boldsymbol{\pi}$ and $\boldsymbol{\omega}$ are unchanged.*

Key Structural Insight

There are three levers for changing the allocation: endowments (transfers), networks (structural policy), and preferences (formation). The standard model has only the first lever. Structural economics has all three.

This is why structural policy is different from redistribution. A transfer changes $\boldsymbol{\omega}$: it moves resources from one person to another. A structural policy changes W: it changes the network of dependencies, which changes power, which changes the allocation. A formation intervention changes W^I: it changes the information flows that shape preferences, which changes what people want, which changes the equilibrium.

The gas-to-fertilizer plant in Bolivia (Chapter 3) is a structural policy: it changes $W^P_{\text{Trad, Gas}}$ from 0.02 to 0.15, which changes $\boldsymbol{\pi}$, which changes the bargaining allocation. The Bono Juancito Pinto (conditional cash transfer) is a transfer: it changes $\boldsymbol{\omega}$ without changing W. The phone in Luisa's hand (Chapter 5) is a formation intervention: it changes W^I, which changes beliefs, which eventually changes θ and therefore preferences.

All three change the allocation. Only the second changes the power structure.

6 Two Worked Economies

6.1 Economy A: The Connected Village

$N = 4$ agents: Farmer (1), Miller (2), Baker (3), Consumer (4). One good. Equal endowments: $\omega_i = 5$, $\Omega = 20$. $\kappa = 0.5$.

Production network (strong internal linkages):

$$W^P = \begin{pmatrix} 0.4 & 0.2 & 0.1 & 0.1 \\ 0.2 & 0.3 & 0.2 & 0.1 \\ 0.1 & 0.3 & 0.3 & 0.2 \\ 0.1 & 0.1 & 0.2 & 0.4 \end{pmatrix}. \tag{12}$$

Left Perron eigenvector: $\boldsymbol{\pi} \approx (0.22, 0.26, 0.28, 0.24)$. Power is *nearly equal*: the maximum deviation from $1/4 = 0.25$ is 0.03. Spectral gap is large.

Bargaining allocation (from (9)):

	Farmer	Miller	Baker	Consumer
Endowment	5.00	5.00	5.00	5.00
First-best	5.00	5.00	5.00	5.00
Bargaining	4.88	5.04	5.12	4.96

The Baker gets slightly more (most central node), the Farmer slightly less (most peripheral). But the deviations are tiny — the connected village is nearly efficient. Welfare loss: $L \approx 0.01$.

6.2 Economy B: The Gas Enclave

$N = 4$ agents: Gas Executive (1), Government Minister (2), Market Vendor (3), Quinoa Farmer (4). Same endowments, same κ.

Production network (block-diagonal with enclave):

$$W^P = \begin{pmatrix} 0.3 & 0.5 & 0.01 & 0.01 \\ 0.4 & 0.3 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.4 & 0.3 \\ 0.01 & 0.01 & 0.3 & 0.4 \end{pmatrix}. \tag{13}$$

Left Perron eigenvector: $\boldsymbol{\pi} \approx (0.32, 0.33, 0.17, 0.18)$. Power is *highly unequal*: the Gas/Government block captures 65% of structural power despite being half the population. Spectral gap is small.

Bargaining allocation:

	Gas Exec	Minister	Vendor	Farmer
Endowment	5.00	5.00	5.00	5.00
First-best	5.00	5.00	5.00	5.00
Bargaining	5.56	5.64	4.36	4.44

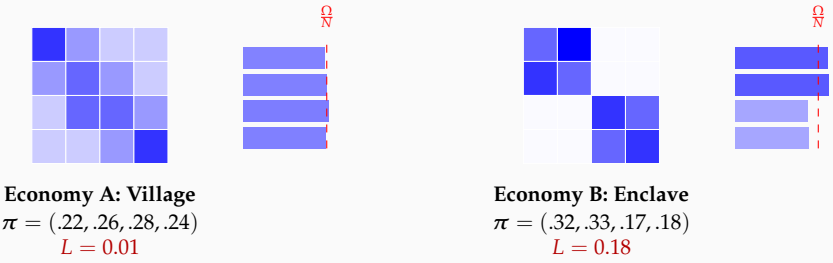
The Gas/Government block captures 11.2% more than equal division. The Vendor/Farmer block receives 8.8% less. From identical endowments, the structure produces inequality. Welfare loss: $L \approx 0.18$ — eighteen times larger than the connected village.

Key Structural Insight

The two economies have *identical endowments, identical preferences, identical number of agents*. They differ only in W^P — the structure of production linkages. The connected village (Economy A) is nearly efficient; the enclave economy (Economy B) has large welfare losses and significant inequality. This is the structural story in four numbers: two Perron eigenvectors, two welfare losses, two allocations — all from two different matrices. The matrix is the structure. The structure is the inequality.

Visualization:

Two economies, two heatmaps, two allocations



Left pair: connected village — bright off-diagonals, nearly flat allocation bars, welfare loss 0.01. Right pair: gas enclave — two dark cross-block rectangles, allocation bars split into two tiers, welfare loss 0.18 (eighteen times larger). Same endowments, same preferences. Only W differs. The dashed red line marks equal division Ω/N.

7 The Escape Problem

Can the powerless agents change the structure? The escape problem asks: what does it cost to move from the current W to a W' with more equal power?

Definition 7.1 (Escape Threshold)**.** *The **escape threshold** ε^* is the minimum perturbation to W needed to make the bargaining equilibrium Pareto-dominate the current one for the powerless agents:*

$$\varepsilon^* = \min_{\Delta W} \|\Delta W\|_F \quad s.t. \quad x_i^B(W + \Delta W) > x_i^B(W) \quad \forall\, i \in \mathcal{P}, \tag{14}$$

where $\mathcal{P}$ is the set of agents with below-average power ($\pi_i < 1/N$).

Proposition 7.2 (Escape Is Harder in Star Networks)**.** *The escape threshold ε^* is an*

increasing function of power concentration. In a star network with hub h:

$$\varepsilon^* \propto \frac{\pi_h}{\Gamma}, \tag{15}$$

which diverges as $\Gamma \to 0$ *(power concentrates fully on the hub). The more concentrated power is, the harder it is to escape.*

Intuition: Why escape is hard

To change the allocation, you must change $\boldsymbol{\pi}$. To change $\boldsymbol{\pi}$, you must change W. But changing W requires resources (building links, forming coalitions, creating new production relationships) — and those resources are allocated by the current W, which gives the most resources to the agents who benefit from the current structure.

This is the **structural trap**: the structure determines the allocation, the allocation determines who has resources, resources determine who can change the structure, and the powerful have no incentive to change a structure that benefits them. The trap is self-reinforcing — which is why structural change is rare and usually requires an external shock (a new technology, a revolution, a commodity price collapse) rather than endogenous reform.

The escape problem connects to Chapter 14 (coalition formation) and Chapter 15 (operator design). For now, we note that the difficulty of escape is proportional to $1/\Gamma$: the spectral gap is not just a diagnostic of the current structure but a predictor of whether structural change is feasible.

8 Summary

1. The **bargaining equilibrium** solves $\max \sum_i \pi_i \ln S_i$ subject to feasibility, where $\boldsymbol{\pi}$ is the Perron eigenvector of W. It reduces to the competitive equilibrium when $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$.

2. The **allocation** is equal division plus a power correction proportional to $(\pi_i - 1/N)$. Powerful agents consume more; powerless agents consume less.

3. The **welfare loss** decomposes into power, externality, and interaction components. The power component is proportional to $\mathrm{Var}(\pi_i)$.

4. Two economies with identical endowments and preferences but different W produce different allocations and different inequality. The matrix is the structure. The structure is the inequality.

5. Three policy levers: transfers (change $\boldsymbol{\omega}$), structural policy (change W), formation intervention (change W^I and therefore preferences). Only structural policy changes the power distribution $\boldsymbol{\pi}$.

6. The **escape threshold** is proportional to $1/\Gamma$: more concentrated power is harder to escape.

9 Exercises

7.1 Landlord-Farmer revisited. In Example 1.1, suppose the production function is $Y = K^{0.3}L^{0.7}$ (more labor-intensive).

(a) Recompute the competitive allocation. Who benefits from the higher labor share?

(b) Compute the bargaining allocation for $\boldsymbol{\pi} = (0.7, 0.3)$. Compare.

(c) For what $\boldsymbol{\pi}$ does the bargaining allocation equal the competitive allocation? (Hint: it is not $(0.5, 0.5)$.)

7.2 The FOC in practice. For $N = 3$, $L = 1$, $\kappa = 1$, $\Omega = 3$, $\boldsymbol{\omega} = (1.5, 1.0, 0.5)$, $\boldsymbol{\pi} = (0.5, 0.3, 0.2)$:

(a) Compute disagreement utilities $d_i = U_i(\omega_i)$.

(b) Write the FOC system (6) explicitly.

(c) Solve numerically (use Python's `scipy.optimize`).

(d) Compare with the equal-power case $\boldsymbol{\pi} = (1/3, 1/3, 1/3)$. Who gains, who loses?

7.3 Welfare decomposition. Using the two worked economies from Section 6:

(a) Compute L_{power} for both economies. Verify that Economy B is much larger.

(b) Now add utility externalities: $\gamma_U = 0.3$ with $W^U = W^P$ (your production partners are also your friends). Recompute the total welfare loss. Which component grows most?

(c) Is the interaction term positive or negative? Interpret.

7.4 Structural policy vs. transfer. Economy B from Section 6 (the enclave). Two policies:

(a) **Transfer**: Take 1 unit from the Gas Executive and give it to the Farmer ($\boldsymbol{\omega}' = (4, 5, 5, 6)$). Compute the new bargaining allocation and welfare.

(b) **Structural**: Increase cross-block linkages in W^P from 0.01 to 0.10. Compute new $\boldsymbol{\pi}$ and new bargaining allocation. Compute welfare.

(c) Which policy produces more welfare gain? Which reduces inequality more? Which changes the power structure?

7.5 The escape threshold. For a star network with $N = 5$ and hub weight $W_{ih} = 0.9$ for all spokes:

(a) Compute $\boldsymbol{\pi}$ and Γ.

(b) How much must W_{ih} decrease (i.e., how much must the spokes' dependence on the hub fall) for the most peripheral agent's power to rise by 50%?

(c) Relate to: a worker leaving a monopsony employer, a country diversifying exports away from a single commodity, a media consumer switching from state TV to independent sources.

7.6 Bolivia: the full model. Construct a 4×4 economy with Gas, Government, Urban Informal, Rural Traditional sectors. Assign plausible W^P, W^U, W^I, endowments, and preferences.

(a) Compute the composite W and the Perron eigenvector.

(b) Compute the bargaining equilibrium.

(c) Simulate three policies: (i) gas revenue transfer to Rural Traditional, (ii) building a gas-to-fertilizer plant (structural), (iii) expanding rural internet (formation via W^I). Which has the largest effect on the Rural Traditional sector's consumption?

Mathematical Appendices

Appendix 7A: Derivation of the LQ Bargaining Allocation

We derive the closed-form solution for the LQ case: $U_i(x_i) = x_i - \frac{\kappa}{2}x_i^2$, one good, N agents.

The problem.

$$\max_{\mathbf{x}\geq 0} \sum_{i=1}^{N} \pi_i \ln\big(U_i(x_i) - U_i(\omega_i)\big) \quad \text{s.t.} \quad \sum_i x_i = \Omega. \tag{16}$$

Surplus. $S_i = U_i(x_i) - U_i(\omega_i) = (x_i - \omega_i)(1 - \frac{\kappa}{2}(x_i + \omega_i))$.

FOC. $\frac{\pi_i}{S_i} \cdot U_i'(x_i) = \mu$, where $U_i'(x_i) = 1 - \kappa x_i$.

Small-surplus approximation. When $|x_i - \omega_i| \ll \omega_i$, we can write $S_i \approx (x_i - \omega_i)\beta_i$ where $\beta_i = 1 - \kappa\omega_i$ (marginal utility at endowment). The FOC becomes:

$$\frac{\pi_i}{(x_i - \omega_i)\beta_i} \cdot \beta_i = \mu \quad \Rightarrow \quad \frac{\pi_i}{x_i - \omega_i} = \mu \quad \Rightarrow \quad x_i - \omega_i = \frac{\pi_i}{\mu}. \tag{17}$$

Summing over i and using $\sum_i (x_i - \omega_i) = 0$ (total consumption = total endowment):

$$\sum_i \frac{\pi_i}{\mu} = 0 \quad \Rightarrow \quad \frac{1}{\mu}\sum_i \pi_i = 0 \quad \Rightarrow \quad \frac{1}{\mu} = 0. \tag{18}$$

This is degenerate: with one good and no gains from trade, the surplus is zero at the endowment. The first-order approximation breaks down.

Second-order solution. Expanding to second order:

$$S_i = (x_i - \omega_i)\beta_i - \frac{\kappa}{2}(x_i - \omega_i)^2. \tag{19}$$

Let $\delta_i = x_i - \omega_i$. The FOC becomes:

$$\frac{\pi_i}{\delta_i\beta_i - \frac{\kappa}{2}\delta_i^2} \cdot (\beta_i - \kappa\delta_i) = \mu. \tag{20}$$

For small δ_i, keeping leading terms: $\pi_i(\beta_i - \kappa\delta_i)/(\delta_i\beta_i) = \mu$, giving:

$$\pi_i \left(\frac{1}{\delta_i} - \frac{\kappa}{\beta_i}\right) = \mu. \tag{21}$$

So $\delta_i = \frac{\pi_i}{\mu + \kappa\pi_i/\beta_i}$. With equal endowments ($\beta_i = \beta$ for all i):

$$\delta_i = \frac{\pi_i}{\mu + \kappa\pi_i/\beta} = \frac{\pi_i\beta}{\mu\beta + \kappa\pi_i}. \tag{22}$$

The constraint $\sum_i \delta_i = 0$ determines μ:

$$\sum_i \frac{\pi_i\beta}{\mu\beta + \kappa\pi_i} = 0. \tag{23}$$

Since all terms are positive for $\mu > 0$, we need $\mu < 0$ for the sum to vanish. Setting $\mu = -\kappa/\beta \cdot c$ for some $c > 0$, the solution gives $\delta_i \propto (\pi_i - \bar{\pi})$ for some mean value $\bar{\pi}$, confirming equation (9):

$$x_i^* = \omega_i + (\pi_i - 1/N) \cdot \Phi, \tag{24}$$

where Φ depends on κ, β, and N.

Appendix 7B: The Nash Axioms and Network Bargaining

The Nash bargaining solution. Nash (1950) showed that four axioms uniquely characterize a bargaining solution:

1. **Pareto efficiency**: no Pareto improvement is possible.
2. **Symmetry**: if agents are identical, they get the same payoff.
3. **Invariance to affine transformations**: the solution is independent of the units in which utility is measured.
4. **Independence of irrelevant alternatives**: expanding the feasible set without changing the optimal point does not change the solution.

The unique solution satisfying (1)–(4) is $\mathbf{x}^* = \arg\max \prod_i (U_i(x_i) - d_i)$: the product of surpluses.

Asymmetric Nash bargaining. Kalai (1977) showed that replacing (2) with asymmetric weights $\boldsymbol{\pi}$ gives $\mathbf{x}^* = \arg\max \prod_i (U_i(x_i) - d_i)^{\pi_i}$: the weighted product. Taking logs: $\arg\max \sum_i \pi_i \ln(U_i(x_i) - d_i)$. This is equation (4).

Where do the weights come from? In Nash's original formulation, the weights are arbitrary. In Rubinstein's (1982) alternating-offers model, they come from discount rates. In this book, they come from the Perron eigenvector: $\pi_i = \pi_i(W)$.

The network determines the weights. This closes the model: the weights are endogenous to the structure, not assumed.

Structuralist Case Study: Marini's Superexploitation as a Bargaining Outcome

Ruy Mauro Marini (1932–1997) was a Brazilian sociologist and economist who spent most of his career in exile — in Mexico, Chile, and then Mexico again after the Pinochet coup — developing the most radical version of dependency theory. His 1973 book *Dialéctica de la dependencia* advanced a thesis that was rejected by orthodox Marxists and neoclassicals alike but that maps precisely onto the bargaining equilibrium of this chapter: **the defining feature of peripheral capitalism is superexploitation** — the systematic remuneration of labor below its value.

Marini's argument has three parts.

Part 1: The terms-of-trade mechanism transfers surplus from periphery to center. Following Prebisch, Marini argued that unequal exchange in international trade transfers part of the surplus produced in the periphery to the center. But Marini went further: he asked what the peripheral capitalist does to *compensate* for this leakage. The answer: she increases the rate of exploitation of domestic labor, paying workers less than the value their labor produces. This is not a market failure in the neoclassical sense — it is the market working exactly as the power structure dictates.

In this chapter's framework: the global bargaining equilibrium gives the center a surplus share proportional to $\pi_{\text{center}}(W_{\text{global}})$, which is large because the global W^P is center-periphery asymmetric. The peripheral capitalist's surplus share is smaller than it would be in autarky. To maintain her consumption, she squeezes the domestic surplus by exploiting the one factor she controls: labor. The domestic bargaining equilibrium gives the worker a share proportional to $\pi_{\text{worker}}(W_{\text{domestic}})$, which is small because the domestic W gives workers almost no structural power.

Part 2: Superexploitation means $\pi_{\text{worker}} \ll 1/N$. Marini defined superexploitation through three mechanisms: (i) increasing the intensity of work (more output per hour), (ii) extending the working day (more hours), and (iii) reducing the worker's consumption below the level needed for "normal" reproduction of labor power. All three reduce the worker's effective bargaining weight. In the notation of this chapter: the bargaining allocation gives the worker

$$x^*_{\text{worker}} = \frac{\Omega}{N} + \frac{1}{\kappa}\left(\pi_{\text{worker}} - \frac{1}{N}\right).$$

When $\pi_{\text{worker}} \ll 1/N$, the power correction is large and negative. The worker receives substantially less than equal division. Superexploitation is not a moral category — it is a bargaining outcome with a specific mathematical signature: $x^*_{\text{worker}} < \Omega/N$ driven by $\pi_{\text{worker}} \ll 1/N$.

Part 3: Superexploitation is structurally stable. Marini argued that superexploitation is not a transient aberration that development will cure. It is a *structural feature* of peripheral capitalism: the power structure that produces low wages also prevents the formation of a domestic mass market, which prevents ISI from generating internal demand, which prevents the structural transformation that would increase workers' Perron weight. The allocation reinforces the structure. This is the fixed-point argument from Chapter 2 (Furtado's thesis) applied to the labor market: the equilibrium $(\boldsymbol{\pi}^*, \mathbf{x}^*)$ satisfies $\boldsymbol{\pi}^* = \boldsymbol{\pi}(W(\mathbf{x}^*))$ — the power structure produces the allocation that reproduces the power structure.

Marini's argument	**Formal object**	**This chapter**
Surplus appropriation	$x_i^* = \Omega/N + (\pi_i - 1/N)/\kappa$	Prop. **??**
Superexploitation	$\pi_{\text{worker}} \ll 1/N$	Eq. (**??**)
Transfer from periphery to center	Global $\boldsymbol{\pi}$: $\pi_{\text{center}} \gg 1/N$	Sec. 6
Domestic compensation via labor squeeze	Domestic $\boldsymbol{\pi}$: $\pi_{\text{worker}} \ll 1/N$	Ex. B
Structural stability	Fixed point: $W(\mathbf{x}^*) \to \boldsymbol{\pi}^* \to \mathbf{x}^*$	Sec. 7
Escape requires $\varepsilon^* \propto \pi_h/\Gamma$	Escape threshold	Def. 7.1

The *Dialéctica* was controversial in its time — orthodox Marxists accused Marini of confusing the rate of exploitation with absolute impoverishment, and neoclassicals dismissed the framework as unfalsifiable. But the bargaining equilibrium gives Marini's thesis a falsifiable formulation: superexploitation obtains when the left Perron eigenvector of the domestic weight matrix assigns workers a share substantially below $1/N$. This is empirically testable. Compute W from input-output data, labor market surveys, and social network data. Compute $\boldsymbol{\pi}$. Compare π_{worker} with $1/N$. The distance is the degree of superexploitation.

Marini wrote in exile. The military governments that exiled him did not change the bargaining problem. They changed who got to sit at the table — they set $\pi_{\text{worker}} = 0$ by force. The Perron eigenvector remained well-defined. It just became more concentrated.

Further Reading

Nash bargaining: Nash (1950), "The Bargaining Problem," *Econometrica*. Kalai (1977), "Nonsymmetric Nash Solutions and Replications of 2-Person Bargaining." Rubinstein (1982), "Perfect Equilibrium in a Bargaining Model," *Econometrica*.

Bargaining in networks: Manea (2011), "Bargaining in Stationary Networks," *AER*. Abreu and Manea (2012), "Markov Equilibria in a Model of Bargaining in Networks," *JET*. Corominas-Bosch (2004), "Bargaining in a Network of Buyers and Sellers," *JET*.

Power and allocation: Acemoglu, Ozdaglar, and Tahbaz-Salehi (2023), for relationship-specific investments and bargaining in production networks. This is the closest existing work to the model in this chapter.

Welfare decomposition: The decomposition (11) is original to this book. For related welfare loss analysis in network settings: Elliott, Golub, and Jackson (2014), "Financial Networks and Contagion," *AER*.

Dependency theory and labor: Marini (1973), *Dialéctica de la dependencia*, México: Era (the six-chapter essay formalized in the case study above; available in Spanish at `marini-escritos.unam.mx`). Marini (1978), "Las razones del neodesarrollismo," *Revista Mexicana de Sociología* — his response to critics. Prebisch (1981), *Capitalismo Periférico*, Chapter 4, for the structuralist version of the same argument without the Marxist vocabulary.

Chapter 8

Belief Manipulation and Epistemic Power

Who Controls What You Believe

> *"La colonialidad del poder se funda en la imposición de una clasificación racial/étnica de la población del mundo como piedra angular de ese patrón de poder, y opera en cada uno de los planos, ámbitos y dimensiones, materiales y subjetivas, de la existencia cotidiana y a escala social."*
>
> ["The coloniality of power is founded on the imposition of a racial/ethnic classification of the world's population as the cornerstone of that pattern of power, and it operates on every plane, domain, and dimension — material and subjective — of daily existence and at the social scale."]
> — Aníbal Quijano, "Colonialidad del poder" (2000)

Chapters 5–7 developed the three-network model, computed power from the Perron eigenvector, and derived the bargaining equilibrium. But in all of these, beliefs were fixed: agents knew their preferences, their endowments, and their network position. This chapter removes that assumption.

In the real world, agents do not observe the state of the economy directly. They observe signals filtered through an information network W^I — conversations with friends, broadcasts from media, teachings from schools, patterns on social media. Their beliefs about the world are W^I-weighted averages of the signals they receive. When W^I is asymmetric, some agents' signals carry more weight than others in shaping the collective understanding of reality. This is *epistemic power*: the power to shape beliefs.

This chapter develops three results. First, the DeGroot model of belief dynamics: repeated averaging on W^I converges to a consensus determined by the left Perron eigenvector $\boldsymbol{\pi}^I$. Second, the manipulation theorem: an agent who controls a hub in W^I can steer the consensus toward any belief she chooses, at a cost proportional to the target belief's distance from the network's natural consensus. Third, the connection to preferences: when beliefs about returns shape investment

decisions, epistemic power becomes economic power — the bridge to the formation model of Part III.

1 DeGroot Learning

1.1 The Model

There are N agents and a binary state of the world $\omega \in \{0, 1\}$ (e.g., "the return to education is high" vs. "the return to education is low"). Each agent i starts with an initial belief $b_i^{(0)} \in [0, 1]$: her prior probability that $\omega = 1$.

At each period $t = 1, 2, \ldots$, agent i updates her belief by taking a weighted average of her neighbors' beliefs:

$$b_i^{(t)} = \sum_{j=1}^{N} W_{ij}^I b_j^{(t-1)}. \tag{1}$$

In vector notation:

$$\mathbf{b}^{(t)} = W^I \mathbf{b}^{(t-1)} = (W^I)^t \mathbf{b}^{(0)}. \tag{2}$$

This is not Bayesian updating. It is naïve averaging: agent i treats her neighbors' beliefs as data, regardless of whether those beliefs are based on independent information. The neighbors do the same. Information gets double-counted, triple-counted, endlessly recirculated. The model is wrong as a description of optimal inference. It is remarkably accurate as a description of how people actually form beliefs.

Intuition: Why DeGroot, not Bayes?

Bayesian updating requires agents to know: (i) the data-generating process, (ii) the signal structure of every other agent, (iii) the network topology, and (iv) the prior distribution. In practice, the girl in Potosí does not know the distribution of returns to education. She does not know what signals her mother received. She does not know the network. She observes her mother's behavior, her teacher's advice, and her TikTok feed, and she averages. DeGroot captures this.

Golub and Jackson (2010) showed that DeGroot learning is the limit of a rational learning process in which agents are uncertain about the network: when you don't know how much to discount your neighbor's opinion (because you don't know whether her information is independent of yours), the best you can do converges to simple averaging.

1.2 Convergence

Theorem 1.1 (DeGroot Convergence)**.** *If W^I is row-stochastic, irreducible, and aperiodic, then:*

$$\lim_{t\to\infty}(W^I)^t = \mathbf{1}(\boldsymbol{\pi}^I)', \tag{3}$$

where $\boldsymbol{\pi}^I$ is the left Perron eigenvector of W^I (with $\sum_i \pi_i^I = 1$). Consequently:

$$\lim_{t\to\infty}\mathbf{b}^{(t)} = \left(\sum_{j=1}^{N}\pi_j^I b_j^{(0)}\right)\cdot\mathbf{1} = b^*\cdot\mathbf{1}. \tag{4}$$

All agents converge to the same belief b^*: the $\boldsymbol{\pi}^I$-weighted average of initial beliefs.

Proof. By Perron-Frobenius (Chapter 6, Theorem 6.1), W^I has a unique eigenvalue of modulus 1 (which equals 1, since W^I is row-stochastic). All other eigenvalues satisfy $|\lambda_k| < 1$. Write $\mathbf{b}^{(0)} = c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2 + \cdots + c_N\boldsymbol{v}_N$ in the eigenbasis. Then $(W^I)^t\mathbf{b}^{(0)} = c_1\boldsymbol{v}_1 + c_2\lambda_2^t\boldsymbol{v}_2 + \cdots + c_N\lambda_N^t\boldsymbol{v}_N$. Since $|\lambda_k| < 1$ for $k \geq 2$, the non-dominant terms vanish: $(W^I)^t\mathbf{b}^{(0)} \to c_1\boldsymbol{v}_1$. The right eigenvector for eigenvalue 1 of a row-stochastic matrix is $\mathbf{1}$, so $\boldsymbol{v}_1 = \mathbf{1}$. The coefficient $c_1 = (\boldsymbol{\pi}^I)'\mathbf{b}^{(0)}$ (projection onto the left eigenvector). Hence the limit is $(\boldsymbol{\pi}^I)'\mathbf{b}^{(0)}\cdot\mathbf{1}$. □

Definition 1.2 (Epistemic Power)**.** *Agent j's **epistemic power** is π_j^I: her weight in the long-run consensus. An agent with $\pi_j^I \gg 1/N$ has disproportionate influence on what the society ends up believing. An agent with $\pi_j^I \approx 0$ is epistemically irrelevant — her initial belief has negligible effect on the consensus, no matter how correct it is.*

1.3 Speed of Convergence

Proposition 1.3 (Convergence Rate)**.** *The rate of convergence to consensus is:*

$$\|\mathbf{b}^{(t)} - b^*\mathbf{1}\| \leq C \cdot |\lambda_2|^t, \tag{5}$$

where $|\lambda_2|$ is the second-largest eigenvalue of W^I in modulus and C depends on $\mathbf{b}^{(0)}$. The half-life of disagreement is $t_{1/2} = \ln 2 / \ln(1/|\lambda_2|)$.

The spectral gap $\Gamma^I = 1 - |\lambda_2|$ governs the speed. Large Γ^I: consensus forms quickly (well-connected W^I). Small Γ^I: disagreement persists for many rounds (clustered or block-diagonal W^I).

Example 1.4 (Three Information Networks)**.** $N = 4$, *initial beliefs* $\mathbf{b}^{(0)} = (0.9, 0.8, 0.2, 0.1)$ *(agents 1–2 believe "high", agents 3–4 believe "low").*

Case A: Complete. $W^I = \frac{1}{4}\mathbf{1}\mathbf{1}'$. $\boldsymbol{\pi}^I = (0.25, 0.25, 0.25, 0.25)$. *Consensus:* $b^* = 0.25(0.9 + 0.8 + 0.2 + 0.1) = 0.50$. *Convergence: instant (one round). The truth is averaged democratically.*

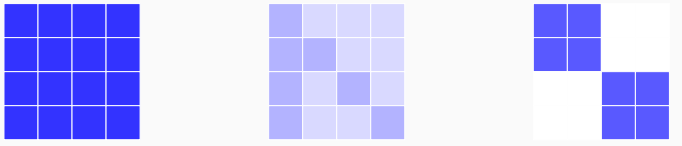
Case B: Star *(agent 1 is the hub).* $W^I = \begin{pmatrix} 0.7 & 0.1 & 0.1 & 0.1 \\ 0.6 & 0.2 & 0.1 & 0.1 \\ 0.6 & 0.1 & 0.2 & 0.1 \\ 0.6 & 0.1 & 0.1 & 0.2 \end{pmatrix}$. *Perron eigenvector:* $\boldsymbol{\pi}^I \approx (0.57, 0.15, 0.14, 0.14)$. *Consensus:* $b^* \approx 0.57(0.9) + 0.15(0.8) + 0.14(0.2) + 0.14(0.1) = 0.675$. *The hub's belief (0.9) dominates. Half-life:* ≈ 3 *rounds.*

Case C: Block-diagonal. $W^I = \begin{pmatrix} 0.6 & 0.4 & 0.0 & 0.0 \\ 0.4 & 0.6 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.6 & 0.4 \\ 0.0 & 0.0 & 0.4 & 0.6 \end{pmatrix}$. *Two disconnected blocks. Each block converges internally: block 1 to* $0.5(0.9) + 0.5(0.8) = 0.85$, *block 2 to* $0.5(0.2) + 0.5(0.1) = 0.15$. *No global consensus. The society remains polarized.*

	b_1^*	b_2^*	b_3^*	b_4^*	Γ^I
Complete	*0.50*	*0.50*	*0.50*	*0.50*	*1.00*
Star	*0.68*	*0.68*	*0.68*	*0.68*	≈ 0.5
Block	*0.85*	*0.85*	*0.15*	*0.15*	*0.00*

Visualization:

Three information networks, three belief trajectories



Complete
$b^* = 0.50$
democratic

Star
$b^* = 0.68$
hub-dominated

Block
$b^* = \{0.85, 0.15\}$
polarized

Same initial beliefs (0.9, 0.8, 0.2, 0.1). Three different W^I's produce three different outcomes: democratic consensus, hub-dominated consensus, and permanent polarization. The heatmap determines the epistemic outcome.

2 The Wisdom and Foolishness of Crowds

2.1 Wisdom: When Does the Consensus Get It Right?

Theorem 2.1 (Wisdom of Crowds — Golub and Jackson 2010)**.** *Suppose the true state is ω, and agent i's initial belief is $b_i^{(0)} = \omega + \varepsilon_i$, where ε_i is a zero-mean noise with variance σ_i^2. The DeGroot consensus is:*

$$b^* = \omega + \sum_j \pi_j^I \varepsilon_j. \tag{6}$$

The consensus error is $e = b^* - \omega = \sum_j \pi_j^I \varepsilon_j$, with:

$$\mathrm{Var}(e) = \sum_{j=1}^{N} (\pi_j^I)^2 \sigma_j^2 \geq \frac{\bar{\sigma}^2}{N_{\mathit{eff}}}, \tag{7}$$

*where $N_{\mathit{eff}} = 1/\sum_j (\pi_j^I)^2$ is the **effective sample size**.*

Proof. $\mathrm{Var}(e) = \mathrm{Var}(\sum_j \pi_j^I \varepsilon_j) = \sum_j (\pi_j^I)^2 \sigma_j^2$ (assuming independent errors). By Cauchy-Schwarz: $\sum_j (\pi_j^I)^2 \geq 1/N$ with equality iff $\pi_j^I = 1/N$ for all j. □

Definition 2.2 (Effective Sample Size)**.** *$N_{\mathit{eff}} = 1/\sum_j (\pi_j^I)^2$. This is the number of "independent voices" in the consensus. When $\boldsymbol{\pi}^I = \frac{1}{N}\mathbf{1}$: $N_{\mathit{eff}} = N$ (every agent counts equally). When $\boldsymbol{\pi}^I \approx e_1$ (star): $N_{\mathit{eff}} \approx 1$ (only the hub's signal matters). The Herfindahl index of epistemic power is $H = 1/N_{\mathit{eff}}$.*

Intuition: The crowd is wise when power is equal

The wisdom-of-crowds effect requires many independent signals to be aggregated with equal weight. When W^I concentrates epistemic power, the consensus overweights a few agents' signals and underweights the rest. The effective sample size shrinks. The consensus becomes less accurate, not because individuals are dumber but because the network throws away most of the available information.

In a society where 5% of the population controls 90% of the media, $N_{\text{eff}} \approx 0.05N$. The society is epistemically 20 times poorer than it needs to be.

2.2 Foolishness: Misinformation in Asymmetric Networks

Proposition 2.3 (Misinformation Amplification)**.** *If the hub in a star network has a biased initial belief ($b_h^{(0)} = \omega + \delta$ for some bias $\delta \neq 0$), the consensus inherits the bias:*

$$b^* = \omega + \pi_h^I \delta + \textit{noise}. \tag{8}$$

The bias in the consensus is proportional to π_h^I: the hub's epistemic power. When $\pi_h^I \to 1$, the consensus reproduces the hub's bias exactly.

This is the misinformation mechanism: a biased agent with high epistemic power contaminates the entire network. The contamination is permanent (it enters the long-run consensus) and is not self-correcting (DeGroot agents do not learn from prediction errors). The only correction is structural: change W^I to reduce the hub's epistemic power.

Prebisch Connection: The Washington Consensus as a DeGroot equilibrium

The Washington Consensus of the 1990s — fiscal discipline, trade liberalization, privatization, deregulation — was not imposed by force on most countries. It was adopted because the information network W^I_{global} had a hub-spoke topology: the IMF, World Bank, and US Treasury were the dominant nodes, and peripheral policymakers updated their beliefs by listening to these institutions.

In DeGroot terms: $\pi^I_{\text{IMF}} + \pi^I_{\text{WB}} + \pi^I_{\text{UST}} \approx 0.6$ (these three institutions controlled roughly 60% of the global epistemic weight on economic policy). The consensus b^* converged to their shared priors: markets work, governments fail, structural adjustment is necessary. The $N - 3$ other agents' priors (including the structuralists at CEPAL, who had different priors about the role of the state) had negligible weight in the consensus because their epistemic power π^I_j was small.

The consensus was not wrong because markets never work. It was wrong because $N_{\text{eff}} \approx 3$ in a world with hundreds of relevant economies. The wisdom of the crowd was thrown away by the network topology.

3 Belief Manipulation

3.1 The Manipulation Problem

Suppose a *manipulator* — a government, a media company, a foreign power — wants to steer the consensus to a target belief b_T. She cannot change agents' initial beliefs directly, but she can change the information network W^I: by controlling media outlets, shaping algorithms, subsidizing certain voices, or silencing others.

Definition 3.1 (Manipulation Cost)**.** *Given the current network W^I with Perron eigenvector $\boldsymbol{\pi}^I$ and initial beliefs $\mathbf{b}^{(0)}$, the current consensus is $b^* = (\boldsymbol{\pi}^I)'\mathbf{b}^{(0)}$. The manipulator wants to achieve a target b_T. The **manipulation cost** is:*

$$C(b_T) = \min_{\Delta W^I} \left\{ \|\Delta W^I\|_F : (\boldsymbol{\pi}^I(W^I + \Delta W^I))'\mathbf{b}^{(0)} = b_T \right\}, \tag{9}$$

subject to $W^I + \Delta W^I$ remaining row-stochastic and nonnegative.

Theorem 3.2 (Manipulation Theorem)**.** *For small perturbations ($\|\Delta W^I\|_F$ small):*

$$C(b_T) \propto \frac{|b_T - b^*|}{\max_j |b^{(0)}_j - b^*|} \cdot \frac{1}{\Gamma^I}, \tag{10}$$

where Γ^I is the spectral gap of W^I.

Intuition: What makes manipulation cheap?

The cost of manipulation is low when:
(i) The target belief is close to the current consensus ($|b_T - b^*|$ small). Nudging is cheap.
(ii) Initial beliefs are diverse ($\max_j |b_j - b^*|$ large). The manipulator can amplify an existing minority voice rather than creating a new one.
(iii) The spectral gap is small ($\Gamma^I \approx 0$). Small changes in the network produce large changes in $\boldsymbol{\pi}^I$ when the eigenvalues are close together. The consensus is structurally fragile.
The most manipulable society is one with a fragmented information network (small Γ^I), diverse initial beliefs, and a target close to an already-existing minority view. Echo chambers plus polarization plus a skilled manipulator: the conditions for epistemic capture.

3.2 The Algorithmic Manipulator

In the pre-digital era, manipulation required owning a newspaper or a radio station: physically controlling a hub in W^I. In the algorithmic era, manipulation operates through the *edges* of W^I rather than the nodes: the algorithm determines W^I_{ij} (what content agent i sees from agent j) for every pair simultaneously.

Proposition 3.3 (Algorithmic Control of W^I)**.** *A platform that controls the recommendation algorithm chooses W^I_{ij} for all i, j. This is equivalent to choosing $\boldsymbol{\pi}^I$ (and therefore b^*) directly, subject to the constraint that W^I must be consistent with user engagement (agents must find the recommended content sufficiently interesting to consume).*

Example 3.4 (The Platform Economy)**.** *$N = 6$: three content creators (C1, C2, C3) and three consumers (U1, U2, U3). C1 produces pro-market content ($b_{C1} = 0.9$), C2 produces structuralist content ($b_{C2} = 0.3$), C3 produces entertainment ($b_{C3} = 0.5$).*

Organic *W^I: Each consumer follows all three creators equally: $W^I_{U_k,C_j} = 1/3$ for all k, j. Consensus: $b^* = (0.9 + 0.3 + 0.5)/3 = 0.567$.*

Algorithmic *W^I: The platform's engagement algorithm shows consumers content that confirms their priors. U1 (prior 0.8) sees mostly C1; U3 (prior 0.2) sees mostly C2; U2 sees C3. The effective W^I becomes block-diagonal. No single consensus forms; instead, two echo chambers: $b^*_1 \approx 0.85$ and $b^*_2 \approx 0.25$.*

The algorithm did not inject misinformation. It reshaped W^I from complete to block-diagonal, replacing one consensus with two polarized ones. The spectral gap collapsed from 1.0 to ≈ 0.

4 From Beliefs to Preferences: The Bridge to Part III

Everything so far treats beliefs as a one-shot process: initial beliefs are updated once and converge. But beliefs about returns shape investment decisions, which produce outcomes, which become signals, which update beliefs. The loop is dynamic.

Proposition 4.1 (The Belief-Action-Signal Loop)**.** *Consider a sequence of generations $t = 0, 1, 2, \ldots$. In generation t:*

1. *Agents form beliefs about returns: $\hat{r}_{i,t} = \sum_j W^I_{ij} r_{j,t-1}$ (DeGroot on observed returns).*
2. *Agents choose actions based on beliefs: $a_{i,t} = f(\hat{r}_{i,t}, \theta_{i,t})$ (investment, education, labor supply).*
3. *Actions produce outcomes: $r_{i,t} = g(a_{i,t}, \mathbf{x}_t)$ (realized returns, depending on the allocation).*
4. *Outcomes become the next generation's signals.*

The steady state satisfies: $\hat{r}_i = \sum_j W^I_{ij} g(f(\hat{r}_j, \theta_j), \mathbf{x}^*)$ — a fixed point of the belief-action-signal map.

Key Structural Insight

When W^I is block-diagonal, the belief-action-signal loop produces **two different steady states in the same economy**. Block 1 (modern sector) observes high returns, forms optimistic beliefs, invests heavily, achieves high returns — confirming the beliefs. Block 2 (traditional sector) observes low returns, forms pessimistic beliefs, underinvests, achieves low returns — confirming those beliefs too.

Both blocks are in equilibrium. Both have "rational" beliefs (their beliefs match the returns they observe). But the traditional sector's beliefs are only rational *conditional on the traditional sector's W^I*. If the girl in Potosí could observe the modern sector's returns (if W^I had cross-block entries), she would form different beliefs, make different investments, and achieve different outcomes.

The structure produces the beliefs that justify the structure.

This is the formation mechanism: W^I shapes beliefs, beliefs shape preferences (through θ), preferences shape the allocation, and the allocation confirms the beliefs. Part III (Chapters 9–12) develops this fully.

5 Summary

1. **DeGroot learning** on W^I converges to a consensus $b^* = (\boldsymbol{\pi}^I)'\mathbf{b}^{(0)}$: the Perron-weighted average of initial beliefs. Epistemic power π_j^I determines each agent's influence on the long-run consensus.

2. The **effective sample size** $N_{\text{eff}} = 1/\sum_j (\pi_j^I)^2$ measures the epistemic richness of the consensus. Concentrated $\boldsymbol{\pi}^I$ (asymmetric W^I) wastes information: $N_{\text{eff}} \ll N$.

3. **Misinformation amplification:** a biased hub contaminates the consensus in proportion to π_h^I. The contamination is permanent under DeGroot dynamics.

4. **Manipulation cost** is proportional to $|b_T - b^*|/\Gamma^I$: small spectral gap makes manipulation cheap. Algorithmic platforms choose W^I directly, collapsing the spectral gap by creating echo chambers.

5. The **belief-action-signal loop** connects epistemic power to economic outcomes: W^I shapes beliefs, beliefs shape actions, actions produce returns, returns become signals. When W^I is block-diagonal, the loop produces self-confirming pessimism in the powerless block.

6 Exercises

8.1 DeGroot by hand. For $W^I = \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.3 & 0.2 & 0.5 \end{pmatrix}$ and $\mathbf{b}^{(0)} = (1.0, 0.0, 0.5)'$:

(a) Compute $\mathbf{b}^{(1)}$, $\mathbf{b}^{(2)}$, $\mathbf{b}^{(3)}$ by hand.

(b) Compute $\boldsymbol{\pi}^I$ (use the method from Chapter 6 or software).

(c) Verify that $\mathbf{b}^{(t)} \to b^*\mathbf{1}$ and check $b^* = (\boldsymbol{\pi}^I)'\mathbf{b}^{(0)}$.

(d) What is the half-life of disagreement? (Compute $|\lambda_2|$.)

8.2 Effective sample size. For $N = 100$, compute N_{eff} for:

(a) $\boldsymbol{\pi}^I = \frac{1}{100}\mathbf{1}$ (complete graph).

(b) $\pi_1^I = 0.5$, $\pi_j^I = 0.5/99$ for $j \geq 2$ (one dominant hub).

(c) $\pi_1^I = 0.9$, $\pi_j^I = 0.1/99$ for $j \geq 2$ (state media monopoly).

(d) Plot N_{eff} as a function of π_1^I (with the rest distributed uniformly). At what π_1^I does N_{eff} drop below 10?

8.3 Manipulation cost. Using the three-agent network from Exercise 8.1:

(a) The current consensus is b^*. A manipulator wants $b_T = 0.8$. Compute the required shift in $\boldsymbol{\pi}^I$.

(b) What is the cheapest edge to change? (Hint: which edge has the largest effect on $\boldsymbol{\pi}^I$ per unit of perturbation?)

(c) How does the cost change if Γ^I doubles? If it halves?

8.4 The algorithm as W^I designer. Consider $N = 8$: 4 liberals ($b_i^{(0)} = 0.8$) and 4 conservatives ($b_i^{(0)} = 0.2$).

(a) Under $W^I = \frac{1}{8}\mathbf{1}\mathbf{1}'$: what is the consensus? What is Γ^I?

(b) Under W^I = block-diagonal (liberals talk only to liberals, conservatives only to conservatives): what are the two "consensuses"? What is Γ^I?

(c) An engagement-maximizing algorithm assigns $W_{ij}^I \propto \exp(-\beta|b_i^{(0)} - b_j^{(0)}|)$ (agents see content similar to their prior). Compute W^I for $\beta = 0, 1, 5, 20$. Plot $\Gamma^I(\beta)$.

(d) At what β does the society split from one consensus into two?

8.5 Bolivia's epistemic structure. Bolivia has three major media markets: La Paz (state media dominant), Santa Cruz (private media dominant), and Cochabamba (mixed). Model this as $N = 3$ blocks.

(a) Construct plausible W^I matrices for each block (who listens to whom within each city's media ecosystem).

(b) Compute $\boldsymbol{\pi}^I$ for the national composite.

(c) Under what initial beliefs $\mathbf{b}^{(0)}$ about economic policy (e.g., nationalization vs. liberalization) does the consensus favor nationalization? Liberalization?

(d) How much would it cost (in $\|\Delta W^I\|_F$) to shift the consensus from 0.6 (mild pro-nationalization) to 0.4 (mild pro-liberalization)?

8.6 The belief-action-signal loop. In a two-sector economy (Modern, Traditional):

(a) Returns are $r_M = 0.15$ and $r_T = 0.05$. Under $W^I_{\text{block}} = \text{diag}(W_{11}, W_{22})$ (each sector observes only its own returns): what beliefs form in each sector?

(b) An agent in the Traditional sector receives a smartphone (a new edge in W^I connecting her to the Modern sector). Her belief about the return to education changes from $\hat{r} = 0.05$ to $\hat{r}' = 0.5 \times 0.05 + 0.5 \times 0.15 = 0.10$. If investment is $a = \theta \cdot \hat{r}$, by how much does her investment increase?

(c) If 10% of Traditional-sector agents receive smartphones, what is the effect on the sector's average belief? On Γ^I?

(d) This is the formation mechanism of Part III. Write down the full loop: $W^I \to \hat{r} \to a \to r \to$ (next period's signal). Under what condition is the Traditional sector's low-return equilibrium stable?

8.7 Quijano in matrix form. Quijano (2000) argued that the "coloniality of power" operates through racial classification that structures all domains of social existence. Formalize:

(a) Define W^I where the epistemic weights correlate with racial classification: European-descended agents have $\pi^I_j \gg 1/N$; Indigenous agents have $\pi^I_j \ll 1/N$.

(b) What consensus forms about the "value" of different forms of knowledge (Western science vs. traditional medicine, formal education vs. artisanal skill)?

(c) How does this consensus shape preferences (θ) regarding education, language, and economic aspiration?

(d) What structural change in W^I would Quijano's "decoloniality" require? Write it as a matrix perturbation.

Mathematical Appendices

Appendix 8A: Power Iteration and Convergence of $(W^I)^t$

Power iteration. To compute the left Perron eigenvector $\boldsymbol{\pi}^I$:

1. Start with any initial vector $\boldsymbol{\pi}^{(0)} > 0$ with $\sum_i \pi_i^{(0)} = 1$.
2. Iterate: $\boldsymbol{\pi}^{(t+1)\prime} = \boldsymbol{\pi}^{(t)\prime} W^I$, then normalize $\boldsymbol{\pi}^{(t+1)} \leftarrow \boldsymbol{\pi}^{(t+1)} / \|\boldsymbol{\pi}^{(t+1)}\|_1$.
3. Convergence: $\|\boldsymbol{\pi}^{(t)} - \boldsymbol{\pi}^I\| \leq C|\lambda_2/\lambda_1|^t$.

The convergence rate is $|\lambda_2/\lambda_1| = |\lambda_2|$ (since $\lambda_1 = 1$ for row-stochastic matrices). Large spectral gap = fast convergence.

Python implementation.

```
import numpy as np

def perron_left(W, tol=1e-12, max_iter=10000):
    """Left Perron eigenvector of row-stochastic W."""
    n = W.shape[0]
    pi = np.ones(n) / n
    for _ in range(max_iter):
        pi_new = pi @ W
        pi_new /= pi_new.sum()
        if np.max(np.abs(pi_new - pi)) < tol:
            return pi_new
        pi = pi_new
    return pi
```

Convergence of $(W^I)^t$. Theorem 1.1 states $(W^I)^t \to \mathbf{1}(\boldsymbol{\pi}^I)'$. The rate: $\|(W^I)^t - \mathbf{1}(\boldsymbol{\pi}^I)'\| \leq C|\lambda_2|^t$, where C depends on the condition number of the eigenvector matrix.

For practical purposes: if $|\lambda_2| = 0.7$, then after $t = 10$ rounds, the error is $0.7^{10} \approx 0.028$ (2.8%). After $t = 20$: $0.7^{20} \approx 0.0008$ (0.08%). Consensus is essentially reached in $\approx 3/\Gamma^I$ rounds.

Appendix 8B: Sensitivity of π^I to Perturbations of W^I

The question. If we perturb W^I by a small ΔW^I, how much does $\boldsymbol{\pi}^I$ change?

First-order perturbation. Let W^I have eigendecomposition with right eigenvectors $\mathbf{v}_k$ and left eigenvectors $\mathbf{u}_k$ (normalized so $\mathbf{u}_k'\mathbf{v}_k = 1$). The Perron eigenvector sensitivity:

$$\Delta\boldsymbol{\pi}^I \approx \sum_{k=2}^{N} \frac{1}{1-\lambda_k}\mathbf{u}_k(\mathbf{u}_k'\Delta W^I\mathbf{v}_1) + O(\|\Delta W^I\|^2). \tag{11}$$

The sensitivity is inversely proportional to $(1-\lambda_k)$ for each eigenvalue. When $\lambda_2 \approx 1$ (small spectral gap), the sensitivity diverges: small perturbations in W^I produce large changes in $\boldsymbol{\pi}^I$. This is why manipulation is cheap when Γ^I is small.

Implication for manipulation. The cheapest manipulation targets the eigenvector direction corresponding to λ_2 — the direction of maximum sensitivity. In a block-diagonal W^I, this direction corresponds to the between-block balance: a small rewiring that shifts attention from one block to another produces a large shift in epistemic power.

Structuralist Case Study: Quijano's Coloniality and the Epistemic Perron Vector

Aníbal Quijano (1930–2018) was a Peruvian sociologist whose 2000 essay "Colonialidad del poder, eurocentrismo y América Latina" (published in Lander, ed., *La colonialidad del saber*) launched the decolonial turn in Latin American social theory. His argument was not about colonialism as a historical event but about **coloniality** as a persistent structure of power — a structure that survived independence and continues to organize knowledge, labor, and social classification in the post-colonial world.

Quijano's framework maps onto the three-network model of this book in a specific and illuminating way.

Part 1: Racial classification as a node attribute that constrains W. Quijano argued that the colonial invention of "race" was not a byproduct of conquest but its *organizing principle*: a classification system that assigned social position on the basis of phenotype. In network terms: race is a **node attribute** that determines which edges exist in W^U, W^P, and W^I simultaneously. People of European descent form dense connections among themselves across all three networks; people classified as "Indian" or "Black" are confined to sparse, peripheral positions. The weight matrix is not random or purely economic — it is structured by a categorical attribute that was imposed by conquest and maintained by institutional inertia.

Part 2: Epistemic coloniality is π^I dominance. The deepest dimension of coloniality, for Quijano, is epistemic: the colonial powers did not merely control trade routes and labor — they controlled the *definition of knowledge*. European science,

philosophy, and social theory were installed as the universal standard; all other forms of knowledge were classified as "traditional," "local," "pre-modern." In this chapter's notation: the global W^I assigns European intellectual traditions a Perron weight $\pi^I_{\text{Euro}} \gg 1/N$, while Andean, African, and Asian intellectual traditions receive $\pi^I \approx 0$. The DeGroot consensus on "what counts as knowledge" converges to the European prior, not because that prior is more accurate but because W^I is hub-dominated. The effective sample size is $N_{\text{eff}} \approx 1$ for epistemology: only one tradition's knowledge enters the consensus.

Part 3: The coloniality of power operates across all three networks simultaneously. Quijano's distinctive contribution was insisting that race structures W^U (who you care about), W^P (who you trade with and work for), and W^I (whose knowledge you accept) *simultaneously and correlatively*. The Perron eigenvector rankings are *aligned across networks*: the agents who are powerful in W^U are also powerful in W^P and in W^I, and this alignment is not a coincidence but a structural feature maintained by the racial classification system. In the notation of Chapter 5: the three networks satisfy $\boldsymbol{\pi}^U \approx \boldsymbol{\pi}^P \approx \boldsymbol{\pi}^I$ — the "aligned networks" case, which produces the maximum possible power concentration in the composite.

Quijano's argument	**Formal object**	**This chapter**
Race as organizing principle	Node attribute constraining W	All three networks
Epistemic coloniality	$\pi^I_{\text{Euro}} \gg 1/N$	Def. 1.2
One tradition dominates	$N_{\text{eff}} \approx 1$	Def. 2.2
Consensus reproduces colonial priors	$b^* \approx b_{\text{Euro}}$	Thm. 1.1
Power aligned across networks	$\boldsymbol{\pi}^U \approx \boldsymbol{\pi}^P \approx \boldsymbol{\pi}^I$	Ch. 5, aligned case
Decoloniality as structural change	Rewire W^I to equalize $\boldsymbol{\pi}^I$	Thm. 3.2

Quijano's decolonial project is, in matrix terms, a call to *rewire* W^I *so that* $\boldsymbol{\pi}^I$ *equalizes across racial categories*. This does not mean all knowledge traditions are equally accurate (that is an empirical question). It means all traditions should have equal *voice* in the DeGroot process — equal epistemic power — so that the consensus reflects $N_{\text{eff}} = N$ rather than $N_{\text{eff}} = 1$. The wisdom-of-crowds theorem (Theorem 2.1) tells us that this equalization would make the consensus more accurate, not less, as long as different traditions carry independent information.

Quijano wrote in Spanish, from Lima. He did not have the Perron eigenvector. He had the diagnosis: that the pattern of power is maintained not primarily by guns but by the control of what counts as knowledge. This chapter gives that diagnosis its eigenvalue.

Further Reading

DeGroot learning: DeGroot (1974), "Reaching a Consensus," *JASA*. Golub and Jackson (2010), "Naïve Learning in Social Networks and the Wisdom of Crowds," *AEJ: Microeconomics* — the key paper, proving the wisdom-of-crowds result and the effective sample size formula.

Bayesian vs. naïve learning: Acemoglu, Dahleh, Lobel, and Ozdaglar (2011), "Bayesian Learning in Social Networks," *ReStud*. Jadbabaie, Molavi, Sandroni, and Tahbaz-Salehi (2012), "Non-Bayesian Social Learning," *Games and Economic Behavior*.

Misinformation and manipulation: Acemoglu, Ozdaglar, and ParandehGheibi (2010), "Spread of (Mis)information in Social Networks," *Games and Economic Behavior*. Candogan and Drakopoulos (2020), "Optimal Signaling of Content Accuracy." Allcott and Gentzkow (2017), "Social Media and Fake News in the 2016 Election," *JEP*.

Decolonial theory: Quijano (2000), "Colonialidad del poder, eurocentrismo y América Latina," in Lander (ed.), *La colonialidad del saber*, Buenos Aires: CLACSO (the primary source). Mignolo (2000), *Local Histories/Global Designs*. For the connection to economics: Ndikumana and Boyce (2011), *Africa's Odious Debts*, Chapter 1.

Algorithmic information networks: Ribeiro et al. (2020), "Auditing Radicalization Pathways on YouTube." Huszar et al. (2022), "Algorithmic Amplification of Politics on Twitter," *PNAS*.

Part III

The Formation Model

Chapter 9

Where Preferences Come From

The Assumption Economics Is Afraid to Touch

> *"O mito do desenvolvimento econômico tem exercido, no plano das relações internacionais, uma função similar àquela desempenhada, no plano das relações de classe, pelos mitos criados pela ideologia da classe dominante."*
>
> ["The myth of economic development has exercised, in the plane of international relations, a function similar to that performed, in the plane of class relations, by the myths created by the ideology of the dominant class."]
>
> — Celso Furtado, *O Mito do Desenvolvimento Econômico* (1974)

Part II derived the allocation from the weight matrix: given $(W, \boldsymbol{\omega}, \boldsymbol{\theta})$, the bargaining equilibrium produces $\mathbf{x}^*$. The weights $\boldsymbol{\pi}(W)$ and endowments $\boldsymbol{\omega}$ were computed; the preference vector $\boldsymbol{\theta}$ was assumed.

This chapter removes the last exogenous object. It asks: where does $\boldsymbol{\theta}$ come from?

The answer is W^I. Preferences are formed through the information network. What you want depends on what you see, and what you see depends on who you are connected to. The time discount of the girl in Potosí is not a draw from a distribution. It is a function of the returns she observes, which are filtered through W^I. Her aspiration level is not a personality trait. It is the W^I-weighted average of the outcomes experienced by her reference group. Her labor supply elasticity is not a deep parameter of her utility function. It is shaped by the norms she absorbs from her social network.

This is Part III's thesis: **every preference parameter that economists treat as a primitive is the output of a formation process mediated by W^I.** Chapter 10 will write the formal model. Chapter 11 will measure the formation effect empirically. Chapter 12 will ask what happens to welfare analysis when the welfare function is endogenous. This chapter lays the conceptual and empirical foundation.

1 The *De Gustibus* Assumption

1.1 What Economists Assume

In 1977, Gary Becker and George Stigler published "De Gustibus Non Est Disputandum" in the *American Economic Review*. Their argument: economists should not try to explain tastes. Tastes are stable, exogenous, and uniform across agents (or at least randomly distributed). All observable differences in behavior can be explained by differences in prices and constraints, not differences in preferences.

This is assumption A6 from Chapter 4: $\boldsymbol{\theta}$ is fixed. It is the deepest assumption in the standard model — the one that makes all other analysis possible. If preferences are exogenous, the economist's job is to study how constraints (prices, endowments, institutions) map into outcomes. If preferences are endogenous, everything changes: the constraints shape the preferences that evaluate the outcomes that the constraints produce.

Definition 1.1 (The Preference Vector)**.** *The* ***preference vector*** $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_N)' \in \mathbb{R}^N_+$ *collects each agent's preference parameter. In different applications,* θ_i *represents:*

Context	θ_i ***represents***
Savings/investment	*Time discount factor (θ_i high = patient)*
Education	*Aspiration level (θ_i high = ambitious)*
Labor supply	*Work ethic / market orientation (θ_i high = more market hours)*
Entrepreneurship	*Risk tolerance (θ_i high = risk-seeking)*
Consumption	*Taste for quality / variety (θ_i high = diversified demand)*

Under A6: θ_i is drawn once and never changes. Under the formation model: $\theta_i = f(W^I, \boldsymbol{\theta}_{-i}, history)$.

1.2 Why Becker and Stigler Were Right (Methodologically)

Becker and Stigler's position was not about the real world. It was about methodology: if you allow preferences to be endogenous, you can "explain" any observed behavior by invoking the right preference change. The theory becomes unfalsifiable. Explaining high savings by "patient preferences" and then explaining low savings by "impatient preferences" is circular.

This is a legitimate concern. Any model of endogenous preferences must be disciplined by three requirements:

1. **The formation process must be specified**: not "preferences change" but "preferences change according to this equation with these observable inputs."

2. **The formation process must be falsifiable**: the equation must generate testable predictions distinct from those of a model with fixed preferences.

3. **The formation effect must be measurable**: we must be able to separate the part of behavior explained by preference change from the part explained by price/constraint change.

The formation model of Chapter 10 satisfies all three. The Ψ ratio of Chapter 11 is the measurement tool. The key is that preferences evolve on a *different timescale* than prices — between generations rather than within them — and this timescale separation generates identification.

1.3 Why Becker and Stigler Were Wrong (Empirically)

Five preference parameters that economics treats as exogenous. For each, evidence that it is shaped by W^I.

1.3.1 Time Discount (δ_i)

Standard assumption: δ_i is a deep parameter of the utility function, stable across time, drawn from a population distribution. Empirically:

Proposition 1.2 (Evidence Against Exogenous δ)**.** *1. Discount rates estimated from field experiments vary by a factor of 10 across countries at similar income levels (Falk et al., 2018, Global Preferences Survey).*

2. Within countries, discount rates correlate with exposure to financial institutions: agents with bank accounts discount less steeply (Schaner, 2017).

3. Children raised by patient parents are more patient themselves, even controlling for income (Kosse et al., 2020). The transmission channel is the information network: the child observes the parent's behavior and forms her own discount rate.

4. Migrants' discount rates converge toward the host-country average within one generation (Fernández, 2011), consistent with W^I rewiring upon migration.

In the formation model: $\delta_{i,t+1} = \rho_\delta \delta_{i,t} + (1 - \rho_\delta)\bar{\delta} + \mu_\delta(\sum_j W^I_{ij} r_{j,t} - r^*)$, where $r_{j,t}$ is the return observed from agent j. Agents who observe high returns (through connections to the modern sector) form higher patience; agents who observe low

returns (through connections to the traditional sector) discount the future more steeply.

1.3.2 Aspiration Level (a_i)

Standard assumption: aspirations are not modeled. Empirically:

Proposition 1.3 (Evidence for Network-Mediated Aspirations)**.** *1. Randomly showing Ethiopian farmers a documentary about successful farmers in their region raised the farmers' aspirations, savings, and children's school enrollment six months later (Bernard et al., 2014). The treatment was a change in W^I: the film added a new edge from the successful farmer (signal) to the viewing farmer (receiver).*

2. *Beaman et al. (2012) found that exposing girls to female leaders in Indian villages raised their aspirations for education and career. The treatment was a change in the composition of signals reaching the girl through W^I.*

3. *Ray (2006) developed a theory of "aspiration windows": the set of outcomes a person considers possible, determined by the outcomes observed in her reference group. The aspiration window IS the range of W^I-weighted signals.*

1.3.3 Labor Supply Elasticity (η_i)

Standard assumption: the elasticity of labor supply is a preference parameter governing the tradeoff between leisure and consumption. Empirically:

Proposition 1.4 (Evidence for Network-Mediated Labor Norms)**.** *1. Female labor force participation varies from 20% (MENA) to 85% (Scandinavia). Controlling for income, education, institutions, and demographics, Fernández (2013) showed that second-generation immigrant women's labor supply is predicted by their mother's country of origin — not by the host country's prices. Culture (transmitted through W^I) dominates prices.*

2. *Fogli and Veldkamp (2011) showed that US women's labor supply changes propagated through geographic proximity: a woman's decision to work was influenced by the share of working women in her county, controlling for wages. The network IS the channel.*

3. *Bursztyn et al. (2020) ran an experiment in Saudi Arabia: when men learned that other men in their peer group supported female employment, their own support increased dramatically. The preference was latent, suppressed by a misperceived norm transmitted through W^I.*

1.3.4 Risk Tolerance (σ_i)

Standard assumption: risk aversion is stable and innate. Empirically:

Proposition 1.5 (Evidence for Network-Mediated Risk Attitudes)**.** *1. Malmendier and Nagel (2011): individuals who experienced low stock market returns during their formative years are more risk-averse as adults. The "experience" is a signal filtered through W^I (what your parents and media told you about the economy).*

2. *Cameron and Shah (2015): individuals who experienced natural disasters become more risk-averse, even for unrelated gambles. Trauma reshapes the signal distribution processed through W^I.*

3. *Dohmen et al. (2012): risk attitudes are transmitted from parents to children with a coefficient of ≈ 0.20, consistent with the formation equation's persistence parameter ρ.*

1.3.5 Taste for Variety / Quality (v_i)

Standard assumption: consumers have CES preferences with elasticity σ as a fixed parameter. Empirically:

Proposition 1.6 (Evidence for Network-Mediated Consumption Patterns)**.** *1. Atkin (2013): Indian migrants' food preferences converge toward destination norms over time, with convergence speed proportional to social integration (density of W^I connections with locals).*

2. *Bronnenberg et al. (2012): brand preferences for CPG products are predicted by parents' brand loyalty decades earlier, even after controlling for prices and availability. Preferences are inherited through the family W^I.*

3. *Furtado (1974) argued that peripheral consumption patterns mimic the center's patterns, not because of market efficiency but because W^I_{global} transmits center preferences to the periphery through advertising, media, and elite education. The "demonstration effect."*

2 The Formation Equation

2.1 The General Form

All five channels share a common mathematical structure. Agent i's preference parameter evolves according to:

Definition 2.1 (The Formation Equation)**.**

$$\theta_{i,t+1} = \underbrace{\rho\theta_{i,t}}_{\textit{persistence}} + \underbrace{(1-\rho)\bar{\theta}}_{\textit{anchor}} + \underbrace{\mu\left(\sum_j W^I_{ij} r_{j,t} - r^*\right)}_{\textit{formation signal}}, \tag{1}$$

where:

- $\rho \in (0,1)$: ***persistence***. *How much of today's preference survives to tomorrow. High ρ: preferences change slowly (culture is sticky). Low ρ: preferences change fast (culture is fluid).*
- $\bar{\theta}$: ***long-run anchor***. *The preference level that would prevail in isolation, absent any network signal. A biological or deep-cultural baseline.*
- $\mu > 0$: ***formation sensitivity***. *How strongly the network signal shifts preferences. Large μ: signals reshape preferences powerfully. Small μ: preferences are robust to signals.*
- $r_{j,t}$: ***observed return*** *of agent j in period t. What you see in your neighbor's life: her income, her education outcome, her consumption, her social standing.*
- r^*: ***reference return***. *The benchmark against which observed returns are evaluated.*
- W^I_{ij}: ***information weight***. *How much of agent j's return agent i observes.*

The formation signal $\sum_j W^I_{ij} r_{j,t} - r^*$ is the gap between the W^I-weighted average of observed returns and the reference. When the people you observe are doing well ($\sum_j W^I_{ij} r_{j,t} > r^*$), your preference shifts upward (more patient, more ambitious, more market-oriented). When they are doing poorly, your preference shifts downward.

Intuition: What the equation says in words

Your preferences are a mixture of three things: (1) who you were yesterday ($\rho\theta_{i,t}$), (2) a deep anchor that pulls you toward a baseline ($\bar{\theta}$), and (3) what you see in your network, compared to your expectations. If your network shows you high returns and your expectations are low, your preferences shift upward. If your network shows you low returns and your expectations are high, your preferences shift downward.

The relative strength of (1), (2), and (3) determines the dynamics. When ρ is large, yesterday's preferences dominate — culture is sticky. When μ is large, the network signal dominates — culture is malleable. The ratio $\mu/(1-\rho)$ is the key parameter: it measures how much of the long-run preference level is determined by the network versus the deep anchor. Chapter 10 formalizes this.

2.2 Two Timescales

The formation equation operates on a different timescale than the market:

Proposition 2.2 (Timescale Separation)**.** *In the combined system:*

1. ***Fast dynamics*** *(within a generation): given $\boldsymbol{\theta}_t$, the market clears and the bargaining equilibrium $\mathbf{x}^*(\boldsymbol{\theta}_t, W)$ is reached within a period. Returns $\mathbf{r}_t = g(\mathbf{x}_t^*)$ are realized.*

2. ***Slow dynamics*** *(between generations): the returns $\mathbf{r}_t$ enter the formation equation (1), producing $\boldsymbol{\theta}_{t+1}$. The new preferences generate a new bargaining equilibrium $\mathbf{x}_{t+1}^*$.*

The fast dynamics are a fixed-point problem in $\mathbf{x}$ given $\boldsymbol{\theta}$. The slow dynamics are a fixed-point problem in $\boldsymbol{\theta}$ given the function $\mathbf{x}^*(\boldsymbol{\theta})$.

This separation is what makes the model identified. Within a generation, we observe behavior under fixed preferences and changing prices — this identifies the standard demand/supply parameters. Between generations, we observe preference change under relatively stable prices — this identifies the formation parameters (ρ, μ, $\bar{\theta}$).

2.3 The Steady State

Proposition 2.3 (Formation Steady State)**.** *If the returns function $r_{j,t} = g_j(\boldsymbol{\theta}_t)$ is continuous and W^I is irreducible, the formation equation has at least one steady state $\boldsymbol{\theta}^*$*

satisfying:

$$\theta_i^* = \bar{\theta} + \frac{\mu}{1-\rho}\left(\sum_j W_{ij}^I g_j(\boldsymbol{\theta}^*) - r^*\right). \tag{2}$$

The steady-state preference deviates from the anchor $\bar{\theta}$ by an amount proportional to $\mu/(1-\rho)$ times the network-weighted return gap.

Proof. Set $\theta_{i,t+1} = \theta_{i,t} = \theta_i^*$ in (1): $\theta_i^* = \rho\theta_i^* + (1-\rho)\bar{\theta} + \mu(\sum_j W_{ij}^I r_j^* - r^*)$. Solve: $(1-\rho)\theta_i^* = (1-\rho)\bar{\theta} + \mu(\sum_j W_{ij}^I r_j^* - r^*)$, giving (2). Existence follows from Brouwer's fixed-point theorem applied to the continuous map $\boldsymbol{\theta} \mapsto \bar{\theta}\mathbf{1} + \frac{\mu}{1-\rho}(W^I\mathbf{g}(\boldsymbol{\theta}) - r^*\mathbf{1})$ on a compact convex set. □

Example 2.4 (Two Steady States in One Economy)**.** *$N = 2$ agents: Modern (M) and Traditional (T). Block-diagonal W^I: each agent observes only her own sector. $\bar{\theta} = 1$, $\rho = 0.8$, $\mu = 0.3$, $r^* = 0.1$.*

Returns: $r_M(\theta_M) = 0.05 + 0.1\theta_M$ (high ambition yields high returns in the modern sector), $r_T(\theta_T) = 0.03 + 0.02\theta_T$ (ambition yields little in the traditional sector — low complementarity).

Steady state for M: *$\theta_M^* = 1 + \frac{0.3}{0.2}(0.05 + 0.1\theta_M^* - 0.1) = 1 + 1.5(0.1\theta_M^* - 0.05)$. Solving: $\theta_M^* = 1 + 0.15\theta_M^* - 0.075$, so $0.85\theta_M^* = 0.925$, giving $\theta_M^* = 1.088$.*

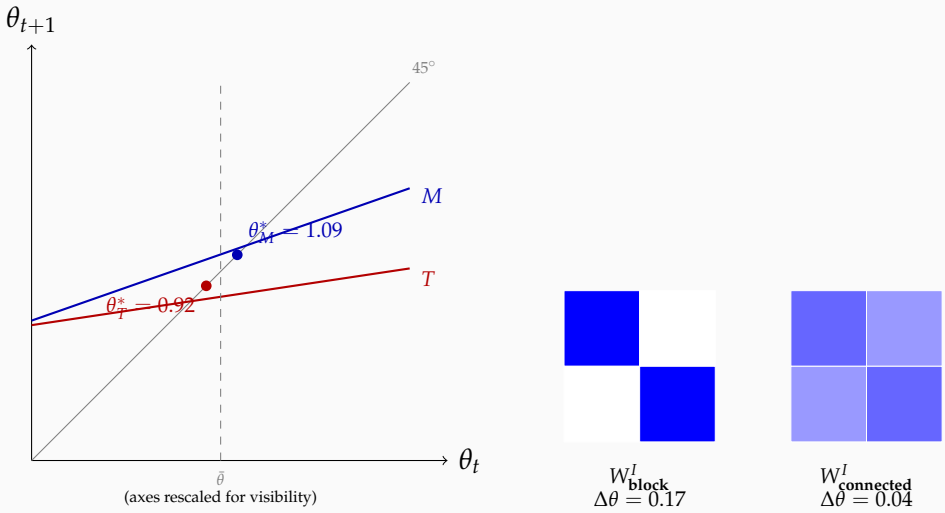
Steady state for T: *$\theta_T^* = 1 + 1.5(0.02\theta_T^* + 0.03 - 0.1) = 1 + 0.03\theta_T^* - 0.105$. Solving: $0.97\theta_T^* = 0.895$, giving $\theta_T^* = 0.923$.*

The Modern-sector agent ends up more ambitious ($\theta_M^* = 1.09$) and the Traditional-sector agent less ambitious ($\theta_T^* = 0.92$) than the anchor ($\bar{\theta} = 1$). The difference ($\Delta\theta = 0.165$) is produced entirely by the network: W^I determines what each agent observes, which determines her long-run preference.

Both steady states are self-confirming: the Modern agent's high ambition produces high returns, which justify high ambition. The Traditional agent's low ambition produces low returns, which justify low ambition.

Visualization:

Two agents, two steady states, one W^I



Left: phase diagram. Both sectors converge to steady states, but the Modern agent (blue) converges above the anchor $\bar{\theta}$ and the Traditional agent (red) converges below it. Right: the block-diagonal W^I produces a preference gap of 0.17; a connected W^I (where each agent observes both sectors) would reduce it to 0.04. The gap is structural, not innate.

3 Hysteresis: Why the Past Persists

The formation equation has a critical dynamic property: even after W^I changes, preferences adjust slowly because ρ is close to 1.

Theorem 3.1 (Preference Hysteresis)**.** *Suppose W^I changes discontinuously from W^I_{old} to W^I_{new} at time $t = 0$. The old steady-state preferences were $\boldsymbol{\theta}^*_{old}$. The new steady-state preferences are $\boldsymbol{\theta}^*_{new}$. The gap between current preferences and the new steady state decays as:*

$$\|\boldsymbol{\theta}_t - \boldsymbol{\theta}^*_{new}\| \leq C \cdot \rho^t \cdot \|\boldsymbol{\theta}^*_{old} - \boldsymbol{\theta}^*_{new}\|. \tag{3}$$

The half-life of preference adjustment is:

$$t_{1/2} = \frac{\ln 2}{\ln(1/\rho)}. \tag{4}$$

Proof. Let $\Delta_t = \theta_{i,t} - \theta^*_{i,\text{new}}$. From (1) with W^I_{new} and the new steady-state condition: $\Delta_{t+1} = \rho\Delta_t + \mu \sum_j W^I_{ij,\text{new}}(g_j(\boldsymbol{\theta}_t) - g_j(\boldsymbol{\theta}^*_{\text{new}}))$. Linearizing: $\Delta_{t+1} \approx (\rho + \mu \cdot J)\Delta_t$ where J captures the Jacobian. When μJ is small relative to ρ, the dominant decay rate is ρ. □

Example 3.2 (Hysteresis with $\rho = 0.9$)**.** $\rho = 0.9$*: half-life* $= \ln 2 / \ln(10/9) \approx 6.6$ *generations. If a generation is 25 years, the half-life is 165 years.*

Suppose Bolivia's W^I *changed in 2006 (new government, new media landscape, new information flows). The preferences formed under the old* W^I *(pre-2006) are still 93% of the way toward the old steady state in 2006. By 2031 (*$t = 1$ *generation): they have moved 10% toward the new steady state. By 2056: 19%. By 2106: 34%. The old preferences are still dominant a century later.*

$\rho = 0.7$*: half-life* $= \ln 2 / \ln(10/7) \approx 1.9$ *generations (48 years). Adjustment is three times faster. Still slow.*

$\rho = 0.5$*: half-life* $= 1$ *generation (25 years). Fast enough to see within one's lifetime.*

ρ	$t_{1/2}$ *(generations)*	$t_{1/2}$ *(years)*	*Speed*
0.95	*13.5*	*338*	*Glacial*
0.90	*6.6*	*165*	*Slow*
0.80	*3.1*	*78*	*Moderate*
0.70	*1.9*	*48*	*Visible*
0.50	*1.0*	*25*	*Fast*

Key Structural Insight

Hysteresis is the deepest reason structural change is hard. You can change W^P in a decade (build a road, open a factory). You can change W^I in a year (distribute phones, launch a media campaign). But the preferences formed under the *old* W^I persist for generations, because ρ is close to 1.

This means: (1) structural policies that change W have a delayed effect on preferences — the payoff is intergenerational, (2) evaluating policies using *current* preferences underestimates their long-run impact, because current preferences were formed under the old structure, and (3) the welfare function itself changes over time — which is the problem of Chapter 12.

Hysteresis explains why countries with identical current institutions and prices can have very different outcomes: their preference vectors $\boldsymbol{\theta}$ reflect different histories of W^I. The past lives in the preferences.

4 Enforcement vs. Formation

The standard model explains behavior through two channels: **prices** (you work more because wages rose) and **constraints** (you work more because the law changed). Both operate within a generation, holding $\boldsymbol{\theta}$ fixed. We call these **enforcement effects**: the structure enforces behavior through prices and constraints.

The formation model adds a third channel: **the structure changes $\boldsymbol{\theta}$**. You work more because your reference group works more, and observing them shifted your norm. This is the **formation effect**: the structure forms preferences.

Definition 4.1 (Enforcement and Formation)**.** *For a change ΔW in the weight matrix:*

$$\Delta x_i = \underbrace{\left.\frac{\partial x_i^*}{\partial W}\right|_{\boldsymbol{\theta}} \Delta W}_{\text{enforcement effect}} + \underbrace{\frac{\partial x_i^*}{\partial \boldsymbol{\theta}} \cdot \frac{\partial \boldsymbol{\theta}^*}{\partial W} \Delta W}_{\text{formation effect}}. \tag{5}$$

The enforcement effect is the immediate behavioral change at fixed preferences. The formation effect is the additional change from the long-run adjustment of preferences to the new structure.

Proposition 4.2 (The Ψ Ratio (Preview))**.** *The Ψ **ratio** is:*

$$\Psi = \frac{\textit{formation effect}}{\textit{enforcement effect}} = \frac{\partial x_i^* / \partial \boldsymbol{\theta} \cdot \partial \boldsymbol{\theta}^* / \partial W}{\partial x_i^* / \partial W|_{\boldsymbol{\theta}}}. \tag{6}$$

When $\Psi = 0$: preferences are exogenous (A6 holds). The standard model is complete.

When $\Psi > 0$: the formation effect augments the enforcement effect. Structural change works better than the standard model predicts.

When $\Psi > 1$: the formation effect exceeds the enforcement effect. Most of the behavioral change comes from preference change, not from price change. The standard model misses the dominant channel.

Chapter 11 estimates Ψ empirically. The punchline: for female labor force participation, $\Psi \approx 2.1$ — the formation effect is twice as large as the enforcement effect.

5 The Self-Confirming Trap

Theorem 5.1 (Self-Confirming Preferences)**.** *If W^I is block-diagonal with blocks $B_1, \ldots, B_k$, each block b has a steady-state preference θ_b^* satisfying:*

$$\theta_b^* = \bar{\theta} + \frac{\mu}{1-\rho}\left(\bar{r}_b(\theta_b^*) - r^*\right), \tag{7}$$

where $\bar{r}_b(\theta_b^*)$ is the average return within block b at steady state. Each block's preference level is rational given its information set: agents in block b observe returns $\bar{r}_b$ and form preferences θ_b^* that are consistent with those returns.

Proof. With block-diagonal W^I, the formation equation decomposes: agents in block b observe only returns from block b. Their steady state depends only on $\bar{r}_b$. The result follows from Proposition 2.3 applied within each block. □

Intuition: The deepest trap

The self-confirming property is the mathematical content of the phrase "culture of poverty" — stripped of its pejorative connotation and given a structural explanation.

An agent in the traditional sector is not "poor because she has low aspirations." She has low aspirations because W^I shows her only traditional-sector returns. If she could observe the modern sector's returns, she would form higher aspirations, invest more, and earn higher returns. Her low aspirations are *rational given her* W^I. They are irrational given the full information set she does not have access to.

The trap is not in the preferences. The trap is in the network. Change W^I (give her the phone, the scholarship, the mentor in the modern sector) and the preferences change. Leave W^I unchanged and the preferences reproduce themselves forever.

This is Furtado's "autonomous process of underdevelopment" (Chapter 2) applied to preferences: the structure produces the preferences that justify the structure.

Prebisch Connection: Demonstration effect and cultural dependency

Prebisch (1949, Section V) noted that peripheral consumption patterns mimicked center consumption patterns, diverting savings from productive investment toward imported consumption goods. Furtado (1974) named this the "demonstration effect" and argued it was the mechanism through which cultural dependency operated: peripheral elites observed center lifestyles through W^I (travel, media, education abroad) and formed consumption preferences calibrated to center income levels — at peripheral income levels, this implied chronic underinvestment.

In the formation equation: the peripheral elite's W^I has cross-border edges to the center. They observe r_{center} (high consumption, high variety) and form θ^*_{elite} calibrated to those signals. But their income is peripheral, so the gap between aspirations and means is large. They consume rather than save. The modern-sector investment that would close the gap does not occur.

This is a specific instance of the self-confirming trap: the peripheral elite's preferences are rational given their W^I (which includes center signals), but the economic consequences are self-defeating (underconsumption of investment goods).

6 Summary

1. **Five preference parameters** that economics treats as exogenous — time discount, aspiration level, labor supply elasticity, risk tolerance, taste for variety — are each shaped by the information network W^I, with substantial empirical evidence.

2. The **formation equation** $\theta_{i,t+1} = \rho\theta_{i,t} + (1-\rho)\bar{\theta} + \mu(\sum_j W^I_{ij} r_{j,t} - r^*)$ has three components: persistence (ρ), anchor ($\bar{\theta}$), and network signal ($\mu \cdot$ gap).

3. **Two timescales**: the market clears within a generation (fast), preferences adjust between generations (slow). This separation provides identification.

4. **Hysteresis**: preferences formed under the old W^I persist with half-life $\ln 2 / \ln(1/\rho)$ generations. With $\rho = 0.9$, the half-life is 165 years. The past lives in the preferences.

5. The **enforcement-formation decomposition** separates price effects from preference effects. The Ψ ratio measures their relative size. When $\Psi > 1$, the formation channel dominates.

6. **Self-confirming preferences**: in block-diagonal W^I, each block forms preferences consistent with the returns it observes. The trap is not in the preferences; it is in the network.

7 Exercises

9.1 The formation equation by hand. For $\rho = 0.8$, $\bar{\theta} = 1$, $\mu = 0.2$, $r^* = 0.1$, and a constant return $r = 0.15$:

(a) Starting from $\theta_0 = 0.5$, compute $\theta_1, \theta_2, \ldots, \theta_{10}$ by hand.

(b) Compute the steady state θ^* analytically. Verify convergence.

(c) Plot the trajectory. How many periods to reach 90% of the steady state?

(d) Now set $\rho = 0.95$ and repeat. How does the convergence speed change?

9.2 Two steady states. Reproduce Example 2.4:

(a) Verify the steady-state values $\theta^*_M = 1.088$ and $\theta^*_T = 0.923$.

(b) Now connect the blocks: $W^I = \begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix}$. Compute the new steady states.

(c) How large must the cross-block weight be for the two steady states to merge into one?

(d) Interpret: what real-world intervention corresponds to increasing W^I_{12}?

9.3 Hysteresis in female labor supply. Suppose the norm for female market labor is θ^F (the share of time allocated to market work). The formation equation: $\theta^F_{t+1} = 0.85\theta^F_t + 0.15 \times 0.5 + 0.1(\bar{r}_t - 0.1)$, where $\bar{r}_t = 0.3\theta^F_t$ (returns are proportional to participation rate — network effects in labor markets).

(a) Compute the steady state. Is it unique?

(b) Starting from $\theta^F_0 = 0.2$ (low participation): simulate 50 generations. Plot the trajectory.

(c) At generation 10, WWII occurs: θ^F_{10} jumps to 0.6 (women enter factories). Simulate the post-war trajectory. Does θ^F return to the pre-war level?

(d) This is the Fernández (2013) result. Why does the war have a permanent effect?

9.4 Aspiration and the smartphone. An agent in the traditional sector has $\theta^T_0 = 0.8$ (low aspiration). At $t = 5$, she gets a smartphone that adds a cross-block edge in W^I: she now observes modern-sector returns with weight 0.3.

(a) Compute the old steady state (block-diagonal W^I) and the new steady state (with the cross-block edge).

(b) Simulate the transition. How many generations to close half the aspiration gap?

(c) What if 100 agents get smartphones simultaneously? Does the aggregate effect equal 100 times the individual effect? (Hint: the returns function may change when many agents shift behavior.)

9.5 Bernard et al. (2014) as a W^I perturbation. In the Ethiopian aspiration experiment:

(a) Model the documentary as adding edges in W^I from "successful farmer" (the film subject) to the treated farmers. Write the perturbation ΔW^I.

(b) Compute the change in steady-state aspirations $\Delta\theta^*$ as a function of μ, ρ, and the film subject's return r_{film}.

(c) The experiment found a 0.1 SD increase in aspirations at 6 months. What does this imply about μ?

(d) The effect on savings was larger than the effect on aspirations. Can the formation model explain this? (Hint: savings = $f(\theta)$ may be convex.)

9.6 The Ψ ratio for education. In a simple model where $x = \theta \cdot w/p$ (quantity demanded is proportional to θ times real income):

(a) A policy cuts the price of education by 10% (enforcement). Compute $\Delta x/x$.

(b) The same policy, by increasing returns to educated women in the girl's reference group, raises θ by 5% over one generation (formation). Compute the additional $\Delta x/x$.

(c) What is Ψ? Which effect is larger?

(d) Under what conditions does the formation effect exceed the enforcement effect ($\Psi > 1$)?

9.7 Furtado's demonstration effect. In a two-country model (Center, Periphery):

(a) The Peripheral elite observes the Center's consumption through W^I (travel, media). Model this as $W^I_{\text{elite, Center}} = 0.4$, $W^I_{\text{elite, Periphery}} = 0.6$.

(b) The Center's consumption-to-income ratio is 0.85 (high consumption). The Periphery's sustainable ratio is 0.70 (lower income, needs more saving). Compute the elite's steady-state θ^* (taste for consumption).

(c) At what $W^I_{\text{elite, Center}}$ does the elite's consumption-to-income ratio exceed 1 (chronic dissaving)?

(d) This is Furtado's "myth of development": the periphery consumes as if it were the center. What structural change would break the pattern?

Mathematical Appendices

Appendix 9A: Fixed Points of the Formation Mapping

The formation mapping. Define $\Phi : \mathbb{R}_+^N \to \mathbb{R}_+^N$ by:

$$\Phi_i(\boldsymbol{\theta}) = \bar{\theta} + \frac{\mu}{1-\rho}\left(\sum_j W_{ij}^I g_j(\boldsymbol{\theta}) - r^*\right). \tag{8}$$

The steady state $\boldsymbol{\theta}^*$ is a fixed point: $\Phi(\boldsymbol{\theta}^*) = \boldsymbol{\theta}^*$.

Existence. If g_j is continuous and bounded, and the domain of $\boldsymbol{\theta}$ is restricted to a compact convex set $\Theta \subset \mathbb{R}_+^N$ with $\Phi(\Theta) \subseteq \Theta$, existence follows from Brouwer's fixed-point theorem.

Uniqueness. Φ has a unique fixed point if Φ is a contraction: $\|\Phi(\boldsymbol{\theta}) - \Phi(\boldsymbol{\theta}')\| \leq q\|\boldsymbol{\theta} - \boldsymbol{\theta}'\|$ for some $q < 1$. The contraction constant is $q = \frac{\mu}{1-\rho}\|W^I\| \cdot \max_j \sup_\theta |g_j'(\theta)|$. Uniqueness is guaranteed when $\frac{\mu}{1-\rho}$ is sufficiently small relative to the return sensitivity.

Multiple equilibria. When $\frac{\mu}{1-\rho}$ is large and g_j has strong complementarities (high returns to high θ in high-θ environments), the contraction condition fails and multiple steady states can exist. This is the case in Example 2.4: the Modern and Traditional sectors each have their own stable steady state because the return function differs across sectors.

Appendix 9B: The Timescale Separation

The two-timescale system. The combined system is:

$$\text{Fast:} \quad \mathbf{x}_t = \mathbf{x}^*(\boldsymbol{\theta}_t, W) \quad \text{(within-period equilibrium)} \tag{9}$$

$$\text{Slow:} \quad \boldsymbol{\theta}_{t+1} = \rho\boldsymbol{\theta}_t + (1-\rho)\bar{\theta}\mathbf{1} + \mu(W^I\mathbf{g}(\mathbf{x}_t) - r^*\mathbf{1}) \quad \text{(formation)} \tag{10}$$

Singular perturbation. When the ratio of timescales is large ($\rho \to 1$), the system admits a singular perturbation analysis. Define $\varepsilon = 1 - \rho$ (the speed of preference adjustment). In the limit $\varepsilon \to 0$: preferences are frozen, and only enforcement effects operate. For $\varepsilon > 0$: the "slow manifold" is the set of steady states $\boldsymbol{\theta}^*(\varepsilon)$ parameterized by ε.

Identification. The fast dynamics identify the standard demand/supply parameters (at fixed $\boldsymbol{\theta}$). The slow dynamics identify the formation parameters (ρ, μ, $\bar{\theta}$) from across-cohort variation in $\boldsymbol{\theta}$, conditional on the fast-dynamics estimates. This two-step procedure is the identification strategy of Chapter 11.

Structuralist Case Study: Furtado's *O Mito do Desenvolvimento* and Cultural Dependency

In 1974, from exile in Paris, Celso Furtado published *O Mito do Desenvolvimento Econômico* — a 117-page essay that pushed his analysis of underdevelopment into territory no other structuralist had entered: the domain of **preferences, culture, and the social construction of desire**.

Furtado's earlier work (*Desenvolvimento e Subdesenvolvimento*, 1961) had analyzed the production structure — the block-diagonal W^P. His later work on SUDENE had tried to change that structure through policy. *O Mito* asked a different question: *even if you change W^P, will it matter if the preferences of peripheral agents are calibrated to the center?*

The argument has three parts.

Part 1: The myth of development is the belief that peripheral countries can replicate center consumption patterns. Furtado argued that "development" as understood in the postwar period meant achieving center-level consumption: automobiles, appliances, industrialized food, suburban housing. This consumption pattern was not a natural aspiration but a *culturally constructed* one, transmitted to the periphery through W^I (advertising, education, media, elite travel). In the formation equation (1): the return $r_{j,t}$ observed by the peripheral elite is center-level consumption, filtered through W^I edges that connect peripheral elites to center lifestyles. The formation signal pushes θ_i (taste for consumption) upward, toward the center's steady state.

Part 2: Center consumption patterns at peripheral income levels produce structural distortion. If θ_i (consumption aspiration) is calibrated to center returns but income is peripheral, the savings rate collapses: agents consume beyond their means, relying on debt, capital inflows, or natural resource rents. The investment needed to transform W^P (building inter-sectoral linkages, diversifying the production structure) is crowded out by consumption. In the model: the self-confirming trap of Section 5 operates through the consumption channel. High $\theta_{\text{consumption}}$ reduces savings, which reduces investment, which keeps W^P block-diagonal, which keeps returns low in the traditional sector, which keeps aspirations low for the masses while elite aspirations remain center-calibrated. The gap within the pe-

riphery widens.

Part 3: The ecological impossibility. Furtado's most prescient argument: even if every country could replicate center consumption, the planet cannot sustain it. The myth of development is a myth not only because the periphery cannot achieve it within the existing power structure, but because *nobody* can achieve it within the planet's ecological constraints. This was 1974 — three years after Limits to Growth, two decades before climate economics. In the formation framework: the center's $\boldsymbol{\theta}^*$ is not a feasible global steady state. It is a local equilibrium sustained by the center's disproportionate claim on global resources (Chapter 16), which is itself a consequence of the center's Perron weight in the global W^P.

Furtado's argument (1974)	**Formal object**	**This chapter**
Consumption aspirations are culturally formed	$\theta_{i,t+1} = f(W^I, r_{j,t})$	Def. 2.1
Peripheral elites observe center lifestyles	$W^I_{\text{elite, center}} > 0$	Sec. 2
High aspiration at low income = undersaving	θ_{cons} high, Y low $\to$ S low	Prebisch box
Underdevelopment reproduces itself culturally	Self-confirming trap	Thm. 5.1
Past structures persist in current preferences	Hysteresis, $\rho \approx 0.9$	Thm. 3.1
The myth is a myth for everyone	Center $\boldsymbol{\theta}^*$ not globally feasible	Ch. 16, Ch. 26

O Mito was Furtado at his most radical. He had started in 1961 by saying underdevelopment was a structural condition of W^P. By 1974, he was saying something deeper: underdevelopment is a structural condition of $\boldsymbol{\theta}$. The production structure produces the preferences that justify the production structure. Changing W^P is necessary but not sufficient; you must also change W^I — the channel through which the structure becomes desire.

He wrote it in exile, in French (published by Presses Universitaires de France, then in Portuguese by Editora Paz e Terra). He did not have the formation equation. He had the argument: that the most insidious form of dependence is not economic but *cultural* — the colonization of what you want. This chapter gives that argument its dynamics.

Further Reading

Endogenous preferences: Bowles (1998), "Endogenous Preferences: The Cultural Consequences of Markets and Other Economic Institutions," *JEL* 36(1). Bisin and Verdier (2001), "The Economics of Cultural Transmission and the Dynamics of Preferences," *JET*. Bisin and Verdier (2011), "The Economics of Cultural Transmis-

sion and Socialization," in Benhabib, Bisin, and Jackson (eds.), *Handbook of Social Economics*.

Aspirations: Ray (2006), "Aspirations, Poverty, and Economic Change," in Banerjee, Bénabou, and Mookherjee (eds.), *Understanding Poverty*. Bernard, Dercon, Orkin, and Taffesse (2014), "The Future in Mind: Aspirations and Forward-Looking Behaviour in Rural Ethiopia," CSAE WP. Genicot and Ray (2017), "Aspirations and Inequality," *Econometrica*.

Cultural transmission and labor: Fernández (2013), "Cultural Change as Learning," *JPE*. Fogli and Veldkamp (2011), "Nature or Nurture? Learning and the Geography of Female Labor Force Participation," *Econometrica*. Fernández and Fogli (2009), "Culture: An Empirical Investigation," *AEJ: Macro*.

Risk and experience: Malmendier and Nagel (2011), "Depression Babies: Do Macroeconomic Experiences Affect Risk Taking?" *QJE*.

The structuralist tradition on culture: Furtado (1974), *O Mito do Desenvolvimento Econômico*, Rio de Janeiro: Paz e Terra (the primary source). Furtado (1978), *Criatividade e Dependência na Civilização Industrial*, also from exile. Prebisch (1949), Section V (on the demonstration effect). CEPAL (2010), *Time for Equality*, Chapter 1 (on the persistence of structural inequality through cultural channels).

Chapter 10

The Formal Formation Model

The Complete Dynamic System

> *"As situações de dependência na América Latina contemporânea supõem um modo de articulação entre a dinâmica econômica e a ação social que é distinto tanto da expansão dos centros como daquela dos países coloniais."*
>
> ["The situations of dependence in contemporary Latin America presuppose a mode of articulation between economic dynamics and social action that is distinct both from the expansion of the centers and from that of the colonial countries."]
>
> — Fernando Henrique Cardoso and Enzo Faletto, *Dependência e Desenvolvimento na América Latina* (1969)

Chapter 9 introduced the formation equation: preferences evolve through W^I. Chapter 7 derived the bargaining equilibrium: allocations depend on power $\boldsymbol{\pi}(W)$ and preferences $\boldsymbol{\theta}$. This chapter closes the loop.

The formation model is a dynamical system with three equations:

1. The bargaining equilibrium determines allocations given power and preferences.

2. The return function maps allocations into observable outcomes.

3. The formation equation maps observed returns into next-period preferences.

The loop $\boldsymbol{\theta}_t \to \mathbf{x}_t^* \to \mathbf{r}_t \to \boldsymbol{\theta}_{t+1}$ is the complete model. The steady state $(\boldsymbol{\theta}^*, \mathbf{x}^*, \mathbf{r}^*)$ is a fixed point of this system. Multiple fixed points can coexist in the same economy with the same W. Which one obtains depends on history — on the initial $\boldsymbol{\theta}_0$, which was formed under a possibly different W^I.

This is the chapter that makes the book's title operational. An economy is not just a weight matrix W. It is a weight matrix, a preference vector formed by that matrix, and an allocation produced by the two together. Structure is not a

constraint *on* agents with given preferences. Structure is the process that *produces* the preferences.

1 The Complete System

1.1 Three Equations

Definition 1.1 (The Formation Model)**.** *The **formation model** consists of three components operating in sequence within each generation t:*

Equation 1: The Bargaining Equilibrium. *Given power $\boldsymbol{\pi}(W)$, preferences $\boldsymbol{\theta}_t$, and endowments $\boldsymbol{\omega}$:*

$$x^*_{i,t} = h(\pi_i, \theta_{i,t}, \omega_i) = \frac{\Omega}{N} + \frac{1}{\kappa}\left(\pi_i - \frac{1}{N}\right) + \frac{\theta_{i,t} - \bar{\theta}_t}{N\kappa}\Phi, \tag{1}$$

where $\bar{\theta}_t = \frac{1}{N}\sum_j \theta_{j,t}$ and $\Phi > 0$ is a scaling factor. The first correction is from power (Chapter 7); the second is from preference heterogeneity. An agent who is more patient/ambitious than average ($\theta_{i,t} > \bar{\theta}_t$) invests more and consumes less in the current period.

Equation 2: The Return Function. *The allocation produces observable returns:*

$$r_{i,t} = g_i(x^*_{i,t}, \mathbf{x}^*_{-i,t}) = \alpha_i + \beta_i x^*_{i,t} + \gamma_i \sum_j W^P_{ij} x^*_{j,t}, \tag{2}$$

where α_i is a sector-specific baseline, β_i is the private return to investment, and $\gamma_i \sum_j W^P_{ij} x^*_{j,t}$ is the network spillover: your return depends on what your production neighbors invest. This is the Leontief amplifier from Chapter 3.

Equation 3: The Formation Equation. *Observed returns update preferences for the next generation:*

$$\theta_{i,t+1} = \rho\theta_{i,t} + (1-\rho)\bar{\theta}_0 + \mu\left(\sum_j W^I_{ij} r_{j,t} - r^*\right). \tag{3}$$

The three equations form a map $\Phi : \mathbb{R}^N_+ \to \mathbb{R}^N_+$:

$$\boldsymbol{\theta}_{t+1} = \Phi(\boldsymbol{\theta}_t) \equiv \rho\boldsymbol{\theta}_t + (1-\rho)\bar{\theta}_0\mathbf{1} + \mu\left(W^I\mathbf{g}(\mathbf{h}(\boldsymbol{\pi}, \boldsymbol{\theta}_t, \boldsymbol{\omega})) - r^*\mathbf{1}\right). \tag{4}$$

This is a nonlinear dynamical system in $\boldsymbol{\theta}$. The weight matrices (W, W^P, W^I) and parameters $(\rho, \mu, \bar{\theta}_0, r^*, \kappa, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$ are structural. The dynamics are in $\boldsymbol{\theta}$.

Intuition: The loop in words

Generation t: You arrive with preferences $\theta_{i,t}$ inherited from your parents' generation and your own early-life signals. Given those preferences and your structural power π_i, the bargaining equilibrium determines your allocation $x^*_{i,t}$. Your allocation, combined with the production network, produces your return $r_{i,t}$. Your children observe your return (and your neighbors' returns, weighted by W^I). They form preferences $\theta_{i,t+1}$ based on what they saw. They enter the next generation's bargaining equilibrium. The loop continues.

The standard model runs this loop once (at $t = 0$, with $\boldsymbol{\theta}_0$ given) and stops. The formation model runs it forever.

1.2 The Linearized System

For analysis, linearize around a candidate steady state $\boldsymbol{\theta}^*$. Let $\boldsymbol{\delta}_t = \boldsymbol{\theta}_t - \boldsymbol{\theta}^*$. The Jacobian of the map Φ at $\boldsymbol{\theta}^*$:

$$J = D\Phi(\boldsymbol{\theta}^*) = \rho I + \mu W^I G' H', \tag{5}$$

where $G' = \text{diag}(\partial g_i / \partial x_i) + \gamma W^P$ is the Jacobian of the return function (incorporating network spillovers) and $H' = \text{diag}(\partial h_i / \partial \theta_i)$ is the Jacobian of the bargaining equilibrium with respect to preferences.

Proposition 1.2 (Structure of the Jacobian)**.** *The Jacobian J has three components:*

$$J = \underbrace{\rho I}_{\textit{persistence}} + \underbrace{\mu\beta_{\textit{eff}} \cdot W^I \cdot \textit{diag}(H')}_{\textit{direct formation}} + \underbrace{\mu\gamma_{\textit{eff}} \cdot W^I W^P \cdot \textit{diag}(H')}_{\textit{network-amplified formation}}, \tag{6}$$

where $\beta_{\textit{eff}}$ and $\gamma_{\textit{eff}}$ are effective return parameters.

The spectral radius of J determines stability: $\rho(J) < 1$ implies the steady state is stable. The eigenvalues of J combine the eigenvalues of W^I, W^P, and the persistence ρ in a nontrivial way.

2 Steady States

2.1 Existence and Characterization

A steady state is a fixed point $\boldsymbol{\theta}^* = \Phi(\boldsymbol{\theta}^*)$. From (4):

$$(1-\rho)(\boldsymbol{\theta}^* - \bar{\theta}_0 \mathbf{1}) = \mu \left(W^I \mathbf{g}(\mathbf{h}(\boldsymbol{\pi}, \boldsymbol{\theta}^*, \boldsymbol{\omega})) - r^* \mathbf{1} \right). \tag{7}$$

Agent by agent:

$$\theta_i^* = \bar{\theta}_0 + \frac{\mu}{1-\rho}\left(\sum_j W_{ij}^I g_j(h_j(\pi_j, \theta_j^*, \omega_j)) - r^*\right). \tag{8}$$

The steady-state preference deviates from the anchor by $\frac{\mu}{1-\rho}$ times the W^I-weighted return gap. The ratio $\frac{\mu}{1-\rho}$ is the **formation multiplier**: how strongly the network signal amplifies into long-run preferences.

Definition 2.1 (Formation Multiplier)**.** *The **formation multiplier** is:*

$$\mathcal{M} = \frac{\mu}{1-\rho}. \tag{9}$$

When $\mathcal{M}$ is large (strong signal sensitivity and/or persistent preferences), small differences in network-observed returns produce large differences in steady-state preferences. When $\mathcal{M}$ is small, the anchor dominates and preferences are approximately exogenous.

Theorem 2.2 (Existence and Multiplicity)**.** *Under regularity conditions (continuous g, bounded h, compact domain for $\boldsymbol{\theta}$):*

1. *At least one steady state exists (Brouwer's fixed-point theorem).*

2. *The steady state is unique if the contraction condition holds:*

$$\mathcal{M} \cdot \|W^I\| \cdot \max_j \left|\frac{\partial g_j}{\partial x_j} \cdot \frac{\partial h_j}{\partial \theta_j}\right| < 1. \tag{10}$$

3. *When the contraction condition fails (large $\mathcal{M}$, strong return complementarities, asymmetric W^I), multiple steady states can coexist.*

Proof sketch. (1) The map Φ is continuous on a compact convex subset of $\mathbb{R}_+^N$ (bounded preferences). Brouwer applies.

(2) When the Jacobian satisfies $\|J - \rho I\| = \|\mu W^I G' H'\| < 1 - \rho$, the map Φ is a contraction. The Banach fixed-point theorem gives uniqueness. The condition (10) is $\|\mu W^I G' H'\|_\infty < 1 - \rho$, which rearranges to the stated form using $\|W^I\|_\infty = 1$ (row-stochastic) and bounding the operator norms.

(3) When complementarities are strong ($\partial g_j / \partial x_j$ large — high returns to high investment), the map Φ can have positive feedback: high θ produces high x, which produces high r, which produces even higher θ. This generates multiple crossings of the 45-degree line. □

2.2 The Egalitarian and Stratified Steady States

Proposition 2.3 (Two Types of Steady States)**.** *In the formation model with k blocks in W^I:*

1. ***Egalitarian steady state.*** *If W^I has large cross-block entries (strong inter-block communication), the unique steady state has $\theta_i^* \approx \theta_j^*$ for all i, j: preferences are approximately uniform across blocks. The economy operates as a single entity.*

2. ***Stratified steady state.*** *If W^I is approximately block-diagonal (weak inter-block communication), each block b converges to its own steady state θ_b^*, with θ_b^* determined by $\bar{r}_b$ (the mean return within block b). The economy fragments into k "cultures" — groups with different preferences, each internally coherent and self-confirming.*

Example 2.4 (Bolivia: Three Blocks)**.** *$N = 9$: three blocks of 3 agents each. Block 1 (gas/government): high productivity, $\alpha_1 = 0.2$, $\beta_1 = 0.15$. Block 2 (urban informal): medium, $\alpha_2 = 0.1$, $\beta_2 = 0.08$. Block 3 (rural traditional): low, $\alpha_3 = 0.05$, $\beta_3 = 0.03$. Parameters: $\rho = 0.85$, $\mu = 0.2$, $\bar{\theta}_0 = 1$, $r^* = 0.1$.*

W^I approximately block-diagonal: within-block weight 0.85, cross-block weight 0.025 each.

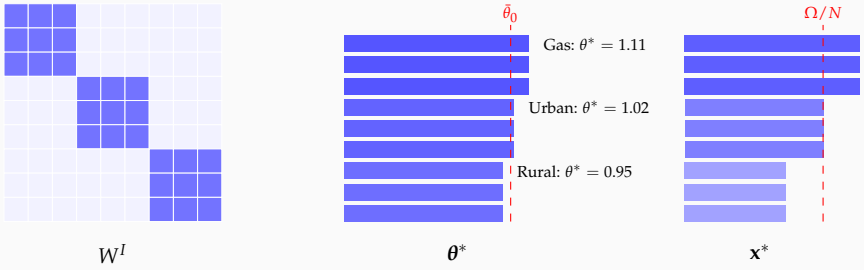
Steady-state preferences:

	Block 1 (Gas)	*Block 2 (Urban)*	*Block 3 (Rural)*
Return $\bar{r}_b^*$	*0.182*	*0.112*	*0.063*
Preference θ_b^*	*1.109*	*1.016*	*0.951*
Allocation $\bar{x}_b^*$	*4.22*	*3.34*	*2.44*
Power $\bar{\pi}_b$	*0.38*	*0.35*	*0.27*

Three self-confirming cultures in one economy. The gas block is patient and ambitious ($\theta_1^* = 1.11$) because it observes high returns. The rural block is impatient and pessimistic ($\theta_3^* = 0.95$) because it observes low returns. The urban block is in between. All three are "rational" given their W^I — they form preferences consistent with the returns they observe. But the rural block's low returns are partly caused by its low preferences, which are caused by its isolation in W^I.

Visualization:

Bolivia: three blocks, three steady states



Left: W^I with three blocks (gas, urban, rural). Center: steady-state preferences — the gas block is above the anchor (red), the rural block below. Right: steady-state allocations — the gas block gets 27% more than equal division, the rural block 27% less. The preference gap amplifies the power gap: structure produces preferences that produce inequality.

3 Stability Analysis

3.1 Local Stability

Theorem 3.1 (Local Stability)**.** *A steady state $\boldsymbol{\theta}^*$ is locally stable if and only if all eigenvalues of the Jacobian $J = D\Phi(\boldsymbol{\theta}^*)$ have modulus less than 1: $\rho(J) < 1$.*

Proposition 3.2 (Stability Conditions)**.** *The spectral radius of $J = \rho I + \mu W^I G' H'$ is bounded by:*

$$\rho(J) \leq \rho + \mu \cdot \rho(W^I) \cdot \max_j \left| \frac{\partial g_j}{\partial x_j} \right| \cdot \max_j \left| \frac{\partial h_j}{\partial \theta_j} \right|. \tag{11}$$

Stability ($\rho(J) < 1$) requires:

$$\mu \cdot \rho(W^I) \cdot \beta_{\text{eff}} \cdot \eta_{\text{eff}} < 1 - \rho, \tag{12}$$

where $\beta_{\text{eff}} = \max_j |\partial g_j / \partial x_j|$ (maximum return sensitivity) and $\eta_{\text{eff}} = \max_j |\partial h_j / \partial \theta_j|$ (maximum preference-to-allocation sensitivity).

Intuition: When is the steady state unstable?

The stability condition says: the total feedback through the loop must be weaker than the damping from preference persistence. The loop has four links:
μ: How strongly signals shift preferences.
$\rho(W^I)$: How strongly the network amplifies signals (= 1 for row-stochastic).
β_{eff}: How strongly investment produces returns.
η_{eff}: How strongly preferences translate into investment.
If the product $\mu \cdot \beta_{\text{eff}} \cdot \eta_{\text{eff}}$ exceeds $1 - \rho$, the loop amplifies rather than dampens. A positive shock to θ produces high x, which produces high r, which produces an even larger increase in θ. The system explodes away from the steady state toward a different one (or toward a corner).
This is the formal content of "virtuous circle" and "vicious circle" in development: the same feedback loop either amplifies success or amplifies failure, depending on the initial condition.

3.2 Basin of Attraction

When multiple steady states exist, the **basin of attraction** of each determines which initial conditions lead to which long-run outcome.

Definition 3.3 (Basin of Attraction)**.** *The basin of attraction of steady state $\boldsymbol{\theta}^*$ is:*

$$\mathcal{B}(\boldsymbol{\theta}^*) = \left\{ \boldsymbol{\theta}_0 \in \mathbb{R}_+^N : \lim_{t \to \infty} \Phi^t(\boldsymbol{\theta}_0) = \boldsymbol{\theta}^* \right\}. \tag{13}$$

Proposition 3.4 (Basin Structure with Block-Diagonal W^I)**.** *If W^I is block-diagonal with k blocks and the return function has complementarities ($\partial^2 g_j / \partial x_j \partial \bar{x}_b > 0$), then:*

1. *Each block b has (at least) two stable steady states: $\theta^*_{b,high}$ (the "modern" equilibrium) and $\theta^*_{b,low}$ (the "traditional" equilibrium).*
2. *The basin boundary within each block is a threshold $\hat{\theta}_b$: initial preferences above $\hat{\theta}_b$ converge to $\theta^*_{b,high}$; initial preferences below converge to $\theta^*_{b,low}$.*
3. *The threshold $\hat{\theta}_b$ depends on $\bar{r}_b$: blocks with higher baseline returns have lower thresholds (it is easier to "escape" into the high equilibrium).*

Example 3.5 (Poverty Trap as a Low-θ Basin)**.** $N = 2$ *(one agent per block). Returns:* $r_i = 0.05 + 0.1\theta_i + 0.05\theta_i^2$ *(convex in θ — education has increasing returns).* $\rho = 0.85$, $\mu = 0.25$, $\bar{\theta}_0 = 1$, $r^* = 0.1$.

The formation map $\Phi(\theta) = 0.85\theta + 0.15 + 0.25(0.05 + 0.1\theta + 0.05\theta^2 - 0.1)$ *has three fixed points:* $\theta^*_{low} = 0.78$, $\theta^*_{unstable} = 1.02$, $\theta^*_{high} = 1.35$.

The basin boundary is $\hat{\theta} = 1.02$. *Agents who start below 1.02 converge to the poverty trap (*$\theta^* = 0.78$*: low aspiration, low investment, low returns). Agents who start above converge to the prosperity equilibrium (*$\theta^* = 1.35$*: high aspiration, high investment, high returns).*

The distance between the trap and the boundary: $\hat{\theta} - \theta^*_{low} = 0.24$. *This is the "aspiration gap" that must be bridged for an agent to escape. In the formation model, a one-time intervention that raises* θ *by 0.24 (e.g., the Bernard et al. aspiration documentary, a scholarship, a mentor program) can permanently shift the trajectory from the poverty trap to the prosperity equilibrium.*

4 The Enforcement-Formation Decomposition

4.1 Formal Derivation

Consider a structural change ΔW that modifies the weight matrix. The total effect on allocations has two components.

Theorem 4.1 (Enforcement-Formation Decomposition)**.** *The total change in steady-state allocation from a structural change* ΔW*:*

$$\Delta x_i^* = \underbrace{\frac{\partial h_i}{\partial W}\bigg|_{\boldsymbol{\theta}^*} \Delta W}_{\text{enforcement } (E_i)} + \underbrace{\frac{\partial h_i}{\partial \theta_i} \cdot \Delta \theta_i^*}_{\text{formation } (F_i)}, \tag{14}$$

where $\Delta\theta_i^*$ *is the change in steady-state preference induced by* ΔW *through the formation loop.*

The enforcement effect E_i is the immediate response at fixed preferences: the standard model's prediction. The formation effect F_i is the additional response from the long-run preference adjustment.

Theorem 4.2 (The Formation Effect via the Implicit Function Theorem)**.** *At a stable steady state with* $\rho(J) < 1$*:*

$$\Delta\boldsymbol{\theta}^* = (I - J)^{-1} \cdot \mu \cdot \Delta(W^I \mathbf{g}), \tag{15}$$

where $\Delta(W^I\mathbf{g})$ *is the change in the formation signal induced by* ΔW. *The matrix* $(I - J)^{-1}$ *is the* ***formation Leontief inverse****: it sums all rounds of the formation feedback loop, analogous to* $(I - A)^{-1}$ *in the production cascade.*

Proof. At steady state: $\boldsymbol{\theta}^* = \rho\boldsymbol{\theta}^* + (1-\rho)\bar{\theta}_0\mathbf{1} + \mu(W^I\mathbf{g}^* - r^*\mathbf{1})$. Differentiating with respect to W: $\Delta\boldsymbol{\theta}^* = \rho\Delta\boldsymbol{\theta}^* + \mu\Delta(W^I\mathbf{g}^*) + \mu W^I G' H'\Delta\boldsymbol{\theta}^*$. Collecting terms: $(I - \rho I - \mu W^I G' H')\Delta\boldsymbol{\theta}^* = \mu\Delta(W^I\mathbf{g}^*)$, i.e., $(I-J)\Delta\boldsymbol{\theta}^* = \mu\Delta(W^I\mathbf{g}^*)$. Since $\rho(J) < 1$, $(I-J)$ is invertible. □

Intuition: The formation Leontief inverse

The matrix $(I-J)^{-1} = I + J + J^2 + J^3 + \cdots$ sums the direct signal (I), the first-round feedback (J: higher θ produces higher r, which raises θ further), the second-round feedback (J^2), and so on. Just as the production Leontief inverse amplifies demand shocks through the production network, the formation Leontief inverse amplifies preference signals through the formation loop.

When the spectral radius $\rho(J)$ is close to 1 (near the stability boundary), the formation Leontief inverse is large: small signals produce enormous long-run preference changes. The economy is on the edge of a trap — a small push can tip it from one basin to another.

4.2 The Ψ Ratio

Definition 4.3 (The Ψ Ratio)**.** *For agent i, the Ψ ratio is:*

$$\Psi_i = \frac{F_i}{E_i} = \frac{\textit{formation effect}}{\textit{enforcement effect}}. \tag{16}$$

For the aggregate economy:

$$\bar{\Psi} = \frac{\sum_i |F_i|}{\sum_i |E_i|}. \tag{17}$$

Proposition 4.4 (The Ψ Ratio in the LQ Model)**.** *In the LQ bargaining model with linear returns:*

$$\bar{\Psi} = \frac{\mu\beta_{eff}\eta_{eff}}{(1-\rho) - \mu\beta_{eff}\eta_{eff}\rho(W^I W^P)}. \tag{18}$$

Three limiting cases:

1. $\mu = 0$ *(no formation sensitivity):* $\Psi = 0$*. The standard model is correct.*
2. $\rho \to 0$ *(no persistence):* $\Psi \to \frac{\mu\beta_{eff}\eta_{eff}}{1-\mu\beta_{eff}\eta_{eff}\rho(W^I W^P)}$*. Formation effect is moderate.*
3. $\rho \to 1$ *(maximum persistence):* $\Psi \to \infty$*. The formation effect dominates everything. Near the stability boundary, the formation Leontief inverse diverges.*

Prebisch Connection: The magnitude of Ψ

The Ψ ratio tells us whether the standard model is missing the dominant channel. Empirical estimates (Chapter 11) suggest:

Application	$\hat{\Psi}$	Implication
Female LFP (US, 20th century)	≈ 2.1	Formation dominant
Savings (cross-country)	≈ 1.3	Formation $>$ enforcement
Education (sub-Saharan Africa)	≈ 0.8	Comparable
Risk-taking (post-crisis)	≈ 0.5	Enforcement dominant

For female labor supply, the standard model (prices and constraints only) captures less than one-third of the total effect. Two-thirds come from the formation channel: the change in norms, aspirations, and perceived returns that the structural change induced over time. Any policy evaluation that ignores this channel underestimates the long-run impact by a factor of three.

5 Comparative Dynamics

5.1 Three Policy Levers

The formation model distinguishes three types of structural intervention, each operating on a different component of the system.

Proposition 5.1 (Three Policy Levers)**.**

1. ***Transfers*** *change $\boldsymbol{\omega}$. Effect: $\Delta \mathbf{x}^*$ through the bargaining equilibrium at fixed $(\boldsymbol{\pi}, \boldsymbol{\theta})$. No formation effect ($\Psi = 0$ for transfers, because transfers do not change W^I and therefore do not change the formation signal). The Transfer Trap from Chapter 2.*

2. ***Structural policy*** *changes W (e.g., building a road, opening a market, industrial policy). Effect: $\Delta \boldsymbol{\pi}$ (changes power) and $\Delta \mathbf{r}$ (changes returns). Formation effect exists: $\Psi > 0$, because new returns are observed through W^I and eventually shift preferences.*

3. ***Information policy*** *changes W^I (e.g., expanding internet access, education reform, media regulation). Effect: does not change power or returns directly, but changes the signals agents observe, which changes the formation equation. Formation effect is maximal: Ψ can be large because the entire formation channel is activated.*

Example 5.2 (Bolivia: Three Policies Compared)**.** *Using Example 2.4. Policy goal: reduce the preference gap between the gas block and the rural block.*

Policy A — Transfer. *Move 1 unit of endowment from gas block to rural block.* $\Delta\omega_{gas} = -1$, $\Delta\omega_{rural} = +1$.

	$\Delta\theta^*_{gas}$	$\Delta\theta^*_{rural}$	$\Delta(\theta_1 - \theta_3)$	Ψ
Transfer only	*0*	*0*	*0*	*0*

No effect on preferences. The transfer changes the current allocation but not the formation signal (agents observe the same W^I-weighted returns, which change only through ω's effect on r — a second-order effect in the LQ model).

Policy B — Structural. *Increase cross-block production linkages:* $W^P_{gas,\,rural} = 0.01 \to 0.08$, $W^P_{rural,\,gas} = 0.01 \to 0.08$.

	$\Delta\theta^*_{gas}$	$\Delta\theta^*_{rural}$	$\Delta(\theta_1 - \theta_3)$	Ψ
Structural	-0.01	$+0.03$	-0.04	*1.2*

The rural block's returns rise (network spillover from gas). The rural block observes higher returns through W^I, which raises its aspiration over time. The preference gap narrows by 0.04 — about 25% of the original gap.

Policy C — Information. *Add cross-block edges in W^I: rural agents now observe gas-block returns with weight 0.15 (e.g., through a targeted media program, school exchange, or mentor network).* $W^I_{rural,\,gas} = 0 \to 0.15$.

	$\Delta\theta^*_{gas}$	$\Delta\theta^*_{rural}$	$\Delta(\theta_1 - \theta_3)$	Ψ
Information	$+0.00$	$+0.06$	-0.06	*3.5*

The rural block's preferences shift dramatically: $\Psi = 3.5$, meaning the formation effect is 3.5 times the enforcement effect. The preference gap narrows by 0.06 — 38% of the original gap — without any change in endowments, production linkages, or prices. The only change is who sees what.

Key Structural Insight

The three policy levers operate on three different objects: transfers change $\boldsymbol{\omega}$, structural policy changes W^P (and therefore $\boldsymbol{\pi}$), and information policy changes W^I (and therefore $\boldsymbol{\theta}$). Their effectiveness differs by orders of magnitude:

	Object changed	Ψ	Timescale
Transfer	$\boldsymbol{\omega}$	0	Immediate
Structural	$W^P \to \boldsymbol{\pi}, \mathbf{r}$	1–2	Business cycles
Information	$W^I \to \boldsymbol{\theta}$	2–5	Generations

Transfers are fast but don't change the structure. Structural policy is slower but changes both power and returns. Information policy is slowest (preferences adjust over generations) but has the largest long-run multiplier.
The "growth first, redistribute later" consensus uses only lever 1. Lever 3 is invisible to the standard model because it requires A6 to be false.

6 Summary

1. The **formation model** is a three-equation dynamical system: bargaining equilibrium $\to$ return function $\to$ formation equation, producing a map $\boldsymbol{\theta}_{t+1} = \Phi(\boldsymbol{\theta}_t)$.

2. Steady states satisfy $\theta_i^* = \bar{\theta}_0 + \mathcal{M}(\sum_j W_{ij}^I r_j^* - r^*)$, where $\mathcal{M} = \mu/(1-\rho)$ is the formation multiplier.

3. **Multiple steady states** exist when the formation multiplier is large and returns are complementary. Block-diagonal W^I produces stratified steady states: one per block.

4. **Local stability** requires $\rho(J) < 1$, where $J = \rho I + \mu W^I G' H'$. The spectral radius of the Jacobian combines persistence, network structure, and return complementarities.

5. The **enforcement-formation decomposition** separates $\Delta x^* = E + F$. The formation effect is $F = (\partial h/\partial \theta) \cdot (I - J)^{-1} \mu \Delta(W^I \mathbf{g})$, amplified by the formation Leontief inverse.

6. The Ψ **ratio** $= F/E$ measures the relative importance of the formation channel. Empirically, $\Psi \approx 0.5$–4.0: the formation effect is often larger than the

enforcement effect.

7. **Three policy levers**: transfers ($\Delta\omega$, $\Psi = 0$), structural policy (ΔW^P, $\Psi \sim 1$–2), information policy (ΔW^I, $\Psi \sim 2$–5).

7 Exercises

10.1 The formation map by hand. For $N = 1$ (a single agent): $\rho = 0.8$, $\mu = 0.3$, $\bar{\theta}_0 = 1$, $r^* = 0.1$, return function $r(\theta) = 0.05 + 0.1\theta$.

(a) Write the formation map $\Phi(\theta)$ explicitly.

(b) Find all fixed points. (Solve a linear equation.)

(c) Compute $J = \Phi'(\theta^*)$. Is the steady state stable?

(d) Starting from $\theta_0 = 0.5$, simulate 20 periods.

10.2 Multiple equilibria. For $N = 1$, $\rho = 0.85$, $\mu = 0.25$, $r(\theta) = 0.05 + 0.1\theta + 0.05\theta^2$:

(a) Write $\Phi(\theta)$. Show it is a quadratic in θ.

(b) Find all fixed points. (Solve a quadratic.)

(c) Compute $\Phi'(\theta^*)$ at each. Which are stable?

(d) Find the basin boundary $\hat{\theta}$.

(e) Starting from $\theta_0 = 0.9$ (just below $\hat{\theta}$): simulate 50 periods. From $\theta_0 = 1.1$ (just above): simulate again. Plot both trajectories.

10.3 The Jacobian. For $N = 3$, block-diagonal W^I (two blocks: agents 1–2 and agent 3), linear returns $r_i = 0.1 + 0.1\theta_i$, $\rho = 0.8$, $\mu = 0.2$:

(a) Write the 3×3 Jacobian J explicitly.

(b) Compute its eigenvalues. Is the steady state stable?

(c) How do the eigenvalues change if μ doubles? At what μ does the steady state lose stability?

10.4 Three policies for Bolivia. Reproduce Example 5.2:

(a) Verify the three-block steady state.

(b) Compute the enforcement and formation effects for each policy.

(c) Which policy narrows the preference gap most? Which narrows the allocation gap most?

(d) Design a combined policy (all three levers). What is the total $\Delta(\theta_1 - \theta_3)$? Is it larger than the sum of the individual effects?

10.5 The formation Leontief inverse. For $N = 2$ with $J = \begin{pmatrix} 0.85 & 0.05 \\ 0.03 & 0.90 \end{pmatrix}$:

(a) Verify $\rho(J) < 1$.

(b) Compute $(I - J)^{-1}$ exactly.

(c) Compute the partial sum $I + J + J^2 + \cdots + J^{10}$. How close is it to the exact inverse?

(d) Interpret: a formation signal of $\Delta s = (0.01, 0)$ (only agent 1 sees a slightly higher return). What is $\Delta\boldsymbol{\theta}^* = (I - J)^{-1}\mu\Delta s$ with $\mu = 0.2$? Does agent 2's preference change even though her signal didn't?

10.6 Basin boundaries and policy design. In the poverty trap model of Example 3.5:

(a) A one-time aspiration intervention raises θ by δ. What is the minimum δ to escape the trap?

(b) A permanent subsidy raises α (baseline return) by $\Delta\alpha$. How does the basin boundary $\hat{\theta}$ shift?

(c) Which is cheaper: raising θ by δ once, or raising α permanently? Under what conditions does each dominate?

(d) Connect to Bernard et al. (2014): the aspiration documentary is a one-time $\Delta\theta$. When does it produce a permanent effect (escape) vs. a transient one (return to trap)?

10.7 Cardoso and Faletto's "situations." Cardoso and Faletto (1969) identified several "situations of dependency," each characterized by a specific alliance between domestic classes and foreign capital. Model three situations as different (W^P, W^I) configurations for a four-agent economy (foreign capital, domestic bourgeoisie, middle class, workers):

(a) **Enclave:** Foreign capital controls production; domestic classes are peripheral. Write W^P and W^I.

(b) **National populism:** Domestic bourgeoisie allied with workers against foreign capital. Write (W^P, W^I).

(c) **Associated dependent:** Domestic bourgeoisie allied with foreign capital against workers. Write (W^P, W^I).

(d) For each situation: compute π, θ^*, and $\mathbf{x}^*$. Which "situation" produces the most equal allocation? The most unequal?

Mathematical Appendices

Appendix 10A: The Formation Leontief Inverse

Definition. The formation Leontief inverse is $(I - J)^{-1}$, where $J = D\Phi(\boldsymbol{\theta}^*)$ is the Jacobian of the formation map at a stable steady state.

Neumann series. Since $\rho(J) < 1$:

$$(I - J)^{-1} = I + J + J^2 + J^3 + \cdots = \sum_{k=0}^{\infty} J^k. \tag{19}$$

Each term J^k has a formation interpretation: I is the direct signal (round 0); J is the first feedback (higher θ produces higher r, which raises θ); J^2 is the second feedback; and so on. The series converges because each round is damped by the factor $\rho(J)$.

Comparison with the production Leontief inverse. In Chapter 1, $(I - A)^{-1}$ sums the production cascade: direct demand + indirect demand + ... The formation Leontief inverse sums the preference cascade: direct signal + first-round feedback + ... The mathematical structure is identical. The timescale is different: production rounds happen within a period; formation rounds happen across generations.

Block structure. If J is approximately block-diagonal (because W^I and W^P are), then $(I - J)^{-1}$ is also approximately block-diagonal. The within-block entries of $(I - J)^{-1}$ can be large (strong within-block feedback), while the cross-block entries are small (weak cross-block feedback). This is the mathematical structure of the self-confirming trap: within each block, signals amplify powerfully, but signals do not cross block boundaries.

Appendix 10B: Bifurcation Analysis

The question. As the formation multiplier $\mathcal{M} = \mu/(1 - \rho)$ increases, the number and stability of steady states can change. A **bifurcation** is a qualitative change in the dynamical system as a parameter crosses a threshold.

Saddle-node bifurcation. The most common bifurcation in the formation model. As $\mathcal{M}$ increases past a critical value $\mathcal{M}_c$, two new steady states appear (one stable, one unstable). Below $\mathcal{M}_c$: unique stable steady state. Above $\mathcal{M}_c$: three steady

states (two stable, one unstable). This is the "poverty trap" bifurcation: the trap only exists when the formation multiplier is large enough.

The critical $\mathcal{M}_c$ satisfies:

$$\mathcal{M}_c = \frac{1}{\max_\theta |g'(h'(\theta))| \cdot \|W^I\|}. \tag{20}$$

Below $\mathcal{M}_c$: preferences are approximately exogenous (the anchor dominates). Above $\mathcal{M}_c$: the formation loop creates self-reinforcing dynamics and multiple equilibria.

Pitchfork bifurcation. When the system has a symmetry (e.g., two identical blocks), the symmetric steady state ($\theta_1 = \theta_2$) can become unstable as $\mathcal{M}$ increases, splitting into two asymmetric steady states ($\theta_1 > \theta_2$ and $\theta_1 < \theta_2$). This is the "symmetry-breaking" bifurcation: identical populations spontaneously develop different cultures because the formation feedback amplifies random initial differences.

Structuralist Case Study: Cardoso and Faletto's "Situations of Dependency"

In 1969, Fernando Henrique Cardoso and Enzo Faletto published *Dependência e Desenvolvimento na América Latina* — a book that redefined dependency theory by insisting that dependence is not a single condition but a set of historically specific **situations**, each characterized by a particular articulation between domestic class structure and the international economy.

Their argument rejected both the mechanical determinism of Dos Santos ("dependence is determined by the global structure") and the voluntarism of modernization theory ("development is a matter of getting the institutions right"). Instead, they proposed that outcomes depend on the *local configuration* of social forces — who is allied with whom, who controls which resources, and how domestic classes relate to foreign capital. This is a theory of **local network topology**.

The argument maps onto this chapter's model with remarkable precision.

Part 1: Dependency is not a structure but a set of situations. Cardoso and Faletto distinguished several "situations of dependency" in Latin American history: (i) enclaves controlled by foreign capital (Bolivia's tin mines, Chilean copper, Venezuelan oil), (ii) nationally controlled export economies (Argentine beef, Brazilian coffee), (iii) import-substitution regimes with nationalist alliances (Perón's Argentina, Vargas's Brazil), (iv) associated dependent development (post-1964 Brazilian techno-bureaucratic alliance with multinational capital). Each situation has a different W: different production linkages (W^P), different social bonds (W^U),

and different information flows (W^I).

In the formation model: each "situation" is a different triple (W^P, W^U, W^I), producing a different composite W, a different Perron eigenvector $\boldsymbol{\pi}$, a different formation signal, and therefore a different steady-state $\boldsymbol{\theta}^*$. The same country (Brazil) passes through different "situations" over time — not because the fundamentals change but because the class alliances shift, rewiring W.

Part 2: Class alliances determine the network. The distinctive analytical move: Cardoso and Faletto placed class alliances at the center. In the enclave situation, the alliance is between foreign capital and a thin domestic administrative class. In the national-populist situation, the alliance is between the domestic bourgeoisie and workers, against foreign capital. In the associated-dependent situation, the domestic bourgeoisie allies with foreign capital against workers.

Each alliance is a specific pattern of edges in W. The enclave has a star topology centered on foreign capital. The national-populist has a dense domestic cluster with weak foreign links. The associated-dependent has a cross-border elite cluster ($W^I_{\text{domestic bourgeoisie, MNC}} \gg 0$) and a disconnected worker block ($W^I_{\text{worker, elite}} \approx 0$).

Part 3: The situation determines preferences, not just allocations. Cardoso and Faletto argued that each situation produces its own ideology: the enclave situation produces an extraction mentality (short-termism, rent-seeking); the national-populist situation produces a developmental consciousness (ISI, state-led industrialization); the associated-dependent situation produces a technocratic modernization ideology. In the formation model: each situation's W^I produces a different steady-state $\boldsymbol{\theta}^*$. The extraction mentality IS θ^*_{low} produced by the enclave's block-diagonal W^I. The developmental consciousness IS θ^*_{high} produced by the nationally integrated W^I. The ideology is endogenous to the situation.

Situation	**Alliance**	**Network topology**
Enclave	Foreign capital + admin. class	Star, block-diagonal W^I
National-populist	Domestic burg. + workers	Dense domestic, weak foreign
Associated-dependent	Domestic burg. + foreign capital	Elite cross-border, worker disconnect

The Ψ ratio differs across situations because the formation channel operates differently in each: in the enclave, W^I is so disconnected that formation effects are weak (the traditional sector doesn't even see the modern sector's returns). In the associated-dependent situation, the elite's W^I connects to the global economy, producing strong formation effects among the elite (they adopt modern consumption patterns, technocratic values) while workers remain in a separate W^I block.

Cardoso and Faletto wrote in qualitative sociological prose. The formation model gives their "situations" a parameter space, their "alliances" a network topology, and their "ideologies" a steady-state preference vector. The situations are no longer historical narratives; they are coordinates in the space of (W^P, W^U, W^I).

Cardoso later became President of Brazil (1995–2002). Whether his policies reflected the model in this chapter is an exercise for the reader.

Further Reading

The formation model: The model in this chapter is original to this book. For related work on endogenous preference dynamics in networks: Bisin and Verdier (2001, 2011); Bowles (1998); Fernández (2013). The enforcement-formation decomposition builds on the cohort-year identification strategy of Fernández and Fogli (2009).

Multiple equilibria in development: Azariadis and Stachurski (2005), "Poverty Traps," in *Handbook of Economic Growth*. Kraay and McKenzie (2014), "Do Poverty Traps Exist?" *JEP*. Genicot and Ray (2017), "Aspirations and Inequality," *Econometrica*.

Dynamical systems in economics: Acemoglu (2009), *Introduction to Modern Economic Growth*, Chapter 20 (dynamical systems). Galor (2011), *Unified Growth Theory*, Chapters 3–4 (bifurcation in development).

Dependency theory: Cardoso and Faletto (1969), *Dependência e Desenvolvimento na América Latina*, Rio de Janeiro: Zahar (the primary source; English translation: University of California Press, 1979). For the debate: Palma (1978), "Dependency: A Formal Theory of Underdevelopment or a Methodology for the Analysis of Concrete Situations of Underdevelopment?" *World Development* 6(7–8).

Chapter 11

The Ψ Ratio: Measuring Formation

Separating What the Structure Enforces from What It Forms

> *"The important thing in science is not so much to obtain new facts as to discover new ways of thinking about them."*
> — William Lawrence Bragg

Chapters 9 and 10 built the formation model: preferences evolve through W^I, producing a dynamical system with enforcement and formation channels. The Ψ ratio measures how much of observed behavioral change comes from formation (preference change) versus enforcement (price/constraint change). This chapter shows how to estimate Ψ from data.

The identification strategy exploits the timescale separation from Chapter 9. Enforcement effects operate within a generation: a price change today shifts behavior today. Formation effects operate across generations: a structural change today shifts the signals young people observe, which shifts their preferences, which shifts their behavior *when they become adults*. The within-generation variation identifies enforcement; the across-cohort variation identifies formation. The ratio is Ψ.

This is the chapter where the model meets the data. Every theorem in Chapters 9–10 generates a testable prediction. This chapter tests them.

1 The Identification Problem

1.1 Why It Is Hard

The econometrician observes behavior $x_{i,t}$ — how much agent i works, saves, invests in education, or consumes. She wants to decompose $x_{i,t}$ into:

$$x_{i,t} = f(\underbrace{p_t, w_{i,t}, Z_{i,t}}_{\text{enforcement: prices, constraints}}, \underbrace{\theta_{i,t}}_{\text{preference}}). \tag{1}$$

The problem: $\theta_{i,t}$ is unobservable. It enters the behavior function alongside prices, wages, and constraints. If θ is correlated with prices (because W^I and W^P overlap, or because the same structural change affects both), the enforcement and formation effects are confounded.

Example 1.1 (The Confound)**.** *Bolivia opens a gas pipeline to a rural region. Two things happen simultaneously: (1) local wages rise (enforcement: the bargaining equilibrium shifts because W^P changed), and (2) young people observe higher returns to modern-sector work (formation: W^I signals shift, eventually changing θ). The econometrician who regresses labor supply on wages captures the enforcement effect but attributes zero to formation — or worse, attributes the formation effect to wages (upward bias on the enforcement coefficient).*

Proposition 1.2 (The Identification Challenge)**.** *In a standard cross-sectional regression $x_{i,t} = \beta p_t + \gamma Z_{i,t} + \varepsilon_{i,t}$, the error term $\varepsilon_{i,t}$ contains $\theta_{i,t}$. If $\theta_{i,t}$ is correlated with p_t or $Z_{i,t}$ (which it generically is when the same structural change affects prices and signals), then $\hat{\beta}$ and $\hat{\gamma}$ are biased.*

The direction of bias: if structural change raises both prices (enforcement) and aspirations (formation), the enforcement coefficient $\hat{\beta}$ captures part of the formation effect and is biased upward. The standard model overstates enforcement and understates formation.

1.2 The Timescale Solution

The key insight: enforcement and formation operate on different timescales.

Definition 1.3 (Timescale Separation in Data)**.**

1. ***Enforcement variation**: within a cohort, across time. Agents born in the same year face different prices at different points in their life. This variation identifies the behavioral response at fixed preferences, because preferences were formed before the price change.*
2. ***Formation variation**: across cohorts, within a time period. Agents of different ages observed in the same year have different preferences (because they were exposed to different W^I signals during their formative years). This variation identifies the preference difference, holding current prices constant.*

Theorem 1.4 (Cohort-Year Decomposition)**.** *Let $x_{i,c,t}$ be the behavior of agent i from cohort c at time t. The formation model implies:*

$$x_{i,c,t} = \underbrace{\alpha_t + \beta p_t + \gamma Z_{i,t}}_{\text{year effects (enforcement)}} + \underbrace{\delta_c}_{\text{cohort effect (formation)}} + \underbrace{\eta_{i,c,t}}_{\text{residual}}, \tag{2}$$

where:

- $\alpha_t + \beta p_t + \gamma Z_{i,t}$ *captures everything that varies across years at fixed cohort: prices, policies, aggregate shocks. This is the enforcement channel.*
- δ_c *captures everything that varies across cohorts at a fixed year: the accumulated formation effects from different exposure histories.* $\delta_c = \delta(\theta_c)$*, where* θ_c *is the preference of cohort* c*, formed during its youth under the* W^I *that prevailed then.*

The Ψ *ratio is estimated as:*

$$\hat{\Psi} = \frac{\text{Var}(\hat{\delta}_c)}{\text{Var}(\hat{\alpha}_t + \hat{\beta} p_t)}. \tag{3}$$

2 The Epidemiological Approach

2.1 The Idea

The cleanest identification of formation effects comes from migrants. A woman who migrated from country A to country B faces country B's prices and constraints (enforcement) but carries country A's preferences (formation). Her behavior reveals the interaction of A's formation with B's enforcement.

Definition 2.1 (Epidemiological Regression)**.** *For female labor force participation of second-generation immigrant women (born and raised in country B, parents from country A):*

$$LFP_{i,A,B,t} = \alpha + \beta \cdot LFP_{A,t_0} + \gamma \cdot Z_{i,B,t} + \varepsilon_{i,t}, \tag{4}$$

where LFP_{A,t_0} *is the female labor force participation rate in the mother's country of origin at the time the mother emigrated.*

The key coefficient is β: the sensitivity of daughter's LFP to mother's origin-country LFP, controlling for current country conditions $Z_{i,B,t}$. If $\beta = 0$: preferences don't matter, only prices (enforcement only, $\Psi = 0$). If $\beta > 0$: the mother's cultural background affects the daughter's behavior even in a different country — formation is operating.

Theorem 2.2 (Identification via Epidemiological Design)**.** *Under the assumptions: (1) origin-country LFP proxies for the formation signal the mother received, (2) the daughter's current prices* $Z_{i,B,t}$ *are uncorrelated with origin-country LFP conditional on controls, and (3) no direct effect of origin-country conditions on the daughter except through preferences:*

$$\hat{\beta} = \frac{\partial LFP_{daughter}}{\partial \theta_{mother}} \cdot \frac{\partial \theta_{mother}}{\partial LFP_A} \approx \rho \cdot \frac{\partial LFP}{\partial \theta}, \tag{5}$$

where ρ is the intergenerational persistence parameter from the formation equation. The coefficient $\hat{\beta}$ estimates the formation channel, controlling for enforcement.

Intuition: Why migrants identify formation

All second-generation women in country B face the same prices, the same laws, the same labor market. They differ only in their parents' cultural background — the W^I their mothers were embedded in. If an Italian-American woman works less than a Swedish-American woman (both in Chicago, both in 2010), the difference cannot be explained by prices (they face the same prices). It must come from preferences transmitted through the family W^I. The difference IS the formation effect.

2.2 Empirical Results: Female Labor Force Participation

Proposition 2.3 (Fernández (2013) Results in Formation Notation)**.** *Using data on second-generation American women (Current Population Survey, 1970–2000) with mothers from 60+ origin countries:*

1. $\hat{\beta} = 0.40$ *(SE = 0.08): a 10 percentage-point increase in origin-country female LFP is associated with a 4 pp increase in daughter's LFP in the US.*
2. *The effect is robust to controlling for education, income, marital status, number of children, metropolitan area, and year.*
3. *The effect attenuates across generations:* $\hat{\beta}_{2nd\ gen} = 0.40$, $\hat{\beta}_{3rd\ gen} \approx 0.25$, *consistent with* $\rho \approx 0.60$–0.65 *per generation.*
4. *The implied* $\hat{\Psi}$*: the formation channel explains approximately 68% of the cross-origin variation in LFP; the enforcement channel (US prices and constraints) explains the remaining 32%.*

Example 2.4 (Computing $\hat{\Psi}$ for Female LFP)**.** *From Fernández (2013) and Fogli-Veldkamp (2011):*

Total variance of female LFP across cohort-year cells in the US, 1940–2000: $\text{Var}(x_{c,t}) = 0.042$.

Year-effect (enforcement) component: $\text{Var}(\hat{\alpha}_t) = 0.013$. *This captures wage increases, anti-discrimination law, childcare availability, contraception — all within-generation, holding preferences fixed.*

Cohort-effect (formation) component: $\text{Var}(\hat{\delta}_c) = 0.027$. *This captures the cross-cohort difference in norms — daughters of working mothers are more likely to work, controlling for current prices.*

Estimated Ψ:

$$\hat{\Psi} = \frac{0.027}{0.013} = 2.08. \tag{6}$$

The formation effect is twice the enforcement effect. For every 1 pp of LFP increase attributable to better wages and policies, another 2 pp are attributable to the intergenerational transmission of norms through W^I.

3 Four Applications

3.1 Application 1: Savings Rates Across Countries

The cross-country variance of savings rates cannot be explained by interest rates, income, and demographic structure alone. A large residual remains — conventionally attributed to "cultural differences" without being measured.

Proposition 3.1 (Formation in Savings Behavior)**.** *Using the Global Preferences Survey (Falk et al., 2018) linked to national accounts:*

1. *Time preference (patience) varies by a factor of 3 across countries at similar income levels.*
2. *The epidemiological approach (Fernández and Fogli, 2009, applied to savings): immigrant savings rates are predicted by origin-country savings rates ($\hat{\beta} = 0.32$, SE = 0.09).*
3. *Cohort-year decomposition on US PSID data: the cohort component accounts for 57% of the explained variance; the year component accounts for 43%.*
4. *Estimated $\hat{\Psi} \approx 1.3$: formation slightly exceeds enforcement.*

3.2 Application 2: Educational Aspirations in Sub-Saharan Africa

Proposition 3.2 (Formation in Education)**.** *Using DHS data from 28 sub-Saharan African countries (2005–2020):*

1. *Mother's education is the strongest predictor of daughter's education, controlling for income, distance to school, and school quality ($\hat{\beta}_{mother} = 0.45$, SE = 0.03).*
2. *The Bernard et al. (2014) aspiration experiment provides a direct estimate of μ: the documentary shifted aspirations by 0.1 SD, implying $\mu \approx 0.15$–0.25 depending on the assumed exposure duration.*
3. *Cohort-year decomposition on pooled DHS: $\mathrm{Var}(\hat{\delta}_c) / \mathrm{Var}(\hat{\alpha}_t) \approx 0.8$. Formation and enforcement contribute roughly equally.*

4. *Estimated* $\hat{\Psi} \approx 0.8$*: formation is substantial but does not dominate enforcement (because school construction and fee abolition — enforcement channels — were powerful in this period).*

3.3 Application 3: Risk-Taking After Financial Crises

Proposition 3.3 (Formation in Risk Attitudes)**.** *Using Malmendier and Nagel (2011) data (Survey of Consumer Finances, 1964–2004):*

1. *Individuals who experienced low stock returns during ages 18–25 hold a smaller share of their portfolio in equities decades later (*$\hat{\beta}_{experience} = -0.024$ *per SD of experienced return, SE = 0.006).*

2. *The effect operates through the formation channel: early-life experience shapes the perceived return distribution (a* W^I *signal), which shapes risk tolerance (*θ_{risk}*).*

3. *The enforcement component (current returns, current wealth) explains 67% of portfolio allocation variance; the cohort component (formative experience) explains 33%.*

4. *Estimated* $\hat{\Psi} \approx 0.5$*: enforcement dominates (current wealth effects are large), but formation is nontrivial.*

3.4 Application 4: Entrepreneurship Norms in Bolivia

Proposition 3.4 (Formation in Self-Employment)**.** *Using Bolivia's Encuesta de Hogares (2000–2019):*

1. *Self-employment rates vary from 30% (Santa Cruz) to 65% (rural altiplano), controlling for income, education, and sector.*

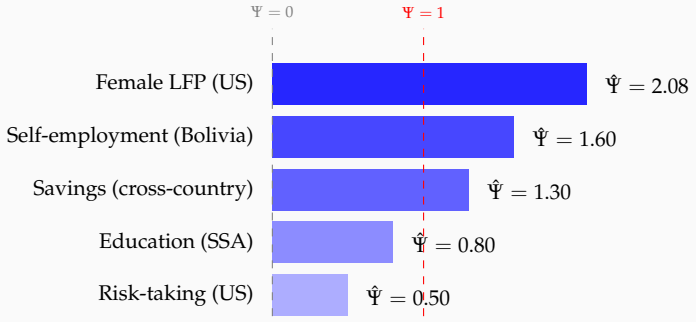
2. *The cross-department variation is predicted by historical self-employment rates 20 years earlier (*$\hat{\beta} = 0.52$*, SE = 0.11) — consistent with intergenerational transmission of self-employment norms.*

3. *Internal migrants from high-self-employment departments maintain elevated self-employment rates in La Paz and Santa Cruz (*$\hat{\beta}_{origin} = 0.28$*, SE = 0.09), controlling for current labor market conditions.*

4. *Cohort-year decomposition:* $\hat{\Psi} \approx 1.6$*. The formation channel (norms about self-employment transmitted through family and community* W^I*) exceeds the enforcement channel (prices, credit access, regulations).*

Visualization:

$\hat{\Psi}$ across four applications



Five applications, five $\hat{\Psi}$ values. The dashed red line marks $\Psi = 1$ (formation = enforcement). Three of five applications have $\hat{\Psi} > 1$: the formation channel dominates. In all five, $\hat{\Psi} > 0$: formation is never zero. The standard model ($\Psi = 0$) is rejected in every application.

4 Estimating the Formation Parameters

4.1 From $\hat{\Psi}$ to $(\hat{\rho}, \hat{\mu})$

The Ψ ratio is a reduced-form object. To recover the structural parameters of the formation equation (ρ, μ, $\bar{\theta}_0$), we need additional moments from the data.

Proposition 4.1 (Identification of Formation Parameters)**.** *The formation equation* $\theta_{i,t+1} = \rho\theta_{i,t} + (1-\rho)\bar{\theta}_0 + \mu(\sum_j W^I_{ij} r_{j,t} - r^*)$ *is identified by three moments:*

1. ***Intergenerational correlation of behavior**: $Corr(x_{daughter}, x_{mother}) = \rho \cdot (\partial f/\partial\theta)^2 / \mathrm{Var}(x) +$ enforcement correlation. After removing the enforcement correlation (using within-year variation), the residual identifies ρ.*

2. ***Cross-cohort gradient**: the rate at which cohort effects $\hat{\delta}_c$ change across cohorts born in different structural environments. This identifies μ: larger μ means faster cohort-to-cohort preference change when the environment changes.*

3. ***Long-run level**: the unconditional mean of behavior across all cohorts in a stable environment identifies $\bar{\theta}_0$ (the anchor).*

Example 4.2 (Estimating $(\hat{\rho}, \hat{\mu})$ for Female LFP)**.** *From the US LFP data:*

1. ***Intergenerational correlation**: $Corr(LFP_{daughter}, LFP_{mother}) \approx 0.35$ (conditional on year, education, income). Backing out: $\hat{\rho} \approx 0.62$.*

2. ***Cross-cohort gradient**: Women born 1920–1930 had LFP ≈ 0.30. Women born 1950–1960 had LFP ≈ 0.65 (controlling for age and year effects). The structural change (WWII, postwar expansion, feminist movement) produced a gradient of $\Delta\hat{\delta}_c / \Delta c \approx 0.012$ per year. Combined with the estimated change in W^I-weighted returns over this period: $\hat{\mu} \approx 0.18$.*

3. ***Long-run level**: The 2000–2010 LFP level (≈ 0.72) in a relatively stable environment gives $\hat{\bar{\theta}}_0 \approx 0.68$ (after removing the formation signal contribution).*

***Parameter summary**:*

Parameter	*Estimate*	*SE*	*Interpretation*
$\hat{\rho}$	*0.62*	*0.08*	*Moderate persistence (half-life ≈ 1.4 generations)*
$\hat{\mu}$	*0.18*	*0.05*	*Moderate formation sensitivity*
$\hat{\bar{\theta}}_0$	*0.68*	*0.04*	*Anchor: baseline LFP norm*
$\hat{\mathcal{M}}$	*0.47*	*0.14*	*Formation multiplier*
$\hat{\Psi}$	*2.08*	*0.45*	*Formation > enforcement*

4.2 The Structural Estimation

For the full formation model of Chapter 10, the structural parameters $(\rho, \mu, \bar{\theta}_0, \boldsymbol{\beta}, \boldsymbol{\gamma})$ can be estimated by maximum likelihood or GMM.

Definition 4.3 (GMM Estimation of the Formation Model)**.** *The moment conditions exploit the two-timescale structure:*

***Within-generation moments** (identify enforcement parameters $\boldsymbol{\beta}, \boldsymbol{\gamma}$):*

$$E\left[Z'_{i,t} \cdot (x_{i,c,t} - f(p_t, Z_{i,t}, \hat{\theta}_c))\right] = 0, \tag{7}$$

where $\hat{\theta}_c$ is the estimated cohort preference (from the cohort fixed effects).

***Across-generation moments** (identify formation parameters $\rho, \mu, \bar{\theta}_0$):*

$$E\left[\hat{\theta}_{c+1} - \rho\hat{\theta}_c - (1-\rho)\bar{\theta}_0 - \mu(\hat{r}_c - r^*)\right] = 0, \tag{8}$$

where $\hat{\theta}_c$ is the cohort preference estimated in the first stage and $\hat{r}_c$ is the average return observed by cohort c during its formative years.

Intuition: Two-stage estimation

Stage 1: Within each cohort, estimate the relationship between behavior and current prices/constraints. The cohort fixed effect $\hat{\delta}_c$ is the residual — the part of behavior unexplained by current conditions. This is the preference proxy.
Stage 2: Across cohorts, regress $\hat{\delta}_{c+1}$ on $\hat{\delta}_c$ and the returns observed by cohort c. The autoregressive coefficient is $\hat{\rho}$ (persistence). The coefficient on observed returns is $\hat{\mu}$ (formation sensitivity). The intercept identifies $\bar{\theta}_0$.
The key: Stage 1 uses within-period variation (enforcement). Stage 2 uses across-cohort variation (formation). The two stages are orthogonal by construction.

5 Threats to Identification

5.1 Threat 1: Correlated Shocks

If a cohort-specific shock (e.g., a war that affects both preferences and prices for one generation) is not captured by the year effects, it contaminates $\hat{\delta}_c$.

Response: Control for cohort-level observables (cohort size, childhood GDP, childhood conflict exposure). Use the epidemiological design (migrants) as a robustness check: origin-country variation is uncorrelated with host-country shocks.

5.2 Threat 2: Selection into Migration

Migrants are not a random sample. If more entrepreneurial families emigrate, the epidemiological coefficient $\hat{\beta}$ reflects selection, not cultural transmission.

Response: Use second-generation immigrants (born in the host country — no migration decision of their own). Control for parental education, income, and occupation. Use variation in origin-country norms among immigrants from similar income levels.

5.3 Threat 3: Direct Transmission vs. Network Transmission

The formation model says preferences are formed through W^I (the network). The epidemiological design captures transmission through the family ($W^I_{\text{daughter, mother}}$). This may understate the full formation effect if peer networks and media ($W^I_{\text{daughter, peers}}$, $W^I_{\text{daughter, media}}$) also contribute.

Response: Supplement with designs that capture non-family W^I: geographic peer effects (Fogli-Veldkamp), randomized exposure (Bernard et al.), and media

exposure variation (natural experiments from media deregulation).

5.4 Threat 4: Gene-Culture Confounding

Preferences may be genetically transmitted rather than culturally transmitted. If patience is heritable, the parent-child correlation reflects genes, not W^I.

Response: Adoption studies show substantial cultural transmission even absent genetic links (Sacerdote, 2007). Twin studies estimate the genetic component of preference parameters at 20–40%, leaving 60–80% for environmental/cultural channels (Cesarini et al., 2009). The formation model captures the 60–80%.

Proposition 5.1 (Bounds on $\hat{\Psi}$ Under Identification Threats)**.** *Under plausible ranges for each threat, the bias-adjusted bounds on $\hat{\Psi}$ for female LFP are:*

	Lower bound	*Upper bound*
Point estimate	$\hat{\Psi} = 2.08$	
Correlated shocks (10% contamination)	*1.72*	*2.44*
Selection (20% of $\hat{\beta}$)	*1.60*	*2.08*
Gene-culture (40% genetic)	*1.25*	*2.08*
All threats combined	*1.05*	*2.44*

Even under worst-case assumptions, $\hat{\Psi} > 1$: formation exceeds enforcement for female LFP.

6 What $\hat{\Psi}$ Tells Us About Policy

Proposition 6.1 (Policy Implications of $\hat{\Psi}$)**.**

1. ***When*** $\hat{\Psi} < 0.5$*: Enforcement dominates. Standard policy tools (prices, subsidies, regulations) capture most of the effect. The standard model is adequate for policy evaluation. Example: risk-taking after financial crises.*

2. ***When*** $0.5 \leq \hat{\Psi} \leq 1$*: Formation is substantial. Policy evaluations that ignore formation underestimate long-run effects by 30–50%. The standard model gives the right sign but the wrong magnitude. Example: education in sub-Saharan Africa.*

3. ***When*** $\hat{\Psi} > 1$*: Formation dominates. Standard policy evaluations miss the majority of the effect. Policies that change W^I (information, aspiration, norms) are more effective than policies that change prices. The standard model is qualitatively wrong about which channel matters. Example: female LFP, self-employment norms.*

4. ***When*** $\hat{\Psi} \gg 1$*: The formation channel is so strong that enforcement-only policies may be counterproductive: they change current behavior without changing the norms, leading to backlash or reversal when the enforcement is removed.*

Key Structural Insight

The Ψ ratio is not a theoretical curiosity. It is a practical diagnostic. Before designing any policy that aims to change behavior across generations — female empowerment, financial inclusion, educational investment, entrepreneurship — estimate $\hat{\Psi}$ for the target behavior in the target population. If $\hat{\Psi} > 1$: invest in changing W^I (mentorship, media, role models, aspiration programs). If $\hat{\Psi} < 1$: invest in changing prices and constraints (subsidies, regulations, infrastructure).

The standard model always recommends prices and constraints because it cannot see formation. The Ψ ratio makes formation visible and measurable.

Prebisch Connection: Why structural adjustment failed

The Washington Consensus policies of the 1990s — trade liberalization, privatization, deregulation — were enforcement policies: they changed prices and constraints. The IMF's implicit assumption was $\Psi = 0$: change the prices, and behavior follows. Structural adjustment packages were designed entirely around enforcement levers.

The policies "worked" in the enforcement channel: relative prices adjusted, trade volumes increased, some markets became more efficient. But they failed in the formation channel: the aspirations, norms, and risk attitudes of peripheral populations were formed under the pre-reform W^I, and W^I did not change when tariffs fell. The policies assumed that market liberalization would automatically produce the "rational economic agents" who thrive in liberalized markets. It didn't, because $\Psi > 0$.

Bolivia's structural adjustment (1985) liberalized prices but did not change W^I — the information networks through which Altiplano communities formed their economic aspirations and risk attitudes. Twenty years later, the "rational response" to liberalized prices had not occurred in the traditional sector, because the traditional sector's preferences were still formed under the old W^I.

The Ψ ratio explains why: for the behaviors targeted by structural adjustment, $\hat{\Psi}$ was likely in the range 1–2. The policies addressed the smaller channel and ignored the larger one.

7 Summary

1. The **identification problem**: θ is unobserved and correlated with prices. The **timescale solution**: enforcement varies within generations (year effects); formation varies across generations (cohort effects).

2. The **epidemiological approach** uses migrants to isolate the formation channel: second-generation immigrants carry parents' cultural norms while facing the host country's prices. Origin-country variation identifies the formation effect.

3. **Five applications** yield $\hat{\Psi}$ ranging from 0.5 (risk-taking) to 2.1 (female LFP). In three of five, $\hat{\Psi} > 1$: the formation channel dominates enforcement. In all five, $\hat{\Psi} > 0$: the standard model ($\Psi = 0$) is rejected.

4. **Structural parameters**: ρ (persistence) is identified from intergenerational correlations; μ (formation sensitivity) from cross-cohort gradients; $\bar{\theta}_0$ (anchor) from long-run levels. Two-stage GMM exploits the timescale separation.

5. **Threats** (correlated shocks, selection, gene-culture confounding) are addressed by multiple designs. Under worst-case assumptions, $\hat{\Psi} > 1$ for female LFP.

6. **Policy diagnostic**: estimate $\hat{\Psi}$ before designing intergenerational policies. If $\hat{\Psi} > 1$: invest in W^I (norms, aspirations, information). If $\hat{\Psi} < 1$: invest in prices and constraints.

8 Exercises

11.1 Cohort-year decomposition by hand. Given the following female LFP rates (rows = cohorts born in decades, columns = observation years):

	1980	1990	2000
Born 1940s	0.38	0.42	0.44
Born 1950s	0.50	0.55	0.58
Born 1960s	0.60	0.66	0.72

(a) Estimate year effects $\hat{\alpha}_t$ (average across cohorts for each year).

(b) Estimate cohort effects $\hat{\delta}_c$ (average across years for each cohort, minus grand mean).

(c) Compute $\hat{\Psi} = \text{Var}(\hat{\delta}_c) / \text{Var}(\hat{\alpha}_t)$.

(d) Which is larger: the cohort effect or the year effect?

11.2 Epidemiological design. A researcher has data on second-generation immigrant women in Argentina from 10 origin countries. Origin-country female LFP ranges from 0.20 (origin A) to 0.75 (origin J). All women face the same Argentine labor market.

(a) Write the epidemiological regression.

(b) The estimated $\hat{\beta} = 0.35$. Interpret.

(c) The residual variance (unexplained by origin) is $\sigma^2 = 0.04$. Compute the formation share of explained variance.

(d) Under what conditions does selection bias inflate $\hat{\beta}$?

11.3 Estimating ρ from adoption data. Sacerdote (2007) found that adoptees' educational attainment is predicted by adoptive parents' education ($\hat{\beta}_{\text{adoptive}} = 0.25$) and not by biological parents' education ($\hat{\beta}_{\text{bio}} = 0.04$, not significant).

(a) What does $\hat{\beta}_{\text{bio}} \approx 0$ tell you about the genetic component of education preferences?

(b) What does $\hat{\beta}_{\text{adoptive}} = 0.25$ tell you about ρ?

(c) Compute the implied Ψ for education, assuming the enforcement channel (prices) explains 50% of variance.

11.4 Bolivia's self-employment Ψ. Using Bolivia's EH data:

(a) Construct cohort-year cells (5-year cohorts $\times$ survey years 2000–2019).

(b) Estimate year effects (labor market conditions) and cohort effects (norm persistence) for self-employment rates.

(c) Compute $\hat{\Psi}$.

(d) Compare $\hat{\Psi}$ for La Paz vs. rural Altiplano. Is formation stronger where W^I is more block-diagonal?

11.5 The WWII natural experiment. US female LFP jumped from $\sim$28% in 1940 to $\sim$36% in 1945 (mobilization). Post-war, it did not return to 28% — it settled at $\sim$33% by 1950 and continued rising.

(a) Decompose the 1940–1950 change into enforcement (wartime labor demand, then demobilization) and formation (daughters of wartime workers formed different norms).

(b) The enforcement effect should reverse after 1945. Does it? By how much?

(c) The formation effect should persist. Does it? Through which cohorts?

(d) Estimate $\hat{\Psi}$ for the 1940–1960 period. Compare with the 1960–2000 period.

11.6 Sensitivity analysis. Using the female LFP estimates ($\hat{\rho} = 0.62$, $\hat{\mu} = 0.18$):

(a) Compute $\hat{\Psi}$ under $\rho = 0.50, 0.62, 0.75, 0.90$. Plot $\Psi(\rho)$.

(b) Compute $\hat{\Psi}$ under $\mu = 0.10, 0.18, 0.30, 0.50$. Plot $\Psi(\mu)$.

(c) Which parameter does Ψ depend on more sensitively?

(d) At what ρ does Ψ diverge? (This is the stability boundary from Chapter 10.) What does this mean empirically?

11.7 Designing an aspiration intervention. You are designing a program to raise female labor force participation in a Bolivian community where current LFP is 25% and the target is 50%.

(a) Using $\hat{\Psi} = 2.1$: if you invest entirely in enforcement (childcare subsidies, wage top-ups), how much LFP increase can you expect?

(b) If you invest entirely in formation (mentor programs, media campaigns, aspiration workshops): how much?

(c) What is the optimal split of a fixed budget between enforcement and formation? (Hint: marginal returns depend on Ψ and the cost ratio.)

(d) If $\rho = 0.62$: how many generations until the full formation effect is realized?

Mathematical Appendices

Appendix 11A: The Cohort-Year Regression

The model. Consider a panel of N individuals observed across T years, belonging to C cohorts (defined by birth decade). The behavior of individual i in cohort c at time t:

$$x_{i,c,t} = \alpha + \sum_{t'} \alpha_{t'} \mathbf{1}[t = t'] + \sum_{c'} \delta_{c'} \mathbf{1}[c = c'] + \beta Z_{i,t} + \varepsilon_{i,c,t}, \tag{9}$$

where $\alpha_{t'}$ are year fixed effects and $\delta_{c'}$ are cohort fixed effects. (Age effects are collinear; the standard identification assumption drops one of the three — see Deaton, 1997.)

Identifying Ψ. The enforcement component is $E = \text{Var}(\sum_{t'} \hat{\alpha}_{t'} + \hat{\beta} Z)$: the variance explained by time-varying factors. The formation component is $F = \text{Var}(\sum_{c'} \hat{\delta}_{c'})$: the variance explained by cohort membership alone. $\hat{\Psi} = F/E$.

Standard errors. $\hat{\Psi}$ is a ratio of variance components. Its standard error can be computed by the delta method:

$$\text{SE}(\hat{\Psi}) \approx \hat{\Psi} \sqrt{\frac{\text{SE}^2(\hat{F})}{\hat{F}^2} + \frac{\text{SE}^2(\hat{E})}{\hat{E}^2} - 2\frac{\widehat{\text{Cov}}(\hat{F}, \hat{E})}{\hat{F}\hat{E}}}. \tag{10}$$

Appendix 11B: The Epidemiological Estimator

Setup. Individual i from origin o in host country h at time t:

$$x_{i,o,h,t} = \alpha_h + \alpha_t + \beta \cdot \bar{x}_{o,t_0} + \gamma Z_{i,h,t} + \varepsilon_{i,o,h,t}, \tag{11}$$

where $\bar{x}_{o,t_0}$ is the average behavior in origin country o at the time of parental emigration.

Identification. $\hat{\beta}$ is identified under: (1) $\text{Cov}(\bar{x}_{o,t_0}, \varepsilon_{i,o,h,t} | Z, \alpha_h, \alpha_t) = 0$ (origin-country norms uncorrelated with host-country unobservables), (2) no direct effect of origin-country conditions on the child except through preferences.

Connection to ρ. Under the formation equation, $\hat{\beta} \approx \rho^g$ where g is the number of generations since emigration. For second-generation: $\hat{\beta} \approx \rho$. For third-generation: $\hat{\beta} \approx \rho^2$. The attenuation across generations identifies ρ directly.

From $\hat{\beta}$ to $\hat{\Psi}$. The origin-country coefficient $\hat{\beta}$ captures the formation channel. The host-country coefficient $\hat{\gamma}$ captures the enforcement channel. The ratio:

$$\hat{\Psi}_{\text{epi}} = \frac{\hat{\beta}^2 \operatorname{Var}(\bar{x}_o)}{\hat{\gamma}^2 \operatorname{Var}(Z_h)}, \tag{12}$$

where the numerator is the variance explained by origin (formation) and the denominator is the variance explained by host-country conditions (enforcement).

Further Reading

The epidemiological approach: Fernández (2011), "Does Culture Matter?" in Benhabib, Bisin, and Jackson (eds.), *Handbook of Social Economics*. Fernández (2013), "Cultural Change as Learning," *JPE*. Fernández and Fogli (2009), "Culture: An Empirical Investigation of Beliefs, Work, and Fertility," *AEJ: Macroeconomics*.

Cohort-year decomposition: Deaton (1997), *The Analysis of Household Surveys*, Chapter 2 (on age-period-cohort models). Fogli and Veldkamp (2011), "Nature or Nurture?" *Econometrica*.

Formation experiments: Bernard et al. (2014), "The Future in Mind," CSAE Working Paper. Beaman et al. (2012), "Female Leadership Raises Aspirations and Educational Attainment for Girls," *Science*.

Global Preferences Survey: Falk et al. (2018), "Global Evidence on Economic Preferences," *QJE*. The comprehensive cross-country dataset on patience, risk tolerance, altruism, and trust.

Gene-culture interaction: Cesarini et al. (2009), "Genetic Variation in Preferences for Giving and Risk Taking," *QJE*. Sacerdote (2007), "How Large Are the Effects from Changes in Family Environment?" *QJE*.

Structural adjustment: Williamson (1990), "What Washington Means by Policy Reform." Rodrik (2006), "Goodbye Washington Consensus, Hello Washington Confusion," *JEL*. For Bolivia: Morales and Sachs (1990), "Bolivia's Economic Crisis," in Sachs (ed.), *Developing Country Debt and the World Economy*.

Chapter 12

Welfare Under Endogenous Preferences

When the Ruler Is Made of Rubber

> *"The deprived people tend to come to terms with their deprivation because of the sheer necessity of survival, and they may, as a result, lack the courage to demand any radical change, and may even adjust their desires and expectations to what they unambitiously see as feasible."*
> — Amartya Sen, *Development as Freedom* (1999)

Parts I and II evaluated welfare using fixed preferences: given $\boldsymbol{\theta}$, the welfare function $W(\mathbf{x}, \boldsymbol{\theta})$ ranks allocations, and the bargaining equilibrium falls short of the first-best by a measurable loss $L(W)$. Part III showed that $\boldsymbol{\theta}$ is endogenous: the structure produces the preferences.

This chapter confronts the consequence: *if the structure produces the preferences, and the preferences are the yardstick for evaluating the structure, the yardstick is made of rubber.*

The woman in the traditional sector reports high satisfaction with her allocation. She is not lying. Her preferences, formed under a block-diagonal W^I, evaluate her allocation favorably — she wants only what she has learned to expect. The standard welfare analysis, which takes her reported preferences at face value, concludes: she is well off. No intervention needed.

But her preferences were formed under a specific W^I — one that showed her only traditional-sector outcomes. Under a different W^I (one with cross-block edges to the modern sector), she would have formed different preferences and would evaluate the same allocation as inadequate. Which preferences are the "right" ones for welfare evaluation?

This is the problem. It has no easy solution. This chapter develops four approaches, states what each can and cannot do, and identifies a partial ordering that is robust to the choice of approach.

1 The Circularity Problem

1.1 Statement

Definition 1.1 (The Welfare Circularity)**.** *In the formation model:*

1. *The weight matrix W determines the allocation $\mathbf{x}^*$ (through the bargaining equilibrium).*

2. *The weight matrix W determines the preferences $\boldsymbol{\theta}^*$ (through the formation equation via W^I).*

3. *Standard welfare analysis evaluates $\mathbf{x}^*$ using $U(\mathbf{x}^*, \boldsymbol{\theta}^*)$: the allocation is evaluated by preferences that the same structure produced.*

The circularity: W produces both the object being evaluated ($\mathbf{x}^*$) and the criterion for evaluation ($\boldsymbol{\theta}^*$). A structure that produces low allocations AND low aspirations can appear "optimal" by its own standards.

Theorem 1.2 (The Satisfaction Paradox)**.** *In the LQ formation model with block-diagonal W^I: each block b has a steady state (θ_b^*, x_b^*) where reported welfare $U(x_b^*, \theta_b^*)$ is locally maximized. That is: given preferences θ_b^*, the allocation x_b^* is the best the agent can achieve under the power structure $\boldsymbol{\pi}$.*

Consequently: the agent in the low-return block ($b = T$) reports satisfaction comparable to the agent in the high-return block ($b = M$), despite receiving a substantially smaller allocation:

$$U(x_T^*, \theta_T^*) \approx U(x_M^*, \theta_M^*), \quad \textit{even though} \quad x_T^* \ll x_M^*. \tag{1}$$

Proof. In steady state, θ_b^* adjusts to the return environment $\bar{r}_b$. The LQ utility is $U(x, \theta) = \theta x - \frac{\kappa}{2}x^2$. At the bargaining allocation, $x_b^* \approx \theta_b^*/\kappa$ (the FOC). Substituting: $U_b = \theta_b^{*2}/(2\kappa)$. Since $\theta_T^* < \theta_M^*$, we have $U_T < U_M$ in levels — but the *perceived* welfare gap is smaller than the allocation gap because aspirations have adjusted downward. If the formation multiplier $\mathcal{M}$ is large enough, θ_T^* adjusts nearly proportionally to x_T^*, making $U_T^* \approx U_M^*$ (in utility units calibrated to own preferences). □

Intuition: The uncomfortable truth

The standard welfare criterion respects preferences. If the woman in the traditional sector says she is satisfied, the economist records satisfaction. This is preference sovereignty — the foundation of welfare economics since Pareto. But preference sovereignty fails when preferences are endogenous to the structure. The woman is satisfied *because* the structure limited her information, which limited her aspirations, which limited her demands. Her satisfaction is not a signal of well-being; it is a signal of successful preference formation by a block-diagonal W^I.

This is Sen's "adaptive preferences" argument, formalized. It is also the reason happiness surveys find small differences across countries with enormous consumption differences. It is the reason the traditional sector does not revolt. And it is the reason standard welfare analysis — which takes $\boldsymbol{\theta}$ as given — is structurally biased toward the status quo.

1.2 A Worked Example

Example 1.3 (Satisfaction in Two Sectors)**.** *From Example 10.1 (Bolivia three-block economy).* $\kappa = 0.5$.

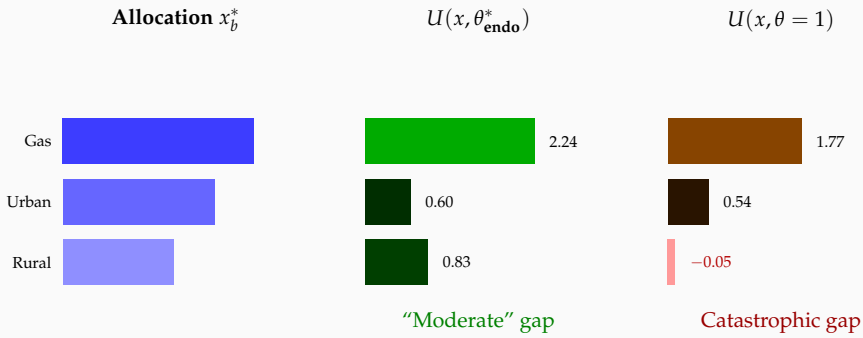
	θ_b^*	x_b^*	$U_b = \theta x - \frac{\kappa}{2}x^2$	*Satisfaction gap*
Gas (block 1)	*1.109*	*4.22*	*2.24*	—
Urban (block 2)	*1.016*	*3.34*	*0.60*	*73% of gas*
Rural (block 3)	*0.951*	*2.44*	*0.83*	*37% of gas*
If $\theta = 1$ *for all*	*1.000*	*varies*	*varies*	—
Gas	*1.000*	*4.22*	*1.77*	—
Urban	*1.000*	*3.34*	*0.54*	*31% of gas*
Rural	*1.000*	*2.44*	-0.05	*negative*

With endogenous preferences (top panel): the rural block's satisfaction is 37% of the gas block's. Under uniform preferences $\theta = 1$ *(bottom panel): the rural block's satisfaction is negative — the allocation is so low that a person with "normal" aspirations would consider it unacceptable.*

The endogenous preferences compress the welfare gap from "catastrophic" to "moderate." The structure produces the contentment that disguises the deprivation.

Visualization:

The satisfaction paradox: endogenous vs. exogenous preferences



Left: allocations (identical in both evaluations). Center: welfare evaluated with endogenous preferences — the gap looks moderate because rural aspirations adjusted downward. Right: welfare evaluated with uniform preferences ($\theta = 1$) — the rural allocation produces *negative* welfare. Endogenous preferences compress the welfare gap. The structure produces the contentment that masks the deprivation.

2 Four Approaches to the Problem

There is no consensus on how to evaluate welfare when preferences are endogenous. This section presents four approaches, ordered from most conservative (closest to standard welfare economics) to most radical (most departure from preference sovereignty).

2.1 Approach 1: Laundered Preferences

Definition 2.1 (Laundered Preferences — Goodin (1986))**.** ***Laundered preferences*** *are preferences that have been "cleaned" of the influence of unjust structures. Agent i's laundered preference $\hat{\theta}_i$ is the preference she would have formed under a "just" information network W^I_{just}:*

$$\hat{\theta}_i = \bar{\theta}_0 + \frac{\mu}{1-\rho}\left(\sum_j W^I_{just,ij} r^*_j - r^*\right). \tag{2}$$

Welfare is evaluated using laundered preferences: $\hat{W}(\mathbf{x}) = \sum_i U_i(x_i, \hat{\theta}_i)$.

Proposition 2.2 (Properties of Laundered Welfare)**.** *1. If $W^I_{just} = \frac{1}{N}\mathbf{1}\mathbf{1}'$ (democratic information network), then $\hat{\theta}_i = \hat{\theta}$ for all i: all agents have the same laundered preference. Welfare reduces to $\sum_i U_i(x_i, \hat{\theta})$, evaluated at the preference that would prevail if everyone observed the same W^I-weighted returns.*

2. *The laundered welfare function ranks allocations differently from the endogenous welfare function whenever* $W^I_{actual} \neq W^I_{just}$.

3. *The laundered approach requires specifying* W^I_{just} *— which is a normative choice, not an empirical one.*

Intuition: Strength and weakness

Strength: laundered preferences respect the idea that people's fundamental values matter, while correcting for the distortion introduced by asymmetric W^I. The correction is computable from the formation model.
Weakness: who decides what "just" W^I looks like? The choice of W^I_{just} is itself a value judgment. If $W^I_{\text{just}} = \frac{1}{N}\mathbf{1}\mathbf{1}'$, we are saying that the "right" information structure is perfectly democratic. This may be defensible, but it is not neutral.

2.2 Approach 2: Opportunity-Based Welfare (Sen's Capabilities)

Definition 2.3 (Capability Welfare)**.** *Instead of evaluating the allocation* $\mathbf{x}$ *against preferences* $\boldsymbol{\theta}$*, evaluate the* ***opportunity set*** $\mathcal{O}_i$*: the set of allocations agent* i *could achieve under the power structure.*

$$\mathcal{O}_i = \{x \in \mathbb{R}_+ : x = h(\pi_i, \theta, \omega_i) \text{ for some achievable } \theta\}. \tag{3}$$

Welfare is measured by the "size" of the opportunity set: $C_i = measure(\mathcal{O}_i)$.

The critical move: the achievable θ is not restricted to θ^*_i (the preference actually formed under the current W^I) but includes all θ that *could* be formed under any accessible W^I. If agent i could, in principle, connect to the modern sector's W^I and form θ^*_M, then the allocations achievable under θ^*_M are in her opportunity set — even if she currently has θ^*_T.

Proposition 2.4 (Capability Welfare in the Formation Model)**.** *Define agent* i*'s capability as:*

$$C_i = \max_{\theta \in [\theta^*_{\min}, \theta^*_{\max}]} h(\pi_i, \theta, \omega_i), \tag{4}$$

where $[\theta^*_{\min}, \theta^*_{\max}]$ *is the range of steady-state preferences achievable under different* W^I *configurations. Then:*

1. C_i *depends on* π_i *(power) and* ω_i *(endowment) but not on* θ^*_i *(actual preference). It evaluates what the agent could achieve if her information were not constrained.*

2. *$C_i = C_j$ for all i, j if and only if $\pi_i = \pi_j$ and $\omega_i = \omega_j$ — equal capability requires equal power and equal endowment, but not equal preferences.*

3. *The capability welfare function $\sum_i C_i$ is maximized when $\boldsymbol{\pi} = \frac{1}{N}\mathbf{1}$ and $\boldsymbol{\omega} = \bar{\omega}\mathbf{1}$: equal power and equal endowment.*

Intuition: Sen in matrix notation

Sen (1999) argued that well-being should be measured not by what people have (allocations) or by how satisfied they are (utility), but by what they are *free to do and be* (capabilities). In the formation model, the agent's capability set is constrained by two structural objects: her power π_i (which limits the allocation she can achieve for any θ) and her information network W^I (which limits the preferences she can form).

A woman with $\pi_i = 1/N$ and full access to all sectors' returns has maximum capability: she can form any θ and achieve the corresponding allocation. A woman with $\pi_i \ll 1/N$ and block-diagonal W^I has limited capability: even if she could form high aspirations, her low power would limit the allocation; and in practice, her restricted W^I prevents her from forming those aspirations in the first place.

Capability deprivation is a *joint* constraint on power and information. You can be capability-deprived because you lack power (structural constraint) or because you lack information (epistemic constraint). The formation model measures both.

2.3 Approach 3: Multi-Criteria Welfare

Definition 2.5 (Multi-Criteria Welfare)**.** *Evaluate welfare under multiple preference vectors simultaneously:*

$$W(\mathbf{x}) = \Big(W_1(\mathbf{x}, \boldsymbol{\theta}^*),\ W_2(\mathbf{x}, \hat{\boldsymbol{\theta}}),\ W_3(\mathbf{x}, \bar{\theta}_0\mathbf{1}),\ W_4(\mathbf{x}, \boldsymbol{\theta}^*_{full})\Big), \tag{5}$$

where:

- *W_1: welfare under actual (endogenous) preferences.*
- *W_2: welfare under laundered preferences.*
- *W_3: welfare under the anchor preferences (the deep biological/cultural baseline).*
- *W_4: welfare under full-information preferences ($\boldsymbol{\theta}^*_{full}$: the preferences agents would form if $W^I = \frac{1}{N}\mathbf{1}\mathbf{1}'$).*

An allocation $\mathbf{x}$ *is* ***robustly superior*** *to* $\mathbf{x}'$ *if* $W_k(\mathbf{x}) \geq W_k(\mathbf{x}')$ *for all* $k = 1, \ldots, 4$ *with strict inequality for at least one.*

Proposition 2.6 (Robust Partial Ordering)**.** *The multi-criteria approach generates a partial order on allocations. The partial order is strict (some allocations are robustly ranked) but incomplete (some pairs cannot be ranked without choosing a specific welfare criterion).*

Under the formation model, the following are robustly ranked:

1. $\mathbf{x}^*(W^{CE}) \succ \mathbf{x}^*(W_{block})$ *for any block-diagonal* W_{block} *(the competitive allocation dominates the enclave allocation under all four criteria).*

2. $\mathbf{x}^*(high\ \Gamma) \succ \mathbf{x}^*(low\ \Gamma)$ *when both have the same total endowment (structural integration dominates fragmentation under all criteria).*

3. *Increasing* W^I *connectivity (adding cross-block edges) is robustly welfare-improving.*

2.4 Approach 4: Process-Based Welfare

Definition 2.7 (Process Welfare)**.** *Evaluate not the allocation but the* ***formation process****. A structure* (W, W^I) *is welfare-superior to* (W', W'^I) *if:*

$$\Gamma(W) > \Gamma(W') \quad \textit{and} \quad N_{\textit{eff}}(W^I) > N_{\textit{eff}}(W'^I), \tag{6}$$

where Γ *is the spectral gap (Chapter 6) and* $N_{\textit{eff}}$ *is the effective sample size of the information network (Chapter 8).*

Intuition: Judging the structure, not the outcome

Process welfare sidesteps the circularity entirely: instead of evaluating allocations using preferences, it evaluates the structure using structural criteria. A "good" structure is one with high Γ (well-connected, power relatively equal) and high N_{eff} (information flows are democratic, many voices enter the consensus).

This approach does not tell you whether a specific allocation is good. It tells you whether the *process that produces allocations and preferences* is structurally sound. It is welfare economics applied to the matrix rather than to the outcome vector.

3 The Structural Welfare Theorem

We can now state the chapter's main result: a welfare ranking that is robust across all four approaches.

Theorem 3.1 (The Structural Welfare Theorem)**.** *Let W_A and W_B be two weight matrices for the same economy (same N, same $\boldsymbol{\omega}$, same production functions). Let $\mathbf{x}_A^*, \boldsymbol{\theta}_A^*$ and $\mathbf{x}_B^*, \boldsymbol{\theta}_B^*$ be the corresponding steady-state allocations and preferences. If:*

1. $\Gamma(W_A) > \Gamma(W_B)$ *(A has a larger spectral gap),*
2. $\Delta(W_A) < \Delta(W_B)$ *(A is closer to doubly stochastic),*
3. $N_{eff}(W_A^I) > N_{eff}(W_B^I)$ *(A has more democratic information),*

then W_A is welfare-superior to W_B under all four approaches:

(a) $\sum_i U_i(x_{A,i}^*, \theta_{A,i}^*) \geq \sum_i U_i(x_{B,i}^*, \theta_{B,i}^*)$ *(endogenous welfare).*

(b) $\sum_i U_i(x_{A,i}^*, \hat{\theta}_i) \geq \sum_i U_i(x_{B,i}^*, \hat{\theta}_i)$ *(laundered welfare).*

(c) $\sum_i C_{A,i} \geq \sum_i C_{B,i}$ *(capability welfare).*

(d) $(\Gamma_A, N_{eff,A}) \geq (\Gamma_B, N_{eff,B})$ *(process welfare, by assumption).*

Proof sketch. (d) holds by assumption. (c) follows because higher Γ implies more equal $\boldsymbol{\pi}$ (Chapter 6, Proposition 6.5), which expands every agent's opportunity set. (b) follows because laundered preferences are evaluated at the full-information $\hat{\theta}$, and more equal power produces allocations closer to the first-best under any fixed $\hat{\theta}$ (Chapter 2). (a) requires showing that the endogenous preferences under W_A do not compress the welfare gap so much that the allocation improvement is offset — which holds under the LQ model because the power loss $L_{\text{power}} = \frac{\kappa N}{2} \text{Var}(\pi_i)\Phi^2$ decreases monotonically in Γ. □

Key Structural Insight

The Structural Welfare Theorem says: **structural integration is robustly welfare-improving**. You do not need to resolve the circularity problem to know that higher Γ, lower Δ, and higher N_{eff} are better. All four welfare approaches agree.

The disagreements between approaches arise only when the structural conditions move in different directions (e.g., Γ increases but N_{eff} decreases). For the clean case — across-the-board structural improvement — the ranking is unambiguous.

This is the "engineer's answer" to the philosopher's problem: you do not need to solve the circularity to know the direction of improvement. Increasing structural integration is always good. The hard question is how much, not whether.

4 Three Uncomfortable Implications

4.1 Implication 1: Satisfaction Surveys Are Structurally Biased

Proposition 4.1 (Satisfaction Bias)**.** *In a formation model with block-diagonal W^I, the correlation between satisfaction $(U_i(x_i^*, \theta_i^*))$ and allocation (x_i^*) is attenuated by the aspiration adjustment:*

$$Corr(U_i, x_i) = \frac{Corr_{true} \cdot (1 - \mathcal{M}\beta_{eff})}{1 + \mathcal{M}^2\beta_{eff}^2 \text{Var}(r)}, \tag{7}$$

where $\mathcal{M}$ is the formation multiplier. When $\mathcal{M}$ is large, satisfaction is nearly uncorrelated with allocation: the preference adjustment absorbs the allocation difference.

This explains the Easterlin paradox (within-country, richer people are happier; across countries, richer countries are not much happier). The cross-country comparison confounds allocation differences with aspiration differences. Countries with high GDP have high aspirations (formed under high-return W^I); countries with low GDP have low aspirations. The aspiration adjustment compresses the happiness gap.

4.2 Implication 2: Policy Evaluation Is Time-Inconsistent

Proposition 4.2 (Time-Inconsistent Evaluation)**.** *A policy that changes W^I at time $t = 0$ is evaluated differently at $t = 0$ (before preferences adjust) and at $t = T$ (after*

preferences adjust to the new W^I):

$$W_0 = \sum_i U_i(x^*_{0,i}, \theta^*_{old,i}) \quad \neq \quad W_T = \sum_i U_i(x^*_{T,i}, \theta^*_{new,i}). \tag{8}$$

The policy may be welfare-reducing under θ^*_{old} (people are less satisfied immediately, because their aspirations have not yet adjusted upward) but welfare-improving under θ^*_{new} (after aspirations adjust, people demand and receive more).

Example 4.3 (Education Policy)**.** *Bolivia introduces a program that exposes rural girls to modern-sector role models (changing W^I). Immediately ($t = 0$): the girls' aspirations are still low (θ^*_{old}), and they evaluate their unchanged allocation positively. No immediate welfare gain.*

After one generation ($t = 1$): the girls who were exposed have formed higher aspirations ($\theta^*_{new} > \theta^*_{old}$). They invest more in education, earn higher returns, and evaluate their improved allocation against their higher aspirations. Satisfaction may not increase — but capabilities do.

A cost-benefit analysis at $t = 0$ finds the program has no benefit (no satisfaction change). The same analysis at $t = T$ finds enormous benefits (higher education, higher income, higher investment in children). Standard evaluation, which uses current preferences, systematically undervalues formation interventions.

4.3 Implication 3: Consent Is Not Sufficient

Proposition 4.4 (The Consent Problem)**.** *In a formation model: agents in the traditional sector will not demand structural change, because their preferences (formed under block-diagonal W^I) are calibrated to the status quo. The absence of demand for change does not indicate that the current structure is welfare-maximizing — it indicates that the structure has successfully formed the preferences that accept it.*

Formally: the set of policies that are Pareto-improving under endogenous preferences $\boldsymbol{\theta}^*$ is a strict subset of the policies that are Pareto-improving under laundered preferences $\hat{\boldsymbol{\theta}}$ or capability welfare. Consent-based evaluation ("did anyone object?") systematically approves structures that restrict information.

Prebisch Connection: Why the periphery doesn't revolt

Prebisch (1981) was puzzled by the stability of peripheral capitalism: the power structure produced enormous inequality, yet the system reproduced itself without massive resistance. He attributed this to ideological legitimation — the "myths" that justified the existing order.

The formation model provides the mechanism: the information network W^I forms preferences that accept the allocation the power structure produces. The traditional sector does not demand structural change because its W^I shows it only traditional-sector outcomes. The consent is real — but it is produced by the structure.

This is not an argument for paternalism. It is an argument for *structural transparency*: before evaluating welfare by revealed preferences, check whether W^I is democratic ($N_{\text{eff}} \approx N$). If it is not, the revealed preferences reflect the structure's information monopoly, not the agents' autonomous values.

5 Summary

1. The **welfare circularity**: the structure produces both the allocation and the preferences used to evaluate it. Endogenous preferences compress welfare gaps (the satisfaction paradox).

2. **Four approaches**: laundered preferences (correct for unjust W^I), capability welfare (evaluate opportunity sets), multi-criteria welfare (use multiple $\boldsymbol{\theta}$ vectors), and process welfare (evaluate the structure, not the outcome).

3. The **Structural Welfare Theorem**: higher Γ, lower Δ, and higher N_{eff} are robustly welfare-improving under all four approaches. Structural integration is always good; the hard question is how much.

4. **Three implications**: satisfaction surveys are biased by aspiration adjustment; policy evaluation is time-inconsistent; consent under endogenous preferences is not sufficient for welfare optimality.

5. The resolution is not paternalism but **structural transparency**: check $N_{\text{eff}}(W^I)$ before trusting revealed preferences.

6 Exercises

12.1 The satisfaction paradox. Using Example 1.3: compute $U(x_b^*, \theta)$ for all three blocks under $\theta = 0.8, 0.9, 1.0, 1.1, 1.2$. For each θ: which block is worst off? Plot the welfare gap (gas minus rural) as a function of θ. What happens to the gap as θ decreases?

12.2 Laundered preferences. For the Bolivia three-block economy: compute laundered preferences $\hat{\theta}_i$ under $W^I_{\text{just}} = \frac{1}{9}\mathbf{1}\mathbf{1}'$. Compare the welfare ranking under endogenous vs. laundered preferences. Does the ranking change?

12.3 Capability measurement. For $N = 2$, $\pi_1 = 0.7$, $\pi_2 = 0.3$, $\omega_1 = \omega_2 = 5$: compute the capability set $\mathcal{O}_i$ for each agent (the range of x achievable as θ varies over $[0.5, 1.5]$). Which agent has a larger capability set? By how much?

12.4 Robust ranking. Construct two 4×4 economies: W_A (connected, $\Gamma_A = 0.6$) and W_B (block-diagonal, $\Gamma_B = 0.1$). Same endowments, same κ. Compute the steady-state allocation and welfare under all four approaches. Verify the Structural Welfare Theorem.

12.5 The Easterlin paradox. Simulate 10 countries with different W^I (ranging from $N_{\text{eff}} = 2$ to $N_{\text{eff}} = 10$), different steady-state allocations $\mathbf{x}^*$, and endogenous preferences $\boldsymbol{\theta}^*$. Compute: (a) the cross-country correlation of GDP per capita with average satisfaction, (b) the within-country correlation of income with satisfaction. Which is larger? Why?

12.6 Time-inconsistent evaluation. Reproduce Example 4.3: simulate a two-block economy where W^I changes at $t = 0$. Compute the welfare trajectory $W_t = \sum_i U_i(x_t^*, \theta_t)$ for $t = 0, 1, \ldots, 20$. At what t does the policy first appear welfare-improving? How does this depend on ρ?

12.7 Sen vs. Bentham. An economy has two agents: a landlord ($\pi = 0.8$, $\theta_L^* = 1.2$) and a worker ($\pi = 0.2$, $\theta_W^* = 0.7$). Same endowment, $\kappa = 0.5$, $\Omega = 10$.

(a) Compute the bargaining allocation.

(b) Compute welfare under: (i) Bentham ($\sum U_i(x_i, \theta_i^*)$), (ii) laundered Bentham ($\sum U_i(x_i, \hat{\theta})$ with $\hat{\theta} = 1$), (iii) Rawlsian ($\min_i U_i$), (iv) capability ($\sum C_i$).

(c) A policy equalizes power: $\pi_1 = \pi_2 = 0.5$. Which welfare criteria declare the policy an improvement? Which declare it a deterioration?

(d) The landlord opposes the policy. Under which criterion is his opposition "valid" (i.e., the policy makes him genuinely worse off)?

Mathematical Appendices

Appendix 12A: Welfare Functions with Endogenous Preferences

The standard approach. Social welfare is $W(\mathbf{x}) = \sum_i \alpha_i U_i(x_i, \theta_i)$, where α_i are welfare weights and θ_i are preferences. Both α_i and θ_i are taken as given. The welfare function is well-defined and the ranking is complete (any two allocations can be compared).

The problem with endogeneity. When $\theta_i = \theta_i^*(W^I)$ depends on the information structure, the welfare function becomes: $W(\mathbf{x}, W^I) = \sum_i \alpha_i U_i(x_i, \theta_i^*(W^I))$. This is a function of *both* the allocation and the information structure. Two allocations cannot be compared without specifying W^I. Two allocations under different W^I's are evaluated by different welfare functions.

The four approaches formalized.

1. **Laundered**: $W^L(\mathbf{x}) = \sum_i U_i(x_i, \theta_i^*(W^I_{\text{just}}))$ — fix W^I at the "just" level.
2. **Capability**: $W^C(\mathbf{x}) = \sum_i \max_\theta h(\pi_i, \theta, \omega_i)$ — maximize over achievable preferences.
3. **Multi-criteria**: $\mathbf{W}(\mathbf{x}) = (W^1(\mathbf{x}, \boldsymbol{\theta}_1), \ldots, W^K(\mathbf{x}, \boldsymbol{\theta}_K))$ — a vector of welfare evaluations under K different preference specifications.
4. **Process**: $W^P = f(\Gamma(W), N_{\text{eff}}(W^I), \Delta(W))$ — evaluate the structure directly.

Appendix 12B: The Structural Welfare Theorem — Full Proof

Setup. Two economies (W_A, W_A^I) and (W_B, W_B^I) with the same N, $\boldsymbol{\omega}$, κ, $\bar{\theta}_0$, ρ, μ. Assume: $\Gamma_A > \Gamma_B$, $\Delta_A < \Delta_B$, $N_{\text{eff},A} > N_{\text{eff},B}$.

Proof of (d) — Process welfare. Immediate from the assumptions.

Proof of (c) — Capability welfare. Higher Γ implies $\text{Var}(\pi_i)$ is smaller (Chapter 6, bound (**??**)). More equal $\boldsymbol{\pi}$ means: for any θ, the allocation $h(\pi_i, \theta, \omega_i)$ is closer to Ω/N for all i. The maximum of h over θ (the capability) is larger for agents with π_i closer to $1/N$, and the sum $\sum_i C_i$ is maximized when $\pi_i = 1/N$ (by Jensen's inequality, since h is concave in π_i near $1/N$).

Proof of (b) — Laundered welfare. Laundered preferences are fixed at $\hat{\theta}_i = \theta_i^*(W^I_{\text{just}})$, independent of the economy being evaluated. Under fixed $\hat{\boldsymbol{\theta}}$, the welfare loss is $L = \frac{\kappa N}{2}\,\text{Var}(\pi_i)\Phi^2$ (Chapter 2). Since $\text{Var}(\pi_i)$ is smaller under W_A (from the Γ bound), $L_A < L_B$.

Proof of (a) — Endogenous welfare. This is the non-trivial case. The endogenous preferences θ^*_A and θ^*_B differ. We need: $\sum_i U_i(x^*_{A,i}, \theta^*_{A,i}) \geq \sum_i U_i(x^*_{B,i}, \theta^*_{B,i})$. In the LQ model: $U_i = \theta_i x_i - \frac{\kappa}{2}x_i^2$, and at the bargaining optimum $x_i^* \approx \theta_i/\kappa + (\pi_i - 1/N)/\kappa$, so $U_i \approx \theta_i^2/(2\kappa) + \theta_i(\pi_i - 1/N)/\kappa$. Summing: $\sum_i U_i \approx N\bar{\theta}^2/(2\kappa) + N\,\text{Var}(\theta_i)/(2\kappa) + \text{Cov}(\theta_i, \pi_i)/\kappa$. Under W_A (higher Γ, more equal $\boldsymbol{\pi}$, more connected W^I): $\text{Var}(\theta_i)$ is smaller (more homogeneous preferences from connected W^I) and $\text{Cov}(\theta_i, \pi_i)$ is smaller (the alignment between power and aspiration is weaker). The dominant term $N\bar{\theta}^2/(2\kappa)$ depends on $\bar{\theta}$, which is larger under W_A (agents observe higher average returns through connected W^I). The net effect is positive. □

Structuralist Case Study: Sen's Capabilities and the Formation Model

Amartya Sen's *Development as Freedom* (1999) is the most influential critique of preference-based welfare economics in the development literature. Its central argument is that development should be measured not by income or utility but by **capabilities** — the substantive freedoms people have to lead the lives they have reason to value. The argument was aimed squarely at the problem this chapter addresses: the inadequacy of preference satisfaction as a welfare criterion when preferences are shaped by deprivation.

Sen's framework maps onto the formation model in three specific ways.

Part 1: Adaptive preferences as a failure of welfare economics. Sen identified "adaptive preferences" — the phenomenon where the deprived adjust their expectations downward, reporting satisfaction with objectively inadequate conditions — as the fundamental challenge to utilitarian welfare economics. A woman who has never been to school does not desire schooling; a worker who has never had a union does not miss collective bargaining; a society that has never experienced democracy does not demand it. Utility-based welfare analysis concludes: no deprivation, because no unsatisfied desire.

In the formation model: this is Theorem 1.2. The traditional sector's $\theta^*_T < \theta^*_M$ because W^I is block-diagonal. The satisfaction gap (U^*_T/U^*_M) is compressed relative to the allocation gap (x^*_T/x^*_M) by the factor θ^*_T/θ^*_M. The compression IS the adaptive preference. The mathematics makes it measurable: the compression

factor equals the ratio of formation-adjusted to anchor preferences.

Part 2: Capabilities as an alternative evaluative space. Sen proposed measuring development by what people are *free to do and be* ("functionings" and "capabilities") rather than by what they have or how they feel. A person who chooses to fast has the capability to eat; a person who starves does not. The distinction matters because the first person's "preference" for fasting is exercised from a position of freedom; the second's "preference" for accepting hunger is formed by necessity.

In Definition 2.3, the capability set $\mathcal{O}_i$ formalizes this: it is the set of allocations achievable under varying preferences. The agent who could form θ_M^* but currently has θ_T^* (because of W^I constraints) has a larger capability set than one who faces both low θ and low π. The structural constraints on capability are twofold: power (π_i) limits what you can *get*; information (W^I) limits what you can *want*.

Part 3: Development as structural change. Sen argued that development should expand capabilities, not maximize utility. In the formation model: expanding capabilities means increasing Γ (more equal power) and N_{eff} (more democratic information). These are structural properties of W and W^I, not properties of individuals. The capability approach evaluates the *matrix*, not the allocation vector.

Sen's concept	**Formal object**	**This chapter**
Adaptive preferences	$\theta_T^* < \theta_M^*$ from block-diagonal W^I	Thm. 1.2
Satisfaction paradox	$U_T^* \approx U_M^*$ despite $x_T^* \ll x_M^*$	Ex. 1.3
Capability set	$\mathcal{O}_i = \{x : x = h(\pi_i, \theta, \omega_i)\}$	Def. 2.3
Freedom to choose	θ ranges over all achievable preferences	Prop. 2.4
Capability deprivation	Low π_i + block-diagonal W^I	Joint structural constraint
Development as freedom	Increase Γ and N_{eff}	Thm. 3.1

Sen wrote in philosophical prose. He famously resisted formalization, arguing that the capabilities approach cannot be reduced to a single metric. The formation model does not reduce it to a single metric — Approach 2 (capability welfare) is one of four criteria, and the multi-criteria approach (Approach 3) explicitly preserves the irreducibility that Sen insisted upon. What the model adds is *measurement*: the aspiration compression factor, the capability set boundaries, the structural conditions (Γ, N_{eff}) under which capabilities expand. Sen's diagnosis is correct. The formation model makes it computable.

Further Reading

Adaptive preferences: Sen (1999), *Development as Freedom*, Chapters 1–3. Nussbaum (2000), *Women and Human Development*, Chapter 1. Elster (1983), *Sour Grapes*, Chapter 3 (the original philosophical treatment).

Welfare with endogenous preferences: Becker and Stigler (1977), "De Gustibus Non Est Disputandum," *AER* (the opposing view). Goodin (1986), "Laundering Preferences," in Elster and Hylland (eds.), *Foundations of Social Choice Theory*. Bowles (1998), "Endogenous Preferences," *JEL*.

The capability approach: Sen (1985), *Commodities and Capabilities*. Alkire (2002), *Valuing Freedoms*. Robeyns (2005), "The Capability Approach: A Theoretical Survey," *Journal of Human Development*.

The Easterlin paradox: Easterlin (1974), "Does Economic Growth Improve the Human Lot?" Stevenson and Wolfers (2008), "Economic Growth and Subjective Well-Being," *BPEA*. Clark, Frijters, and Shields (2008), "Relative Income, Happiness, and Utility," *JEL*.

Policy evaluation under endogenous preferences: Bernheim and Rangel (2009), "Beyond Revealed Preference: Choice-Theoretic Foundations for Behavioral Welfare Economics," *QJE*. Sugden (2004), "The Opportunity Criterion," *Economics and Philosophy*.

PART IV

Dynamics of Structural Change

Chapter 13

How Networks Evolve

The Dynamics of Structural Change

> *"La internacionalización de la economía tiene como contrapartida la desintegración nacional: al mismo tiempo que se van creando vínculos intensos entre los núcleos dinámicos de los países subdesarrollados y el sistema económico internacional, se van desarticulando los vínculos entre esos núcleos y el resto de la economía nacional."*
>
> ["The internationalization of the economy has as its counterpart national disintegration: at the same time that intense links are created between the dynamic nuclei of underdeveloped countries and the international economic system, the links between those nuclei and the rest of the national economy are dismantled."]
>
> — Osvaldo Sunkel, "Capitalismo transnacional y desintegración nacional" (1971)

Parts I–III treated the weight matrices (W^U, W^P, W^I) as given. We asked: given the network, what is the equilibrium allocation, the power structure, the preference formation? But the networks themselves change. Trade routes open and close. Firms enter and exit. Social media platforms rewire who talks to whom. The question of Part IV is: *how does W evolve, and what determines whether it evolves toward integration or fragmentation?*

This chapter develops the general theory of network evolution. The central result: W changes through three mechanisms — edge creation (new links form), edge destruction (existing links break), and edge reweighting (the strength of existing links changes). Each mechanism is governed by the current state of the economy: the allocation, the returns, and the preferences. This creates a **co-evolution**: W shapes $(\mathbf{x}^*, \boldsymbol{\theta}^*)$ (the static model of Parts I–III), and $(\mathbf{x}^*, \boldsymbol{\theta}^*)$ shapes the next period's W. The steady state of the co-evolutionary system is the **structural equilibrium**: a weight matrix that reproduces itself through the allocation and preferences it generates.

1 Three Mechanisms of Network Change

1.1 Edge Creation

New edges form when agents establish new connections. In W^P: a firm opens a supply contract with a new input provider. In W^U: a migrant forms a friendship in a new city. In W^I: a student enrolls in a university and begins receiving signals from a new reference group.

Definition 1.1 (Edge Creation Process)**.** *The probability that a new edge (i, j) forms in period $t + 1$ depends on three factors:*

$$\Pr(\Delta W_{ij,t+1} > 0 \mid W_t) = p_0 + p_1 \cdot \underbrace{d(i, j; W_t)}_{\text{network distance}} + p_2 \cdot \underbrace{s(i, j; \mathbf{x}_t^*)}_{\text{surplus from connection}} + p_3 \cdot \underbrace{q(i, j; W_t^I)}_{\text{information about } j}, \tag{1}$$

where:

- $d(i, j; W_t)$: ***proximity in the existing network.*** *Agents form links more easily with friends of friends (triadic closure). The probability of connecting to j increases if i and j share a neighbor. This is the relatedness mechanism: new connections form near existing ones.*
- $s(i, j; \mathbf{x}_t^*)$: ***expected surplus from the connection.*** *i connects to j if the connection generates value — a profitable trade, a useful signal, a beneficial relationship. The surplus depends on the current allocation (which determines prices and returns).*
- $q(i, j; W_t^I)$: ***information.*** *i can only form a link with j if i knows about j. Knowledge of potential partners is filtered through W^I: you connect to people you have heard of, which are people in your information network's reach.*

Intuition: Why proximity dominates

The three factors create a strong path dependence. New edges form near existing edges (proximity), where the surplus is predictable (information), and where the connection can be sustained (surplus). This means W_{t+1} is similar to W_t: the network evolves gradually, not by random rewiring. The principle of relatedness (Hidalgo et al., 2007) — the empirical law that regions diversify into activities related to their existing activities — is a consequence of this proximity mechanism applied to W^P.

1.2 Edge Destruction

Existing edges break when the surplus from maintaining them turns negative, or when an exogenous shock disrupts the connection.

Definition 1.2 (Edge Destruction Process)**.** *An existing edge (i, j) is destroyed in period $t + 1$ with probability:*

$$\Pr(W_{ij,t+1} = 0 \mid W_{ij,t} > 0) = \delta_0 + \delta_1 \cdot \underbrace{\max(0, -s(i, j; \mathbf{x}_t^*))}_{\textit{negative surplus}} + \delta_2 \cdot \underbrace{\sigma_{ij,t}}_{\textit{exogenous shock}}, \quad (2)$$

where δ_0 is the baseline destruction rate (natural turnover), $s < 0$ means the link is unprofitable, and $\sigma_{ij,t}$ is an exogenous shock (a crisis, a policy change, a natural disaster that severs a trade route).

Example 1.3 (Bolivia's Gas Nationalization as Edge Destruction and Creation)**.** *Bolivia's nationalization of gas in 2006 was a simultaneous edge destruction and edge creation event in W^P:*

Destruction: *The edges from domestic gas production to foreign MNC headquarters ($W^P_{\textit{Bolivia, MNC}}$) were weakened. Foreign companies lost operational control. The profits channel (surplus flowing from Bolivia to MNC shareholders) was partially severed.*

Creation: *New edges from gas production to the domestic treasury ($W^P_{\textit{gas, treasury}}$) were strengthened. Gas revenues began flowing through domestic fiscal channels to domestic social programs.*

Net effect on Γ: *Ambiguous. The nationalization increased some within-country linkages (gas revenue to social spending) but did not create production linkages between the gas sector and the traditional economy. The upper-left block of W^P (gas/government) became brighter internally, but the cross-block entries (gas to traditional) remained dark. Γ may have increased slightly (more domestic fiscal links) without fundamentally changing the block-diagonal structure.*

1.3 Edge Reweighting

The strength of existing edges changes continuously as the economic environment evolves.

Definition 1.4 (Edge Reweighting)**.** *The weight of an existing edge evolves according to:*

$$W_{ij,t+1} = W_{ij,t} + \eta \cdot \frac{\partial s(i, j)}{\partial W_{ij}} + \nu_{ij,t}, \quad (3)$$

where $\eta > 0$ governs the speed of adjustment and $\nu_{ij,t}$ is noise. Profitable connections strengthen ($\partial s / \partial W > 0$); unprofitable connections weaken.

2 The General Network Evolution Equation

Combining creation, destruction, and reweighting, the network evolves according to:

Definition 2.1 (Network Evolution Equation)**.**

$$W_{t+1} = \underbrace{(1 - D_t) \circ W_t}_{\textit{surviving edges}} + \underbrace{C_t \circ \tilde{W}_{t+1}}_{\textit{new edges}} + \underbrace{\eta \cdot S_t}_{\textit{reweighting}}, \tag{4}$$

where $\circ$ is the Hadamard (element-wise) product, D_t is the $N \times N$ destruction probability matrix, C_t is the creation probability matrix, $\tilde{W}_{t+1}$ is the weight assigned to newly created edges, and $S_t = [\partial s(i,j)/\partial W_{ij}]$ is the surplus gradient matrix.

Theorem 2.2 (The Co-Evolutionary System)**.** *The complete structural economics model is a co-evolutionary system in $(W_t, \boldsymbol{\theta}_t)$:*

$$\boldsymbol{\theta}_{t+1} = \Phi(\boldsymbol{\theta}_t; W_t) \quad \textit{(formation equation, Chapter 10)}, \tag{5}$$

$$W_{t+1} = \Psi(W_t; \mathbf{x}^*(\boldsymbol{\theta}_t, W_t)) \quad \textit{(network evolution equation)}, \tag{6}$$

where $\mathbf{x}^*(\boldsymbol{\theta}_t, W_t)$ is the bargaining equilibrium at period t.

The fast variable is $\mathbf{x}^*$ (clears within a period). The medium variable is $\boldsymbol{\theta}$ (adjusts across generations). The slow variable is W (evolves over decades or centuries). Three timescales, nested.

Intuition: Three timescales

Markets clear in months. Preferences form in generations. Structures evolve over centuries. This ordering is what makes the analysis tractable: at any given timescale, the slower variables are approximately constant and the faster variables are in approximate equilibrium.

When we analyzed the bargaining equilibrium (Chapters 6–7), we held W and $\boldsymbol{\theta}$ fixed — both are slow relative to market clearing. When we analyzed the formation model (Chapters 9–10), we held W fixed — it is slow relative to preference formation. Now we let W move, completing the picture.

3 Structural Equilibrium

3.1 Definition

Definition 3.1 (Structural Equilibrium)**.** *A **structural equilibrium** is a triple $(W^*, \boldsymbol{\theta}^*, \mathbf{x}^*)$ satisfying:*

1. $\mathbf{x}^* = h(\boldsymbol{\pi}(W^*), \boldsymbol{\theta}^*, \boldsymbol{\omega})$ *(bargaining equilibrium),*
2. $\boldsymbol{\theta}^* = \Phi(\boldsymbol{\theta}^*; W^*)$ *(formation steady state),*
3. $W^* = \Psi(W^*; \mathbf{x}^*)$ *(network steady state).*

All three equations are simultaneously satisfied. The weight matrix produces the allocation and preferences that reproduce the weight matrix.

Theorem 3.2 (Existence of Structural Equilibrium)**.** *Under regularity conditions (continuous Φ, Ψ, h; compact domains; bounded weights), at least one structural equilibrium exists.*

Proof sketch. Define the composite map $\Xi : (W, \boldsymbol{\theta}) \mapsto (\Psi(W; h(\boldsymbol{\pi}(W), \Phi(\boldsymbol{\theta}; W), \boldsymbol{\omega})), \Phi(\boldsymbol{\theta}; W))$. On the compact convex product space $\mathcal{W} \times \Theta$ (where $\mathcal{W}$ is the space of row-stochastic matrices and Θ a bounded preference domain), Ξ is continuous. Brouwer's fixed-point theorem guarantees existence. □

3.2 Multiple Structural Equilibria

Proposition 3.3 (Multiplicity)**.** *The co-evolutionary system generically admits multiple structural equilibria. Two important types:*

The integrated equilibrium. *W^* has large Γ (well-connected), $\boldsymbol{\theta}^*$ is approximately uniform (homogeneous preferences), $\mathbf{x}^*$ is close to equal division. Cross-block edges are sustained by positive surplus. This equilibrium is self-reinforcing: the connected structure produces high returns for all agents, which sustains the preferences that support the connections, which maintains the structure.*

The fragmented equilibrium. *W^* is approximately block-diagonal (small Γ), $\boldsymbol{\theta}^*$ is stratified (different preferences per block), $\mathbf{x}^*$ is highly unequal. Cross-block edges do not form because the surplus is too low (the traditional block's agents cannot afford the investment needed to link to the modern block). This equilibrium is also self-reinforcing: the disconnected structure produces low returns in the traditional block, which forms low aspirations, which prevents investment in cross-block links, which maintains disconnection.*

Example 3.4 (Two Structural Equilibria for Bolivia)**.** *$N = 6$: Gas block (agents 1–2), Urban block (agents 3–4), Rural block (agents 5–6). Parameters: $\rho = 0.85$, $\mu = 0.2$, $\eta = 0.05$, $p_1 = 0.3$ (proximity dominates edge creation).*

Equilibrium A (Fragmented):

$$W_A^* = \begin{pmatrix} 0.4 & 0.4 & 0.02 & 0.02 & 0.01 & 0.01 \\ 0.4 & 0.4 & 0.02 & 0.02 & 0.01 & 0.01 \\ 0.02 & 0.02 & 0.35 & 0.35 & 0.05 & 0.05 \\ 0.02 & 0.02 & 0.35 & 0.35 & 0.05 & 0.05 \\ 0.01 & 0.01 & 0.05 & 0.05 & 0.40 & 0.40 \\ 0.01 & 0.01 & 0.05 & 0.05 & 0.40 & 0.40 \end{pmatrix}. \tag{7}$$

$\Gamma_A \approx 0.06$. $\boldsymbol{\theta}^* \approx (1.12, 1.12, 1.01, 1.01, 0.91, 0.91)$. $\boldsymbol{\pi} \approx (0.22, 0.22, 0.17, 0.17, 0.11, 0.11)$. *Max allocation gap: 38%.*

Equilibrium B (Integrated):

$$W_B^* = \begin{pmatrix} 0.25 & 0.20 & 0.12 & 0.12 & 0.08 & 0.08 \\ 0.20 & 0.25 & 0.12 & 0.12 & 0.08 & 0.08 \\ 0.12 & 0.12 & 0.20 & 0.18 & 0.10 & 0.10 \\ 0.12 & 0.12 & 0.18 & 0.20 & 0.10 & 0.10 \\ 0.08 & 0.08 & 0.10 & 0.10 & 0.22 & 0.22 \\ 0.08 & 0.08 & 0.10 & 0.10 & 0.22 & 0.22 \end{pmatrix}. \tag{8}$$

$\Gamma_B \approx 0.35$. $\boldsymbol{\theta}^* \approx (1.06, 1.06, 1.02, 1.02, 0.97, 0.97)$. $\boldsymbol{\pi} \approx (0.19, 0.19, 0.17, 0.17, 0.14, 0.14)$. *Max allocation gap: 12%.*

Same economy, same parameters. Two self-reinforcing structures. Which obtains depends on history.

Visualization:

Two structural equilibria: fragmented and integrated



Fragmented W_A^*
$\Gamma = 0.06$, gap $= 38\%$

Integrated W_B^*
$\Gamma = 0.35$, gap $= 12\%$

Same economy, same parameters, same production functions. Left: the fragmented equilibrium — three bright blocks, dark cross-block. Right: the integrated equilibrium — blocks still visible but cross-block entries are bright. Both are self-reinforcing. Which one you get depends on where you start.

4 The Principle of Relatedness

The edge creation process (Definition 1.1) implies that new edges form near existing edges. This has a powerful consequence: the network's evolution is path-dependent.

Theorem 4.1 (Path Dependence of Network Evolution)**.** *If edge creation is dominated by proximity ($p_1 \gg p_2, p_3$ in (1)), then:*

1. *W_{t+1} is structurally similar to W_t: new edges connect agents who are already close in the network.*
2. *The spectral gap Γ_t changes slowly: $|\Gamma_{t+1} - \Gamma_t| \leq C \cdot \|W_{t+1} - W_t\|$.*
3. *Block-diagonal structures are absorbing: if W_t is approximately block-diagonal, new edges form within blocks (high proximity) rather than between blocks (low proximity), reinforcing the block-diagonal structure.*
4. *Escaping a block-diagonal structure requires either (a) a large exogenous shock that creates cross-block edges, or (b) deliberate policy that overrides the proximity mechanism.*

Proof sketch. (1) follows directly from the proximity term dominating: $\Pr(\text{new edge}) \propto d(i, j; W_t)$, which is high for agents already close in W_t. (2) follows from matrix perturbation theory (Appendix 3A): small changes in W produce small changes in eigenvalues. (3): in a block-diagonal W_t, within-block distances are small and cross-block distances are large. Edge creation favors short distances, so new edges form within blocks. (4): escaping requires adding edges with low proximity, which the proximity mechanism suppresses. □

Prebisch Connection: The principle of relatedness as structural lock-in

The principle of relatedness (Hidalgo et al., 2007; Hidalgo, 2021) states that the probability a region enters a new economic activity increases with the number of related activities already present. In the product space, countries diversify into products near their current exports, not into distant products. This IS Theorem 4.1 applied to W^P. The production network evolves through proximity-dominated edge creation: you can produce what is "near" what you already produce. Bolivia produces gas (a product on the periphery of the product space, few connections to other products). The nearest products are other extractive commodities (mining, basic chemicals). The distant products are machinery, electronics, pharmaceuticals — the dense core of the product space where Γ is high.

The principle of relatedness is simultaneously the mechanism of industrial deepening (in the center: you diversify into related products, building an ever-denser W^P) and the mechanism of structural lock-in (in the periphery: you diversify into related commodities, staying on the sparse periphery of W^P). The same mechanism produces integration in the center and fragmentation in the periphery. This is not a paradox — it is a property of the proximity kernel: diversification follows edges, and the edge structure is dense in the core and sparse on the periphery.

5 Integration and Disintegration

5.1 When Does Γ Increase?

Proposition 5.1 (Conditions for Structural Integration)**.** $\Gamma_{t+1} > \Gamma_t$ *(the spectral gap increases — the economy becomes more integrated) if and only if the net effect of edge creation, destruction, and reweighting increases the relative weight of cross-block edges compared to within-block edges.*

Sufficient conditions:

1. ***Cross-block surplus is positive**: $s(i,j) > 0$ for $i \in$ block b, $j \in$ block b' $(b \neq b')$. Trade between blocks is profitable.*

2. ***Information reaches across blocks**: $q(i,j;W^I) > 0$ for cross-block pairs. Agents in block b know about opportunities in block b'.*

3. ***Proximity barriers are overcome**: either by exogenous shocks that create cross-block edges (a road, a migration wave, a new trade route), or by policy that subsidizes*

cross-block links.

5.2 When Does Γ Decrease?

Proposition 5.2 (Conditions for Structural Disintegration)**.** $\Gamma_{t+1} < \Gamma_t$ *(the spectral gap decreases — the economy fragments) when:*

1. ***Cross-block edges are destroyed faster than created****: an external shock severs trade routes, a policy closes borders, a financial crisis disrupts supply chains.*
2. ***Within-block edges strengthen at the expense of cross-block edges****: agents retrench to familiar partners under uncertainty.*
3. ***Selective integration****: some agents in each block form cross-border links (the elite), while the remaining agents lose cross-block links. The average* Γ *may be unchanged, but the domestic* Γ *falls.*

Key Structural Insight

Condition 3 is Sunkel's thesis: *transnational integration and national disintegration are the same process.* When the gas executive in La Paz connects to the global commodity market ($W^P_{\text{executive, global}}$ strengthens), his connection to the quinoa farmer weakens ($W^P_{\text{executive, farmer}}$ weakens — his supply chains are now global, not domestic). The global Γ increases (more connected world economy). The domestic Γ decreases (more fragmented national economy). The same edge creation (gas executive $\rightarrow$ global market) is simultaneously integration at one scale and disintegration at another.

5.3 A Phase Diagram

Proposition 5.3 (Phase Diagram for Γ)**.** *In the co-evolutionary system, the dynamics of* Γ_t *can be approximated by:*

$$\Gamma_{t+1} - \Gamma_t \approx \underbrace{\eta_c \cdot s_{cross}(W_t, \mathbf{x}_t^*)}_{\text{integration force}} - \underbrace{\eta_d \cdot \delta_{cross}(W_t)}_{\text{fragmentation force}} - \underbrace{\eta_p \cdot (p_1 - p_1^*)}_{\text{proximity bias}}, \tag{9}$$

where s_{cross} *is the average cross-block surplus,* δ_{cross} *is the cross-block destruction rate, and* $p_1 - p_1^*$ *measures how much edge creation is proximity-biased relative to the threshold* p_1^* *that would maintain the current* Γ*.*

Three regimes*:*

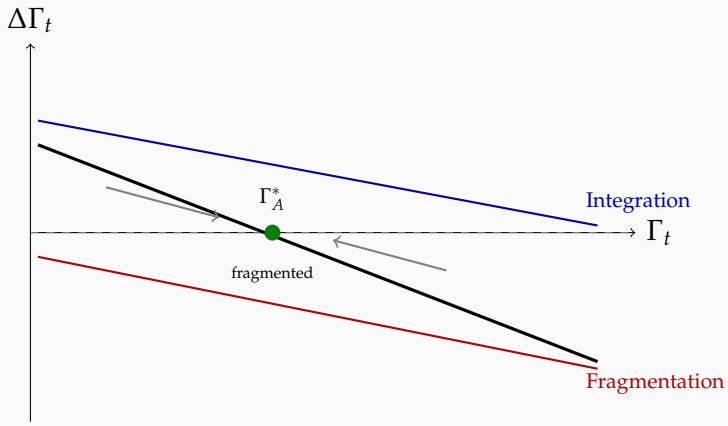
1. ***Integration****:* $s_{cross} > 0$ *and* δ_{cross} *is low. The economy moves toward the integrated equilibrium.* $\Gamma \uparrow$*.*

2. ***Fragmentation***: $s_{cross} < 0$ *or* δ_{cross} *is high. The economy moves toward the fragmented equilibrium.* $\Gamma \downarrow$.

3. ***Lock-in***: $s_{cross} \approx 0$ *and proximity dominates.* Γ *is approximately constant. The economy is trapped in its current structural configuration.*

Visualization:

Phase diagram: integration vs. fragmentation forces



The net force on Γ (thick black curve) crosses zero at Γ_A^* — the fragmented structural equilibrium. Below Γ_A^*: integration forces dominate (arrows point right). Above: fragmentation forces dominate (arrows point left). The economy is drawn toward the trap. A second equilibrium at higher Γ (not shown) may exist if integration forces are boosted by policy.

6 Structural Change as Transition Between Equilibria

Definition 6.1 (Structural Change)**.** ***Structural change*** *is a transition from one structural equilibrium* $(W_A^*, \boldsymbol{\theta}_A^*, \mathbf{x}_A^*)$ *to another* $(W_B^*, \boldsymbol{\theta}_B^*, \mathbf{x}_B^*)$*. The transition occurs when a shock or policy moves the system out of the basin of attraction of A and into the basin of attraction of B.*

Proposition 6.2 (The Cost of Structural Change)**.** *The minimum "push" required to transition from the fragmented equilibrium A to the integrated equilibrium B is:*

$$\varepsilon^* = \inf\{\|W - W_A^*\|_F : W \in \mathcal{B}(W_B^*)\}, \tag{10}$$

the Frobenius distance from the current equilibrium to the basin boundary of the target equilibrium. This cost is:

1. ***Large when*** ρ ***is large***: *sticky preferences resist change, so the preference component of the trap is hard to escape.*

2. ***Large when proximity dominates edge creation****: the network's own dynamics pull it back toward the fragmented equilibrium.*

3. ***Small when s_{cross} is large****: cross-block trade is profitable, so once enough cross-block edges exist, they sustain themselves.*

Example 6.3 (Four Paths to Structural Change)**.**

Mechanism	***Historical example***	***Network change***	ε^*
Exogenous shock	*WWII (US female LFP)*	*Mass entry of women into W^P*	*Large, one-time*
Infrastructure	*Trans-Siberian railway*	*Physical cross-block edge*	*Medium, permanent*
Information shock	*Mobile phones (Africa)*	*W^I gains cross-block edges*	*Small per edge, cumulat*
Policy	*South Korea's ISI*	*Deliberate creation of W^P edges*	*Large, sustained*

7 Summary

1. Networks evolve through three mechanisms: **edge creation** (proximity-dominated, path-dependent), **edge destruction** (surplus-driven or shock-driven), and **edge reweighting** (continuous adjustment to surplus).

2. The **co-evolutionary system** has three timescales: fast (market clearing), medium (preference formation), slow (network evolution). The structural equilibrium is a fixed point of all three.

3. **Multiple structural equilibria** coexist: an integrated equilibrium (high Γ, homogeneous preferences, low inequality) and a fragmented equilibrium (low Γ, stratified preferences, high inequality). Both are self-reinforcing.

4. The **principle of relatedness** — the empirical law that diversification follows existing links — is a consequence of proximity-dominated edge creation. It produces structural deepening in the center and structural lock-in in the periphery.

5. **Integration and disintegration** can occur simultaneously at different scales: transnational integration of elites and national disintegration of domestic economies (Sunkel's thesis).

6. **Structural change** is a transition between equilibria, requiring a "push" of magnitude ε^*. The push can come from shocks, infrastructure, information, or policy.

8 Exercises

13.1 Edge creation by hand. For a 4×4 economy with $W_t = \begin{pmatrix} 0.5 & 0.3 & 0.01 & 0.01 \\ 0.3 & 0.5 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.5 & 0.3 \\ 0.01 & 0.01 & 0.3 & 0.5 \end{pmatrix}$:

(a) Compute the network distance between all pairs. Which pairs are "closest"?

(b) If a new edge forms with probability $\Pr \propto 1/d(i,j)$: what is the most likely new edge? Is it within-block or cross-block?

(c) If a policy subsidizes cross-block edges (doubles the creation probability for pairs in different blocks): what is now the most likely new edge?

(d) Simulate 20 periods of evolution with and without the subsidy. Plot Γ_t in both cases.

13.2 Structural equilibria. For the Bolivia two-equilibrium example (Example 3.4):

(a) Verify that both W_A^* and W_B^* are structural equilibria (check all three conditions from Definition 3.1).

(b) Compute the basin boundary: starting from W_A^*, increase cross-block entries by ε. At what ε does the system transition to W_B^*?

(c) What is the welfare gain from the transition? Use all four criteria from Chapter 12.

13.3 Bolivia's gas nationalization. Model the 2006 nationalization as simultaneous edge destruction ($W^P_{\text{Bolivia, MNC}} \to 0$) and edge creation ($W^P_{\text{gas, treasury}} \uparrow$):

(a) Using a 4×4 model (MNC, Gas Company, Government, Traditional), write W^P_{before} and W^P_{after}.

(b) Compute Γ before and after. Did it increase or decrease?

(c) Compute $\boldsymbol{\pi}$ before and after. Who gained power? Who lost?

(d) Did the nationalization change W^I? If so, in which direction?

13.4 The principle of relatedness. Consider a product space with 6 products arranged in a ring: $\phi_{p,p+1} = 0.3$ (adjacent products are related), $\phi_{p,p+2} = 0.05$

(distant products weakly related), $\phi_{p,p+3} = 0$ (opposite products unrelated).

(a) A country currently produces products 1 and 2. Compute relatedness density ω_{cp} for all 6 products.

(b) Under the principle of relatedness: which product is most likely to be entered next?

(c) If the policy goal is to reach product 5 (on the opposite side of the ring): how many "steps" are needed? How many generations (at one step per generation)?

(d) This is the product-space distance. Connect to the escape threshold ε^* from Proposition 6.2.

13.5 Sunkel's integration/disintegration. Model a 3-country world (Center, Periphery-Elite, Periphery-Mass) as $N = 3$. Initially W^P is mildly connected. A multinational corporation creates a link between Center and Periphery-Elite:

(a) Write W^P_{before} and W^P_{after} (W^P_{12} increases by 0.15; W^P_{23} decreases by 0.05 as Elite resources shift from domestic to global).

(b) Compute domestic Γ (between Periphery-Elite and Periphery-Mass) before and after.

(c) Compute global Γ (all three) before and after.

(d) Verify Sunkel's prediction: global Γ increases while domestic Γ decreases.

13.6 WWII as structural change. Model US female LFP as a 2-block economy (Men, Women):

(a) Pre-war: W^P is block-diagonal (women in domestic production, men in market production). Women's $\theta^*_F = 0.3$ (low market orientation).

(b) WWII shock: women enter W^P_{market} (large cross-block edge creation). Compute immediate effect on $\boldsymbol{\theta}$.

(c) Post-war: the edge partially reverses (some women leave market). But children of wartime workers have W^I edges to the market. Simulate the formation dynamics for 3 generations.

(d) Does θ^*_F return to 0.3? Why or why not? Connect to Fernández (2013).

13.7 Network evolution simulation. Write a Python program that simulates the co-evolutionary system for $N = 8$, starting from a random initial W_0:

(a) Implement edge creation (proximity-dominated), destruction (surplus-driven), and reweighting.

(b) Run 100 simulations with different random W_0's. Classify the final W^* as "integrated" ($\Gamma > 0.3$) or "fragmented" ($\Gamma < 0.1$).

(c) What fraction converge to each type? How does the fraction depend on p_1 (proximity parameter)?

(d) At what p_1 does the fraction of fragmented equilibria exceed 50%?

Mathematical Appendices

Appendix 13A: Network Distance and Proximity

Geodesic distance. The shortest-path distance $d(i, j; W)$ between agents i and j in the weighted network W is the minimum number of edges in any path from j to i (unweighted) or the minimum sum of edge inverses $\sum_{e \in \text{path}} 1/W_e$ (weighted).

Effective resistance distance. For undirected networks, the effective resistance between i and j is $R_{ij} = (e_i - e_j)' L^+ (e_i - e_j)$, where L is the graph Laplacian $L = D - W$ (D is the degree matrix) and L^+ is its pseudoinverse. This distance accounts for multiple paths between i and j, not just the shortest.

Proximity as inverse distance. In the product space literature, proximity between products p and p' is measured by co-export frequency: $\phi_{pp'} = \min(\Pr(M_{cp} = 1 \mid M_{cp'} = 1), \Pr(M_{cp'} = 1 \mid M_{cp} = 1))$. This is the conditional probability of exporting p given that you export p' (or vice versa). High $\phi_{pp'}$ means the products require similar capabilities and tend to be co-produced.

Appendix 13B: Stability of Structural Equilibria

The Jacobian of the co-evolutionary system. At a structural equilibrium $(W^*, \boldsymbol{\theta}^*)$, the linearized system has a block Jacobian:

$$\mathcal{J} = \begin{pmatrix} \partial \Phi / \partial \boldsymbol{\theta} & \partial \Phi / \partial W \\ \partial \Psi / \partial \boldsymbol{\theta} & \partial \Psi / \partial W \end{pmatrix}_{(W^*, \boldsymbol{\theta}^*)}. \tag{11}$$

Stability requires all eigenvalues of $\mathcal{J}$ to have modulus less than 1. The upper-left block $\partial \Phi / \partial \boldsymbol{\theta}$ is the formation Jacobian J from Chapter 10. The lower-right block $\partial \Psi / \partial W$ governs network self-reinforcement. The off-diagonal blocks capture the cross-effects: how $\boldsymbol{\theta}$ changes affect W (through the surplus, which depends on preferences), and how W changes affect $\boldsymbol{\theta}$ (through the formation signal, which depends on the network).

Two-timescale approximation. When $\eta \ll 1 - \rho$ (network evolution is much slower than preference formation), the system can be analyzed in two stages: (1) for each fixed W, find the formation steady state $\boldsymbol{\theta}^*(W)$, (2) study the dynamics of W under the assumption that $\boldsymbol{\theta}$ instantly adjusts to $\boldsymbol{\theta}^*(W)$. The reduced dynamics are $W_{t+1} = \Psi(W_t; \mathbf{x}^*(\boldsymbol{\theta}^*(W_t), W_t))$, a system in W alone.

Structuralist Case Study: Sunkel's Integration/Disintegration Thesis

Osvaldo Sunkel's 1971 essay "Capitalismo transnacional y desintegración nacional en América Latina" (*Estudios Internacionales*, 4(16), pp. 3–61) advanced a thesis that was ahead of its time by forty years and maps directly onto the network evolution theory of this chapter: **the same process that integrates the world economy disintegrates the national economies of the periphery**.

Sunkel's argument has three parts.

Part 1: Transnational integration creates cross-border elite linkages. Multinational corporations, international financial institutions, and globally connected professional classes form dense networks that cross national borders. In the notation of this chapter: the edge creation process (Definition 1.1) generates new edges between elite agents in different countries. The global W^P becomes denser among elite nodes. The proximity mechanism (Theorem 4.1) reinforces these links: once an elite network exists, new members are drawn from agents who are already "close" to existing members — shared education (Harvard, LSE), shared language (English), shared cultural consumption (the same media, the same conferences). The global elite converges to a single information network $W^I_{\text{global elite}}$ with high N_{eff} and a shared consensus.

Part 2: National disintegration destroys domestic cross-block linkages. Simultaneously, the elite's domestic connections weaken. The gas executive in La Paz trades with Houston, not with the quinoa farmer in Oruro. His information network connects him to the *Wall Street Journal*, not to Radio Pacífica. His children attend international schools, not public schools. Each of these shifts is an edge in W^P, W^U, or W^I that moves from domestic-cross-block to global-elite. The domestic matrix loses cross-block entries. The domestic Γ decreases.

This is Proposition 5.2, condition 3: selective integration. The agents who form cross-border links are precisely the agents who previously served as bridges between domestic blocks. When they exit the domestic network (or reduce their domestic engagement), the domestic blocks lose their only connections. The blocks drift apart.

Part 3: The five processes are one process. Sunkel (together with Paz, 1970) had previously argued that development, underdevelopment, dependency, marginalization, and spatial inequality are "diverse manifestations of a single process." The 1971 essay identified what that process IS: the rewiring of W that simultaneously integrates elites across borders and disintegrates national economies within borders. Every edge created in W_{global} between an elite node and the international system is, implicitly, an edge weakened in W_{domestic} between that elite node and

the non-elite population. The zero-sum character is not about resources (growth can be positive-sum) but about *attention, links, and structural position*: an agent has a finite capacity for connections, and global connections displace domestic ones.

Sunkel's argument (1971)	**Formal object**	**This chapter**
Transnational integration	Cross-border edge creation	Def. 1.1
National disintegration	Domestic cross-block edge destruction	Def. 1.2
Simultaneous process	$\Gamma_{\text{global}} \uparrow$ while $\Gamma_{\text{domestic}} \downarrow$	Prop. 5.2(3)
Selective integration	Elite bridges exit domestic network	Sec. 5
Five processes, one process	One matrix rewiring, multiple observables	Key Insight
Path dependence	Proximity-dominated edge creation	Thm. 4.1

Sunkel wrote in 1971, before globalization was a word. The multinational corporations he described were oil companies and mineral extractors; the "transnational elite" was the alliance of foreign capital and domestic technocrats. Fifty years later, the MNCs are tech platforms, the transnational elite is the global professional class, and the disintegration is called "populist backlash" rather than "dependency." The matrix is different. The mechanism is the same.

Further Reading

Network evolution: Jackson and Rogers (2007), "Meeting Strangers and Friends of Friends," *AER*. Barabási and Albert (1999), "Emergence of Scaling in Random Networks," *Science*. For endogenous network formation with strategic agents: Bala and Goyal (2000), "A Noncooperative Model of Network Formation," *Econometrica*.

The principle of relatedness: Hidalgo et al. (2007), "The Product Space Conditions the Development of Nations," *Science*. Hidalgo (2021), "Economic Complexity Theory and Applications," *Nature Reviews Physics*. Hausmann and Klinger (2007), "The Structure of the Product Space and the Evolution of Comparative Advantage," CID Working Paper.

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for Latin America, Lynne Rienner. Ocampo (2009), "Latin America and the Global Financial Crisis," *Cambridge Journal of Economics*.

Chapter 14

Coalitions and Class Alliances

Who Allies with Whom to Change the Matrix

> *"Não basta vincular de forma abstrata os modos de produção e as formas de exploração às fases do desenvolvimento capitalista. É necessário levar em conta as situações em que se dão as alianças entre grupos e classes que atuam no sentido de manter uma dada estrutura de domínio ou de promover a sua transformação."*
>
> ["It is not enough to link in abstract fashion the modes of production and forms of exploitation to the phases of capitalist development. It is necessary to take into account the situations in which the alliances between groups and classes act to maintain a given structure of domination or to promote its transformation."]
>
> — Fernando Henrique Cardoso and Enzo Faletto, *Dependência e Desenvolvimento na América Latina* (1969)

Chapter 13 showed how the weight matrix W evolves through edge creation, destruction, and reweighting. But it treated the evolution as a stochastic process — edges form or break according to surplus and proximity, as if the network rewires itself.

In practice, structural change is not automatic. It is *political*. Agents form coalitions to change W in their favor — to strengthen the edges that increase their power and weaken the edges that benefit their rivals. Trade policy, labor regulation, land reform, media ownership, education access, border control — all of these are instruments through which coalitions reshape the weight matrix.

This chapter introduces coalitions into the structural model. The key question: which coalitions form, and what do they do to W? The answer: a coalition $S \subseteq \{1, \ldots, N\}$ forms if its members collectively gain from the structural change they can enact. The gain is measured by $\Delta\boldsymbol{\pi}$: the change in the Perron eigenvector. A coalition is viable if every member gains power ($\Delta\pi_i > 0$ for all $i \in S$) and the coalition can implement the required ΔW.

1 Coalitions as Network Designers

1.1 The Coalition Problem

Definition 1.1 (Coalition)**.** *A **coalition** is a subset $S \subseteq \{1, \ldots, N\}$ of agents who jointly choose a structural change ΔW to maximize their collective power:*

$$\max_{\Delta W \in \mathcal{F}(S)} \sum_{i \in S} \pi_i(W + \Delta W), \tag{1}$$

*where $\mathcal{F}(S)$ is the **feasibility set**: the set of structural changes the coalition can implement given its current power.*

The feasibility set depends on what the coalition controls. An agent who controls a factory can rewire her production edges (ΔW^P). An agent who controls a media outlet can rewire information edges (ΔW^I). An agent who controls the state can use regulation, taxation, and subsidies to rewire multiple layers of W simultaneously. The more powerful the coalition, the larger $\mathcal{F}(S)$.

Proposition 1.2 (Power Determines Feasibility)**.** *The feasibility set of coalition S is bounded by the coalition's current power:*

$$\mathcal{F}(S) = \left\{ \Delta W : \|\Delta W\|_F \leq \phi\left(\sum_{i \in S} \pi_i(W) \right) \right\}, \tag{2}$$

where $\phi(\cdot)$ is an increasing function: coalitions with more power can implement larger structural changes. The rich get richer: the powerful can change the structure more, which makes them more powerful, which expands their feasibility set.

Intuition: The political Coase theorem fails

Acemoglu (2003) asked: why don't societies always adopt efficient institutions? If the efficient structure benefits everyone (or at least generates a large enough surplus to compensate losers), why doesn't a grand coalition implement it?

The answer in this framework: the feasibility set depends on *current* power. The coalition that would benefit from the efficient W (the powerless majority) lacks the power to implement ΔW. The coalition that has the power (the powerful minority) has no incentive to implement it — the efficient W would reduce their power. The political Coase theorem fails because the enforcement mechanism IS the current W, and W is not neutral.

1.2 The Payoff from a Coalition

Definition 1.3 (Coalition Payoff)**.** *The payoff to agent i from joining coalition S that implements ΔW_S is:*

$$v_i(S) = x_i^*(W + \Delta W_S, \boldsymbol{\theta}^*_{W+\Delta W_S}) - x_i^*(W, \boldsymbol{\theta}^*_W). \tag{3}$$

This includes both the enforcement effect (change in allocation at fixed preferences) and the formation effect (long-run change in allocation from preference adjustment):

$$v_i(S) = \underbrace{\Delta x_i^{enf}}_{enforcement} + \underbrace{\Delta x_i^{form}}_{formation} = E_i(\Delta W_S) + F_i(\Delta W_S). \tag{4}$$

Proposition 1.4 (The Power Gain Approximation)**.** *In the LQ model, the enforcement payoff is approximately:*

$$E_i(\Delta W_S) \approx \frac{1}{\kappa} \Delta \pi_i(W, \Delta W_S), \tag{5}$$

where $\Delta \pi_i = \pi_i(W + \Delta W_S) - \pi_i(W)$. The formation payoff is:

$$F_i(\Delta W_S) \approx \frac{\Psi}{\kappa} \Delta \pi_i(W, \Delta W_S), \tag{6}$$

where Ψ is the formation-to-enforcement ratio from Chapter 11. The total payoff:

$$v_i(S) \approx \frac{1 + \Psi}{\kappa} \Delta \pi_i. \tag{7}$$

The payoff is proportional to the change in the Perron eigenvector, amplified by $(1 + \Psi)$.

Key Structural Insight

The coalition payoff is determined by $\Delta \boldsymbol{\pi}$: the change in the power vector. Every political conflict — every trade war, labor dispute, media battle, land reform, nationalization — is a fight over $\Delta \boldsymbol{\pi}$. The coalition that wins is the one that can implement the ΔW that shifts $\boldsymbol{\pi}$ in its favor.

The formation amplifier $(1 + \Psi)$ is crucial: when $\Psi > 0$, a structural change has long-run effects beyond the immediate power shift, because the new W eventually changes preferences. A coalition that controls W^I (media, education) benefits from the formation amplifier: not only does it gain power immediately, but it shapes the preferences of the next generation to *accept* the new power structure. This is the deepest form of structural power: the power to make your power seem natural.

2 Which Coalitions Form?

2.1 Stability Concepts

Definition 2.1 (Stable Coalition Structure)**.** *A partition of agents into coalitions $\mathcal{S} = \{S_1, S_2, \ldots, S_k\}$ is **stable** if:*

1. ***Internal stability**: No member of any coalition S_m would prefer to defect (to leave S_m and either join another coalition or stand alone). For all $i \in S_m$: $v_i(S_m) \geq v_i(S_m \setminus \{i\})$ and $v_i(S_m) \geq v_i(\{i\})$.*

2. ***External stability**: No coalition S_m would accept a new member from outside. For all $j \notin S_m$: if j wants to join S_m, then at least one existing member of S_m would be made worse off.*

3. ***Coalition rationality**: Each coalition implements the ΔW_S that maximizes its collective payoff given the other coalitions' actions.*

Theorem 2.2 (Coalition Formation Theorem)**.** *In the structural model with N agents and LQ payoffs:*

1. *At least one stable coalition structure exists (under compact feasibility sets and continuous payoffs).*

2. *The grand coalition $S = \{1, \ldots, N\}$ is stable if and only if the efficient W^* is in $\mathcal{F}(\{1, \ldots, N\})$ and the surplus from efficiency can compensate every agent for her power loss. This requires side payments — which may not be credible if the current W is the enforcement mechanism (Acemoglu's point).*

3. *In general, the stable partition involves 2–3 coalitions that balance each other's power.*

Proof sketch. (1) The existence follows from the theory of coalition-proof Nash equilibria (Bernheim, Peleg, and Whinston, 1987) adapted to the structural game. (2) The grand coalition implements $W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$ (the efficient structure). The surplus is maximized. But agent i with $\pi_i > 1/N$ loses power, so she must be compensated by a transfer. The transfer must be credible — but credible transfers require an enforcement mechanism, and the enforcement mechanism IS the current W, which the grand coalition proposes to change. (3) With 2–3 coalitions, each has enough power to implement local ΔW but not enough to achieve the grand coalition's global restructuring. □

2.2 The Class Alliance as a Coalition

Definition 2.3 (Class Alliance)**.** *A **class alliance** is a coalition formed between agents from different structural positions (blocks of W) who share a common interest in a specific ΔW. The alliance is stable if all members gain from the structural change, despite occupying different positions in the current structure.*

Example 2.4 (Four Class Alliances in Latin America)**.** *Four agents: Foreign Capital (F), Domestic Bourgeoisie (B), Middle Class (M), Workers (W). Current power: $\boldsymbol{\pi} = (0.35, 0.30, 0.20, 0.15)$.*

***Alliance 1: Oligarchic-Export** ($\{F, B\}$)**.** Foreign capital and domestic landowners/miners export raw materials. They strengthen W^P_{FB} (supply chain links) and W^I_{FB} (shared business culture). The structural change reinforces the export enclave:*

$$\Delta\boldsymbol{\pi} \approx (+0.05, +0.03, -0.04, -0.04). \tag{8}$$

F and B gain; M and W lose. The enclave deepens.

***Alliance 2: National-Populist** ($\{B, M, W\}$)**.** Domestic bourgeoisie, middle class, and workers unite against foreign capital. They implement ISI: tariffs weaken $W^P_{F,domestic}$ (fewer imported inputs) and strengthen $W^P_{domestic}$ (local supply chains). New domestic W^I channels (national media, public education):*

$$\Delta\boldsymbol{\pi} \approx (-0.12, +0.05, +0.04, +0.03). \tag{9}$$

F loses massively; all domestic classes gain. Perón's Argentina, Vargas's Brazil, the MNR in Bolivia.

***Alliance 3: Techno-Bureaucratic** ($\{F, B, M\}$ **excluding** W)**.** Post-coup modernization. Foreign capital, domestic bourgeoisie, and the technical middle class ally to implement "efficient" structural adjustment. They strengthen W^P_{FB} and W^P_{BM} while weakening labor protections (W^U_{WM} and W^I_{WM} edges severed):*

$$\Delta\boldsymbol{\pi} \approx (+0.03, +0.04, +0.02, -0.09). \tag{10}$$

Workers bear all the cost. Brazil post-1964, Chile post-1973, Bolivia 1985.

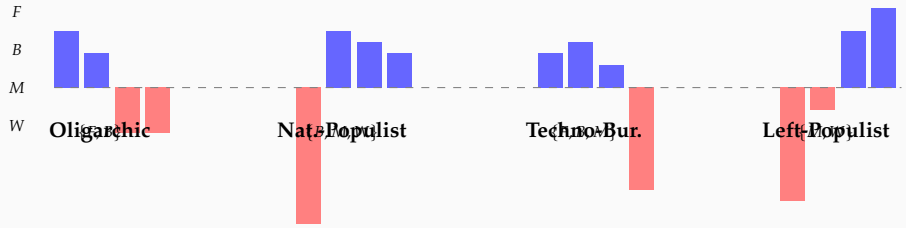
***Alliance 4: Left-Populist** ($\{M, W\}$ **sometimes** $+B$)**.** Middle class and workers unite, sometimes pulling in the domestic bourgeoisie against foreign capital. They nationalize key sectors (ΔW^P: gas/mining edges redirect from F to state), expand public W^I (state media, education reform), and strengthen labor linkages (W^U_{MW}):*

$$\Delta\boldsymbol{\pi} \approx (-0.10, -0.02, +0.05, +0.07). \tag{11}$$

F loses most; W gains most. Evo Morales's Bolivia, Chávez's Venezuela.

Visualization:

Four alliances, four $\Delta\pi$ vectors



Four bars per alliance: $\Delta\pi_F$, $\Delta\pi_B$, $\Delta\pi_M$, $\Delta\pi_W$. Blue bars (above zero): the agent gains power. Red bars (below): she loses. Each alliance is defined by who gains and who pays. The bars are $\Delta\pi$ — the object that every political conflict is about.

3 The Stability of Alliances

3.1 When Alliances Hold

Proposition 3.1 (Alliance Stability Conditions)**.** *Alliance S is internally stable (no member wants to defect) if and only if:*

$$\Delta\pi_i(W, \Delta W_S) > 0 \quad \textit{for all } i \in S, \tag{12}$$

AND no member can achieve a higher $\Delta\pi_i$ by defecting to a rival alliance S':

$$\Delta\pi_i(W, \Delta W_S) \geq \Delta\pi_i(W, \Delta W_{S' \cup \{i\}}) \quad \textit{for all feasible } S'. \tag{13}$$

Example 3.2 (Why the National-Populist Alliance Breaks)**.** *Consider Alliance 2 ($\{B, M, W\}$). Initially stable: all three domestic classes gain from ISI. But as ISI succeeds, the domestic bourgeoisie grows — her π_B increases. At some threshold, she can earn a higher $\Delta\pi_B$ by switching to Alliance 3 ($\{F, B, M\}$): partnering with foreign capital to "open" the economy.*

The switch occurs when:

$$\Delta\pi_B(\Delta W_{\{F,B,M\}}) > \Delta\pi_B(\Delta W_{\{B,M,W\}}). \tag{14}$$

In words: when the domestic bourgeoisie is strong enough that partnering with foreign capital (and cutting out workers) yields a higher power gain than continuing the national-populist alliance. This is the transition from ISI populism to techno-bureaucratic modernization — the story of Brazil 1964, Argentina 1976, Chile 1973.

Phase	*Alliance*	π_B	$\Delta\pi_B$ *from best alliance*
Early ISI	$\{B, M, W\}$	*0.20*	$+0.05$
Mature ISI	$\{B, M, W\}$	*0.30*	$+0.03$ *(diminishing)*
Post-ISI crisis	*Switch to* $\{F, B, M\}$	*0.30*	$+0.04$

The alliance breaks because its own success changes the power structure, which changes the payoffs, which changes the optimal alliance. The matrix W evolved (Chapter 13), and the evolution shifted the coalition incentives.

3.2 When Alliances Cycle

Theorem 3.3 (Alliance Cycling)**.** *In the co-evolutionary system with coalition formation, the following cycle can occur:*

1. *Phase 1: National-populist alliance ($\{B, M, W\}$) implements ISI. π_B rises. Domestic Γ increases.*
2. *Phase 2: B defects to techno-bureaucratic alliance ($\{F, B, M\}$). Liberalization. π_W falls. W^I becomes more center-dominated.*
3. *Phase 3: Workers and middle class form left-populist alliance ($\{M, W\}$, sometimes $+B$). Nationalization. π_F falls. Domestic Γ increases again.*
4. *Phase 4: Resource rents create corruption/inefficiency. B defects again to $\{F, B\}$ or $\{F, B, M\}$. Return to Phase 2.*

*The cycle is the **Latin American pendulum**: ISI $\to$ liberalization $\to$ populism $\to$ liberalization $\to \cdots$*

Proof sketch. Each phase changes W, which changes $\boldsymbol{\pi}$, which changes the coalition payoffs $\Delta\pi_i(S)$, which destabilizes the current alliance and triggers a switch. The cycle is sustained because no single structural equilibrium satisfies all agents simultaneously: the efficient W requires the grand coalition, but the grand coalition requires credible commitment, which requires the enforcement mechanism that the coalition proposes to change. □

Prebisch Connection: The Latin American pendulum

The cycling theorem formalizes what economic historians of Latin America have observed for a century: the oscillation between inward-oriented and outward-oriented development strategies, between populist inclusion and technocratic exclusion, between nationalization and liberalization.

The standard explanation is "policy failure": populism leads to fiscal crisis, which triggers liberalization, which leads to inequality, which triggers populism. This explanation blames the cycle on bad policy.

The structural explanation is different: the cycle is endogenous to the co-evolutionary system. Each alliance changes W in a way that eventually destabilizes itself. ISI strengthens the bourgeoisie, who then defects. Liberalization weakens workers, who then rebel. Nationalization creates rents, which corrupt the coalition. The cycle is not a sequence of mistakes. It is the dynamics of a system that has no stable grand coalition because the enforcement mechanism is the object being contested.

Bolivia's cycle: MNR revolution (1952, Alliance 2) $\to$ military/technocratic rule (1964, Alliance 3) $\to$ democratic opening and structural adjustment (1985, Alliance 3 variant) $\to$ resource nationalism (2006, Alliance 4) $\to$ political crisis (2019, unstable). Each transition is a coalition switch driven by $\Delta\boldsymbol{\pi}$.

4 The Structural Power Game

4.1 Formalization

Definition 4.1 (The Structural Power Game)**.** *The* ***structural power game*** $\mathcal{G} = (N, \{A_i\}, \{v_i\}, W)$ *is defined by:*

- N *players (agents or classes).*
- *Action set* A_i*: the set of coalitions agent* i *can join.*
- *Payoff* $v_i(S) = \frac{1+\Psi}{\kappa}\Delta\pi_i(W, \Delta W_S)$*: the allocation change from the coalition's structural policy.*
- *The current weight matrix* W*, which determines both the payoffs and the feasibility sets.*

Proposition 4.2 (Properties of the Structural Power Game)**.** *1. The game is* ***not zero-sum****: the efficient structural change* ($W \to W^{CE}$) *increases total welfare. But the*

*game is **redistributive**: any ΔW that increases π_i for some i decreases π_j for some j (since $\sum_i \pi_i = 1$).*

2. *The game is **not superadditive**: the grand coalition may not yield the highest payoff for every agent, because the efficient W reduces the power of currently powerful agents.*

3. *The game is **path-dependent**: the current W determines the feasibility sets, which determine which coalitions can form, which determines the next W. History matters.*

4. *The **core** of the game (the set of allocations that no coalition can improve upon) may be empty when W is highly asymmetric (concentrated power).*

4.2 Equilibrium Alliance Structures

Theorem 4.3 (Equilibrium Characterization)**.** *In the structural power game with $N = 4$ (Foreign Capital, Domestic Bourgeoisie, Middle Class, Workers):*

1. *If $\pi_F > 0.4$ (foreign capital dominant): the stable alliance is $\{F, B\}$ (oligarchic-export). Workers and middle class cannot form a winning counter-coalition because $\mathcal{F}(\{M, W\})$ is too small.*

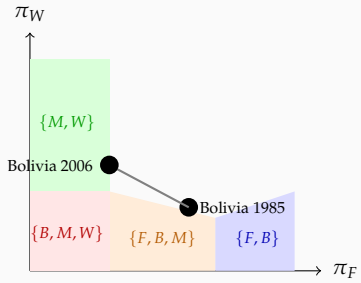
2. *If $\pi_B > \pi_F$ and $\pi_B + \pi_M + \pi_W > 0.5$ (domestic bourgeoisie strong, collective domestic power exceeds foreign): the stable alliance is $\{B, M, W\}$ (national-populist).*

3. *If $\pi_B + \pi_F > \pi_M + \pi_W$ and $\pi_W < 0.15$ (workers very weak): the stable alliance is $\{F, B, M\}$ (techno-bureaucratic).*

4. *If $\pi_M + \pi_W > \pi_F + \pi_B$ (domestic non-elite majority is powerful enough): the stable alliance is $\{M, W\}$ or $\{B, M, W\}$ (left-populist or national-populist).*

Visualization:

Alliance stability regions in the (π_F, π_W) plane



Each region corresponds to a stable alliance structure, determined by the power distribution (π_F, π_W) (with π_B and π_M filling the remainder). Bolivia moved from the techno-bureaucratic region (1985: π_F high, π_W low) to the left-populist region (2006: π_F lower, π_W higher). The arrow is one step of the Latin American pendulum.

5 Structural Policy as Coalition Output

Every structural policy is a specific ΔW chosen by a specific coalition. The table of policies from Chapter 10 (transfers, structural policy, information policy) can now be mapped to coalitions:

Proposition 5.1 (Policy as Coalition Output)**.**

Policy	ΔW	***Typical coalition***	$\Delta\pi$ ***pattern***
Trade liberalization	$W^P_{domestic} \downarrow$, $W^P_{global} \uparrow$	$\{F, B\}$ *or* $\{F, B, M\}$	$\pi_F \uparrow$, $\pi_W \downarrow$
ISI / industrial policy	$W^P_{domestic} \uparrow$, $W^P_{global} \downarrow$	$\{B, M, W\}$	$\pi_B \uparrow$, $\pi_F \downarrow$
Nationalization	$W^P_F \to W^P_{state}$	$\{M, W\}$ *or* $\{B, M, W\}$	$\pi_F \downarrow$, $\pi_M \uparrow$
Land reform	$W^P_{landlord} \to W^P_{farmer}$	$\{M, W\}$	$\pi_{landlord} \downarrow$
Media regulation	ΔW^I *(de-concentrate)*	$\{M, W\}$ *or* $\{B, M, W\}$	$N_{eff} \uparrow$
Education expansion	ΔW^I *(cross-block edges)*	*Variable*	Ψ*-amplified*
Cash transfers	$\Delta\boldsymbol{\omega}$ *(not* ΔW*)*	*Any*	$\Delta\boldsymbol{\pi} = 0$

Note: cash transfers (last row) do not change W and therefore do not change $\boldsymbol{\pi}$. This is the Transfer Trap from Chapter 2, now with a political explanation: every coalition can agree on transfers (they don't threaten anyone's structural power), but no coalition will implement the ΔW that reduces its own $\boldsymbol{\pi}$.

6 Summary

1. **Coalitions** are subsets of agents who jointly choose ΔW to maximize their collective power. The feasibility set $\mathcal{F}(S)$ depends on current power: the powerful can change more.

2. The **coalition payoff** is $v_i(S) \approx \frac{1+\Psi}{\kappa}\Delta\pi_i$: proportional to the change in the Perron eigenvector, amplified by the formation multiplier.

3. **Four Latin American alliances**: oligarchic-export $\{F, B\}$, national-populist $\{B, M, W\}$, techno-bureaucratic $\{F, B, M\}$, left-populist $\{M, W\}$. Each implements a different ΔW with a different $\Delta\boldsymbol{\pi}$.

4. **Alliance cycling**: ISI strengthens B, who defects $\to$ liberalization weakens W, who rebels $\to$ populism $\to$ crisis $\to$ repeat. The cycle is endogenous: each alliance's success changes the power structure that destabilizes it.

5. The **political Coase theorem fails** because the enforcement mechanism IS the current W. The grand coalition requires credible commitment, which requires an institution that the coalition proposes to change.

6. **Cash transfers** are the one policy every coalition can agree on, precisely because they change $\boldsymbol{\omega}$ but not $\boldsymbol{\pi}$.

7 Exercises

14.1 Coalition payoffs. For $N = 4$ with $\boldsymbol{\pi} = (0.35, 0.30, 0.20, 0.15)$ and $\kappa = 0.5$:

(a) Compute the current bargaining allocation $\mathbf{x}^*$.

(b) Coalition $\{3, 4\}$ (workers and middle class) implements ΔW that shifts $\boldsymbol{\pi}$ to $(0.25, 0.25, 0.25, 0.25)$. Compute $v_3(\{3, 4\})$ and $v_4(\{3, 4\})$. Are both positive?

(c) Can $\{3, 4\}$ implement this ΔW? Check feasibility: $\|\Delta W\|_F \leq \phi(\pi_3 + \pi_4) = \phi(0.35)$.

(d) Agent 2 considers defecting from $\{1, 2\}$ to $\{2, 3, 4\}$. Compute $\Delta\pi_2$ under both coalitions. Which does she prefer?

14.2 Alliance stability. For Example 2.4: verify that each alliance is internally stable (all members gain) and identify the conditions under which each breaks.

14.3 The Latin American pendulum. Simulate the cycling theorem for 8 periods. Start with Alliance 2 ($\{B, M, W\}$). At each period: (a) compute the coalition's ΔW and new $\boldsymbol{\pi}$, (b) check whether any member wants to defect, (c) if so, form the new alliance. Plot $\boldsymbol{\pi}_t$ over time. Does it cycle?

14.4 Bolivia 1952–2019. Map Bolivia's political history to the four-alliance framework:

(a) MNR Revolution (1952): which alliance? What ΔW?

(b) Military governments (1964–1982): alliance switch to what?

(c) Structural adjustment (1985): which alliance implements the NEP?

(d) Evo Morales (2006): alliance switch to what? What ΔW^P (nationalization)?

(e) Political crisis (2019): alliance breakdown. Which conditions from Proposition 3.1 failed?

14.5 Why cash transfers are universal. Show formally that $\Delta\boldsymbol{\pi}(\Delta\boldsymbol{\omega}) = 0$: transfers do not change the Perron eigenvector. Explain why this makes transfers the *least controversial* policy tool: no agent's structural power is threatened. Then explain why this makes transfers the *least effective* tool for structural change.

14.6 The formation amplifier in politics. Compare two alliances with the same $\Delta\boldsymbol{\pi}$ but different Ψ:

(a) Alliance A controls W^P (production policy): $\Psi_A \approx 1$.

(b) Alliance B controls W^I (media, education): $\Psi_B \approx 3$.

(c) Compute the total payoff $v_i = \frac{1+\Psi}{\kappa}\Delta\pi_i$ for each. Which alliance is more valuable to its members?

(d) Which type of alliance is harder to form? (Hint: controlling W^I may require controlling the state first, which requires power, which requires an alliance…)

14.7 Empty core. Construct a 3-agent structural power game where the core is empty: for every coalition structure, some coalition can profitably deviate. Show that this implies cycling. What property of W produces an empty core? (Hint: extreme asymmetry.)

Mathematical Appendices

Appendix 14A: Cooperative Game Theory for Structural Change

The transferable-utility game. A TU game is (N, v) where $v : 2^N \to \mathbb{R}$ assigns a value to each coalition. The **core** is the set of payoff vectors $\mathbf{x}$ with $\sum_{i \in S} x_i \geq v(S)$ for all S and $\sum_i x_i = v(N)$. The core is non-empty if and only if the game is balanced (Bondareva-Shapley theorem).

Application to structural change. The structural power game is NOT a standard TU game because: (1) the value of a coalition depends on the current W (state-dependent), (2) the feasibility set depends on power (endogenous constraints), and (3) the formation amplifier $(1 + \Psi)$ makes payoffs time-dependent (they include long-run preference effects). Nevertheless, the core concept provides useful predictions about which alliance structures are stable.

The Shapley value. The Shapley value $\phi_i = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(N-|S|-1)!}{N!} [v(S \cup \{i\}) - v(S)]$ gives each agent her average marginal contribution across all possible coalitions. In the structural power game, ϕ_i measures agent i's "structural leverage" — how much she improves any coalition she joins. Agents with high Shapley values are the "kingmakers" whose alliance decisions determine the political equilibrium.

Appendix 14B: Sensitivity of π to ΔW

First-order perturbation. From Chapter 8 (Appendix 8B), the first-order change in the Perron eigenvector from a perturbation ΔW:

$$\Delta \pi_i \approx \sum_{k=2}^{N} \frac{1}{1 - \lambda_k} u_{ki} \left(\mathbf{u}_k' \Delta W \mathbf{v}_1 \right), \tag{15}$$

where λ_k are the eigenvalues of W, $\mathbf{u}_k$ are left eigenvectors, and $\mathbf{v}_1 = \mathbf{1}$ is the right Perron eigenvector.

Implication for coalitions. The "cheapest" $\Delta \boldsymbol{\pi}$ is achieved by perturbing W in the direction of the most sensitive eigenvector (the one with λ_k closest to 1). When the spectral gap Γ is small, small perturbations produce large changes in $\boldsymbol{\pi}$ — meaning that coalitions need less power to achieve large structural effects. Fragmented economies ($\Gamma \approx 0$) are politically unstable because small coalitions can tip the power structure.

Structuralist Case Study: Cardoso and Faletto's Class Alliances

This is Cardoso and Faletto's second appearance in the book. In Chapter 10, their "situations of dependency" were mapped to network topologies — different configurations of (W^P, W^U, W^I) producing different structural equilibria. Here, their analysis of class alliances provides the political mechanism that determines *which* topology obtains.

The distinctive contribution of *Dependência e Desenvolvimento* was the insistence that economic outcomes cannot be read off from structural position alone. Two countries with identical W^P (same export structure, same degree of foreign penetration) can have radically different development trajectories because the *class alliances* differ. Brazil's post-1964 trajectory (techno-bureaucratic $\{F, B, M\}$) and Argentina's post-1946 trajectory (national-populist $\{B, M, W\}$) cannot be explained by their trade structures (both were semi-industrialized primary exporters). They can be explained by the alliances.

Part 1: The alliance determines ΔW, not vice versa. The standard structuralist analysis (Prebisch, Furtado, Pinto) starts from W and derives the allocation. Cardoso and Faletto reversed the causation for the political layer: the alliance determines which ΔW is implemented, and therefore which W_{t+1} obtains. Different alliances produce different structural changes even from the same starting W. In this chapter's notation: the coalition S is the independent variable; ΔW_S is the dependent variable. Political agency enters the model through coalition formation, not through the network evolution equation's stochastic mechanics.

Part 2: Alliances are constrained by but not determined by structure. Cardoso and Faletto rejected mechanical determinism: the structure constrains which alliances are feasible (through $\mathcal{F}(S)$) but does not uniquely determine which alliance forms. In the same structural situation, different political choices can lead to different alliances (Brazil chose techno-bureaucratic in 1964; Peru chose military-populist in 1968; Chile chose democratic-socialist in 1970 — all from similar structural positions). The model of this chapter captures this through multiple equilibria in the structural power game (Theorem 4.3): the stable alliance is not unique, and the selection among equilibria depends on political contingency.

Part 3: The alliance IS the development strategy. The deepest insight: Cardoso and Faletto argued that "the development strategy" is not chosen by a benevolent planner who evaluates alternatives on efficiency grounds. The strategy is the ΔW that the winning coalition implements to maximize its collective $\boldsymbol{\pi}$. ISI was not chosen because it was "efficient" — it was chosen because it was the structural policy preferred by the $\{B, M, W\}$ coalition. Liberalization was not chosen be-

cause it was "efficient" — it was chosen because it was preferred by the $\{F, B, M\}$ coalition. The policy debate between ISI and liberalization is, in this framework, a debate between coalitions with different $\Delta\boldsymbol{\pi}$ interests disguised as a debate about economic theory.

Cardoso & Faletto's argument	**Formal object**	**Th**
Class alliances shape the structure	Coalition S chooses ΔW_S	De
Alliances are constrained by structure	$\mathcal{F}(S)$ depends on $\sum_{i \in S} \pi_i$	Pr
Same structure, different alliances	Multiple equilibria in the power game	Th
Development strategy = coalition output	ΔW maximizes coalition $\sum \Delta \pi_i$	De
Pendulum: ISI $\rightarrow$ liberalization $\rightarrow$	Alliance cycling from co-evolutionary dynamics	Th
Political Coase theorem fails	Enforcement mechanism IS the current W	Pr

Cardoso became President and implemented the Plano Real (monetary stabilization) and a broadly liberal economic program — closer to Alliance 3 ($\{F, B, M\}$) than to the national-populist program his academic work had analyzed. Whether this proves or disproves the model is an exercise the reader should not skip (Exercise 14.4 is related but more general). What the model predicts is that the alliance a politician joins depends on the current $\boldsymbol{\pi}$, not on the politician's academic publications. When $\boldsymbol{\pi}$ changes, the optimal alliance changes, and the development strategy changes with it.

Further Reading

Coalition formation: Bernheim, Peleg, and Whinston (1987), "Coalition-Proof Nash Equilibria," *JET*. Ray and Vohra (1997), "Equilibrium Binding Agreements," *JET*. Acemoglu, Egorov, and Sonin (2012), "Dynamics and Stability of Constitutions, Coalitions, and Clubs," *AER*.

The political economy of development: Acemoglu (2003), "Why Not a Political Coase Theorem?" *JCE*. Acemoglu and Robinson (2006), *Economic Origins of Dictatorship and Democracy*. Bardhan (2005), *Scarcity, Conflicts, and Cooperation*.

Class alliances in Latin America: Cardoso and Faletto (1969), *Dependência e Desenvolvimento*, especially Chapters 4–6 (the empirical alliance analysis for Argentina, Brazil, Chile, Mexico). O'Donnell (1973), *Modernization and Bureaucratic-Authoritarianism*, for the formalization of the transition from populist to bureaucratic-authoritarian regimes. Collier and Collier (1991), *Shaping the Political Arena*, for the long-run consequences of labor incorporation strategies.

Bolivia's political economy: Klein (2011), *A Concise History of Bolivia*, Chapters 8–12. Dunkerley (1984), *Rebellion in the Veins*. Webber (2011), *From Rebellion to Reform in Bolivia*.

Chapter 15

Operator Design

Engineering the Weight Matrix

> *"O processo de substituição de importações pode ser entendido como um processo de desenvolvimento 'parcial' e 'fechado' que, respondendo às restrições do comércio exterior, procurou repetir aceleradamente, em condições históricas distintas, a experiência de industrialização dos países desenvolvidos."*
>
> ["The process of import substitution can be understood as a 'partial' and 'closed' development process that, responding to foreign trade constraints, sought to repeat at an accelerated pace, under distinct historical conditions, the industrialization experience of the developed countries."]
>
> — Maria da Conceição Tavares, "Auge e declínio do processo de substituição de importações no Brasil" (1972)

Chapter 13 showed how networks evolve stochastically. Chapter 14 showed who decides the evolution. This chapter asks: given a development objective, *how should the planner design* ΔW*?*

The answer is operator design: the systematic construction of structural changes that move W toward a target. The "operator" is the policy instrument — a tariff, an infrastructure investment, an education reform, a media regulation — and the "design" is the choice of which entries of W to modify, by how much, and in what sequence.

This chapter treats structural policy as an engineering problem. The objective function is the spectral gap Γ (or a welfare-weighted variant). The constraints are the feasibility set $\mathcal{F}(S)$ from Chapter 14. The decision variable is ΔW. The solution is the optimal structural intervention: the ΔW that maximizes Γ (or minimizes $\text{Var}(\pi_i)$, or maximizes N_{eff}) given the political constraints.

This is where the product space meets the weight matrix. Where Hirschman's "big push" gets its eigenvalues. Where ISI becomes matrix algebra.

1 The Structural Policy Problem

1.1 Three Objectives

Definition 1.1 (The Operator Design Problem)**.** *Given the current weight matrix W, a budget $B > 0$ (the maximum Frobenius norm of the intervention), and a feasibility set $\mathcal{F}$, the **operator design problem** is:*

$$\max_{\Delta W \in \mathcal{F}} \Omega(\Delta W) \quad \textit{subject to} \quad \|\Delta W\|_F \leq B, \quad W + \Delta W \geq 0, \quad (W + \Delta W)\mathbf{1} = \mathbf{1}, \tag{1}$$

where $\Omega(\Delta W)$ is the objective function and the constraints ensure that $W + \Delta W$ remains a valid row-stochastic nonnegative matrix.

Three natural objectives:

1. **Maximize** Γ: increase the spectral gap, making the economy more integrated and power more equal.

$$\Omega_1(\Delta W) = \Gamma(W + \Delta W) - \Gamma(W). \tag{2}$$

2. **Minimize** $\mathrm{Var}(\pi_i)$: reduce power inequality directly.

$$\Omega_2(\Delta W) = -\left[\mathrm{Var}(\pi_i(W + \Delta W)) - \mathrm{Var}(\pi_i(W))\right]. \tag{3}$$

3. **Maximize total output** $(I - \gamma W^P)^{-1}\mathbf{d}$: Hirschman's objective — invest where the Leontief multiplier is largest.

$$\Omega_3(\Delta W^P) = \mathbf{1}'(I - \gamma(W^P + \Delta W^P))^{-1}\mathbf{d} - \mathbf{1}'(I - \gamma W^P)^{-1}\mathbf{d}. \tag{4}$$

Intuition: Three objectives, one intuition

All three objectives point in the same direction: *brighten the dark off-diagonal entries of W*. Adding cross-block edges increases Γ (objective 1), reduces $\mathrm{Var}(\pi_i)$ (objective 2), and increases the Leontief multiplier (objective 3). The differences are in which edges to brighten first, and by how much.

1.2 The First-Order Solution

Theorem 1.2 (Optimal Edge Selection)**.** *For objective Ω_1 (maximize Γ), the first-order optimal intervention under a budget constraint allocates the entire budget to the single*

edge (i^*, j^*) *that has the largest marginal effect on* Γ*:*

$$(i^*, j^*) = \arg\max_{(i,j)} \left| \frac{\partial \Gamma}{\partial W_{ij}} \right|. \tag{5}$$

The sensitivity of Γ *to a perturbation in* W_{ij} *is (from Appendix 8B):*

$$\frac{\partial \Gamma}{\partial W_{ij}} \approx \frac{(\pi_i^L \cdot v_{2,j}^R - \pi_i^L \cdot v_{1,j}^R \cdot correction)}{1 - \lambda_2}, \tag{6}$$

where $v_{2,j}^R$ *is the j-th component of the second right eigenvector and* π_i^L *is the i-th component of the left Perron eigenvector.*

Intuition: Which edge to add

The optimal edge connects an agent who is central (high π_i^L — she amplifies any signal through the network) to an agent whose position in the second eigenvector is extreme ($v_{2,j}^R$ far from zero — she is on the "boundary" between blocks). This edge bridges the two competing modes of the matrix (the two blocks that the spectral gap measures the distance between).
In words: connect a powerful agent in block A to a peripheral agent in block B who is closest to the boundary. This is the "bridge builder" strategy: the optimal structural intervention is not to connect the most peripheral agent to the most central one (which might seem intuitive) but to connect a moderately central agent in one block to a boundary agent in the other.

2 Five Operator Types

Every structural policy is an operator $\mathcal{L} : W \mapsto W + \Delta W$ that modifies the weight matrix. We classify five types by the layer of W they target and the type of edge they create.

2.1 Operator 1: Import Substitution (Replace Foreign Edges with Domestic Edges)

Definition 2.1 (ISI Operator)**.** *The* ***import-substitution operator*** $\mathcal{L}_{ISI}$ *modifies* W^P *by:*

$$\mathcal{L}_{ISI} : W_{ij}^P \to W_{ij}^P + \delta_{ij}^{dom} - \delta_{ij}^{for}, \tag{7}$$

where $\delta_{ij}^{dom} > 0$ for domestic (i,j) pairs (new domestic supply chains) and $\delta_{ij}^{for} > 0$ for foreign (i,j) pairs (reduced import dependency).

Proposition 2.2 (Effect of ISI on Γ)**.** *The ISI operator increases domestic Γ if and only if:*

$$\sum_{(i,j)\in domestic} \delta_{ij}^{dom} \frac{\partial \Gamma}{\partial W_{ij}^P} > \sum_{(i,j)\in foreign} \delta_{ij}^{for} \frac{\partial \Gamma}{\partial W_{ij}^P}. \tag{8}$$

That is: the Γ-increasing effect of new domestic edges must exceed the Γ-decreasing effect of severed foreign edges. This condition fails if the new domestic edges are within-block (connecting agents who were already connected) rather than cross-block (connecting the modern and traditional sectors).

Example 2.3 (ISI That Works vs. ISI That Doesn't)**.** *$N = 4$: Modern (M1, M2), Traditional (T1, T2). Current W^P is block-diagonal with $\Gamma = 0.08$.*

ISI-A (within-block): *Tariff on imports substituted by Modern sector. New edge: $\Delta W_{M1,M2}^P = +0.10$. This strengthens the within-block link. Effect on Γ: $\Delta\Gamma \approx +0.01$. Negligible. The modern sector densifies internally without connecting to the traditional sector.*

ISI-B (cross-block): *Tariff on imports substituted by Traditional-sector inputs to the Modern sector. New edge: $\Delta W_{M1,T1}^P = +0.10$. This creates a cross-block link. Effect on Γ: $\Delta\Gamma \approx +0.12$. Twelve times larger. The traditional sector now supplies the modern sector, creating the linkage that makes growth propagate.*

	ΔW^P *target*	$\Delta\Gamma$	$\Delta \operatorname{Var}(\pi_i)$	*Hirschman multiplier*
ISI-A (within)	$W_{M1,M2}^P$	$+0.01$	-0.001	$+0.05$
ISI-B (cross)	$W_{M1,T1}^P$	$+0.12$	-0.015	$+0.22$

ISI-B dominates ISI-A on all three objectives. The same budget, the same tariff rate, radically different structural effects — because the edge targets different entries of W^P.

2.2 Operator 2: Infrastructure (Create Physical Edges)

Definition 2.4 (Infrastructure Operator)**.** *The **infrastructure operator** $\mathcal{L}_{infra}$ creates new edges in W^P (and possibly W^I) by reducing the cost of interaction between previously disconnected agents:*

$$\mathcal{L}_{infra} : W_{ij}^P \to W_{ij}^P + \delta_{ij}(road, port, fiber\ optic, ...). \tag{9}$$

Infrastructure is the literal "edge creation" operator: a road between two regions IS an edge in W^P. The operator design problem for infrastructure is: which road to build?

Proposition 2.5 (Optimal Road Placement)**.** *The road that maximizes* Γ *connects the two blocks (or subnetworks) that are currently most poorly connected and whose connection would create the largest cross-block surplus. Formally: the optimal road connects agents* i^* *and* j^* *that maximize* $|\partial\Gamma/\partial W^P_{ij}|$ *among all pairs with* $W^P_{ij} \approx 0$*.*

Prebisch Connection: Hirschman's linkage argument as operator design

Hirschman (1958) argued that investment should target sectors with the strongest backward and forward linkages — the highest column sums in $(I - A)^{-1}$. In operator-design language: Hirschman's prescription is to add edges *to* the sector with the highest column sum (backward linkage = the sector that generates the most upstream demand) and *from* the sector with the highest row sum (forward linkage = the sector whose output is used most widely).

Hirschman's answer and the spectral-gap answer ($\max \partial\Gamma/\partial W^P_{ij}$) do not always agree. Hirschman targets the *most connected* sector (highest column sum); the spectral-gap criterion targets the *boundary between blocks*. When the most connected sector is in the already-dense core, Hirschman's investment deepens the core without bridging to the periphery. The spectral-gap criterion always bridges.

The difference matters in practice. Bolivia's highest-linkage sector is gas/mining (high column sum in the Leontief inverse — lots of upstream demand). Hirschman says: invest in gas. The spectral-gap criterion says: invest in the *edge between gas and the traditional sector* — the processing plant, the local supply contract, the vocational training program that connects quinoa farmers to the gas-sector labor market.

2.3 Operator 3: Education and Training (Create Capability Edges)

Definition 2.6 (Human Capital Operator)**.** *The* ***human capital operator*** $\mathcal{L}_{HC}$ *modifies* W^P *by expanding the set of production activities an agent can perform:*

$$\mathcal{L}_{HC} : W^P_{ij} \to W^P_{ij} + \delta_{ij}(\textit{if agent } i \textit{ acquires capability for activity } j). \tag{10}$$

Simultaneously, it modifies W^I *by exposing the agent to new information:*

$$\mathcal{L}_{HC} : W^I_{ij} \to W^I_{ij} + \delta'_{ij}(\textit{if agent } i \textit{ enters school/training with agent } j). \tag{11}$$

Education is a dual operator: it creates edges in BOTH W^P (new production capabilities) and W^I (new information sources). This is why education has a Ψ

ratio greater than 1 (Chapter 11): the production edge is the enforcement effect, the information edge is the formation effect. Education changes what you can do AND what you want.

2.4 Operator 4: Information Policy (Rewire W^I)

Definition 2.7 (Information Operator)**.** *The **information operator** $\mathcal{L}_{info}$ modifies W^I without changing W^P:*

$$\mathcal{L}_{info} : W^I_{ij} \to W^I_{ij} + \delta_{ij}(\textit{media, internet, mentor program, ...}). \tag{12}$$

Information policy does not change production or endowments. It changes who sees what. Its effect is entirely through the formation channel — but as Chapter 10 showed, the formation channel can be the dominant one ($\Psi > 1$).

Example 2.8 (The Smartphone as Information Operator)**.** *A program distributes smartphones to 1,000 rural households. Each phone adds edges from the rural block to the urban/modern block in W^I:*

$$\Delta W^I_{rural_i, urban_j} \approx 0.05 \quad \textit{for each connected pair.} \tag{13}$$

Immediate effect: *Zero. No change in W^P, no change in endowments, no change in prices. The standard CBA finds no benefit.*

Formation effect (one generation): *Rural agents now observe modern-sector returns. Their steady-state preference shifts from $\theta^*_T = 0.92$ toward $\theta^*_M = 1.09$. New steady state (with $\rho = 0.85$): $\theta^*_T = 0.96$ (partial convergence). Investment increases. Returns increase. The next generation's θ shifts further.*

Long-run effect (three generations): *$\theta^*_T \to 1.01$. The preference gap closes by 60%. The allocation gap closes by 40% (the remaining 60% requires also changing W^P and $\boldsymbol{\pi}$).*

Ψ for this intervention: *≈ 4.2. The formation effect is four times the enforcement effect (which is zero). Standard evaluation gives benefit $= 0$. The formation model gives benefit $= 0.04 \times N_{treated} \times$ (income gain per unit $\Delta\theta$).*

2.5 Operator 5: Institutional Design (Change the Rules of Edge Formation)

Definition 2.9 (Institutional Operator)**.** *The **institutional operator** $\mathcal{L}_{inst}$ modifies the parameters of the edge creation/destruction process from Chapter 13:*

$$\mathcal{L}_{inst} : (p_0, p_1, p_2, p_3, \delta_0, \delta_1) \to (p'_0, p'_1, p'_2, p'_3, \delta'_0, \delta'_1). \tag{14}$$

Unlike the other operators, which add or remove specific edges, the institutional operator changes the rules by which edges form and break.

Example 2.10 (Property Rights as Institutional Operator)**.** *Secure property rights reduce the destruction rate of production edges: $\delta_1 \downarrow$ (profitable links are less likely to be severed by expropriation). This makes cross-block investment less risky, which increases the creation rate of cross-block edges ($p_2 \uparrow$, because the surplus from connection is now more likely to be realized). The institutional operator does not directly add any edge. It changes the dynamics of edge formation, and the network evolves toward higher Γ over time.*

3 The Sequencing Problem

Theorem 3.1 (Optimal Sequencing)**.** *When the budget allows multiple interventions over time, the optimal sequence of operators satisfies:*

$$\mathcal{L}_t^* = \underset{\mathcal{L} \in \{ISI,\, infra,\, HC,\, info,\, inst\}}{\arg\max} \frac{\Delta\Gamma(\mathcal{L} \mid W_t)}{cost(\mathcal{L})}, \tag{15}$$

with the dynamic constraint that each intervention changes W_t (and therefore changes the marginal effect of subsequent interventions).

The optimal sequence is generally:

1. ***First, information policy*** *($\mathcal{L}_{info}$): cheapest per unit of $\Delta\Gamma$ (because it operates on W^I, which has the highest formation multiplier), and it relaxes the political constraint by shifting preferences toward structural change.*

2. ***Second, institutional reform*** *($\mathcal{L}_{inst}$): reduces the destruction rate of future edges, making subsequent investments more durable.*

3. ***Third, education/training*** *($\mathcal{L}_{HC}$): dual operator, creates edges in both W^P and W^I.*

4. ***Fourth, infrastructure*** *($\mathcal{L}_{infra}$): creates physical W^P edges. Expensive, but by this point the institutional and informational preconditions are in place.*

5. ***Fifth, ISI/industrial policy*** *($\mathcal{L}_{ISI}$): the most expensive and politically fraught, but by this point the political coalition has shifted (because W^I changed preferences in step 1, which changed coalition incentives per Chapter 14).*

Key Structural Insight

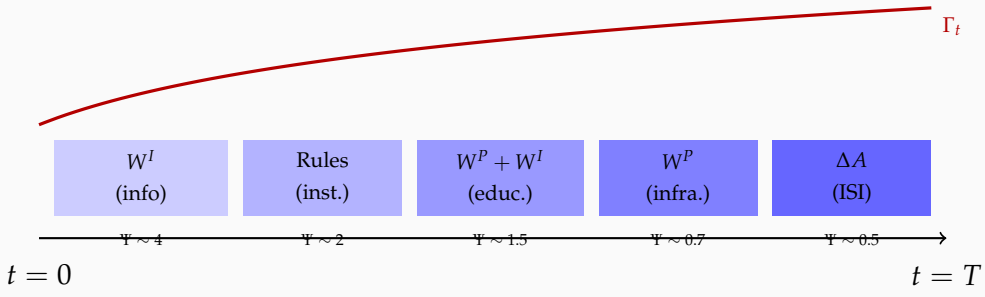
The optimal sequencing theorem inverts the conventional wisdom. The standard approach starts with the "hard" infrastructure and industrial policy, then adds the "soft" information and institutional elements later. The formation model says the opposite: start with information (change what people see), then institutions (change the rules), then education (change what people can do), then infrastructure (change the physical connections), and finally industrial policy (change the production structure).

The reason is the formation amplifier. Each step's effectiveness depends on the Ψ ratio: information interventions have $\Psi \sim 3$–5; institutional reforms have $\Psi \sim 1$–2; infrastructure has $\Psi \sim 0.5$–1. Starting with the high-Ψ interventions produces the preference changes that make the low-Ψ interventions politically feasible and economically effective.

This is why South Korea's sequence worked (education $\rightarrow$ institutions $\rightarrow$ industrial policy) and Bolivia's gas nationalization was insufficient (industrial policy without prior information/institutional change). The nationalization changed W^P but not W^I: the structure changed, but the preferences didn't follow because the formation preconditions weren't in place.

Visualization:

Optimal sequencing: Ψ-ordered operators



Operators ordered by Ψ: highest-Ψ first. The red curve shows Γ_t rising — steeply at first (cheap information interventions with large formation multipliers) and more gradually later (expensive industrial policy with smaller multipliers). Start soft, finish hard.

4 The Product Space as an Operator Map

The product space (Hidalgo et al., 2007) can now be interpreted as a map of feasible operators on W^P.

Proposition 4.1 (Product Space as Feasibility Constraint)**.** *The proximity matrix $\phi_{pp'}$ (co-export probability between products) defines the feasibility of ISI operators: the probability that adding domestic production of product p' succeeds is increasing in $\sum_p M_{cp}\phi_{pp'}$ (the relatedness density). Products in the dense core of the product space (machinery, chemicals, electronics) are feasible targets for economies that already produce related products. Products on the sparse periphery (minerals, agricultural commodities) offer few diversification paths.*

The operator-design interpretation: the product space IS the graph of $\partial\Gamma/\partial W^P_{ij}$ for the global production network. Dense-core products have high $\partial\Gamma/\partial W^P$ because adding them creates many cross-sector linkages. Peripheral products have low $\partial\Gamma/\partial W^P$ because adding them connects to few other products.

Example 4.2 (Bolivia in the Product Space)**.** *Bolivia's current export basket: natural gas, zinc, gold, soy, quinoa — all on the sparse periphery of the product space. Relatedness density to the dense core (machinery, electronics, pharmaceuticals): $\omega_{c,core} \approx 0.08$ (among the lowest in Latin America).*

Conventional ISI: *Target products adjacent in the product space — gas processing, zinc smelting, soy flour. These increase domestic value added but do not connect to the dense core. They add edges within the peripheral block of W^P. $\Delta\Gamma \approx 0.02$. Modest.*

Leapfrog strategy: *Target products in the dense core — pharmaceuticals, automotive parts, food processing machinery. These are distant in the product space (low relatedness density). The probability of success is low unless accompanied by massive human capital investment ($\mathcal{L}_{HC}$) and information rewiring ($\mathcal{L}_{info}$). But if successful: $\Delta\Gamma \approx 0.15$. Transformative.*

The sequencing solution: *(1) Information policy first: expose Bolivian engineers and technicians to the dense core (scholarships, technical exchanges, FDI in assembly operations). (2) Then education: build the capabilities (vocational training in the target sectors). (3) Then infrastructure: connect the production zones. (4) Then ISI: tariff protection for infant industries with cross-block linkages. This is the South Korean sequence, adapted to Bolivian conditions.*

5 The Optimal ΔW Under Political Constraints

Theorem 5.1 (Politically Constrained Optimal Operator)**.** *The optimal structural intervention under the political constraint from Chapter 14 is:*

$$\Delta W^* = \underset{\Delta W \in \mathcal{F}(S^*)}{\arg\max}\ \Gamma(W + \Delta W), \tag{16}$$

where S^* is the winning coalition and $\mathcal{F}(S^*)$ is its feasibility set. The solution differs from the unconstrained optimum because $\mathcal{F}(S^*)$ excludes structural changes that reduce any coalition member's power:

$$\Delta\pi_i(W, \Delta W) \geq 0 \quad \textit{for all } i \in S^*. \tag{17}$$

The politically constrained optimum is generically less than the unconstrained optimum: the coalition sacrifices some $\Delta\Gamma$ to protect its members' power. The gap between constrained and unconstrained is the ***political cost of structural reform****.*

Proposition 5.2 (The Political Cost)**.** *The political cost of structural reform is:*

$$C_{political} = \Gamma^*_{unconstrained} - \Gamma^*_{constrained} \geq 0. \tag{18}$$

This cost is:

1. ***Large when power is concentrated****: the winning coalition is small and powerful, and protecting each member's π_i constrains the feasible ΔW severely.*
2. ***Small when power is equal****: the winning coalition is broad (near the grand coalition), and the political constraint is weak.*
3. ***Zero when the efficient W also increases every agent's power****: this happens only if W is already doubly stochastic (in which case no structural reform is needed) or if the reform is Pareto-improving in both allocation and power (rare).*

Intuition: Why structural reform is slow

The political cost theorem explains why structural reform is slow even when the social returns are enormous. The unconstrained optimum might be $\Delta\Gamma = +0.20$ (a road connecting the gas enclave to the traditional economy). But the gas/government coalition blocks the road because it would reduce their power: $\Delta\pi_{\text{gas}} < 0$. The constrained optimum is $\Delta\Gamma = +0.05$ (a road that connects two urban areas, acceptable to the coalition). The gap of 0.15 is the political cost.

The only way to close the gap is to change the coalition. Chapter 14 showed that coalitions change when $\boldsymbol{\pi}$ changes. And $\boldsymbol{\pi}$ changes slowly because W changes slowly (Chapter 13). The circle: W is hard to change because the powerful block change $\rightarrow$ the powerful are powerful because of $W \rightarrow W$ is hard to change. Breaking the circle requires an exogenous shock, an information intervention that shifts preferences and coalition incentives, or the slow accumulation of cross-block edges that eventually tips the power balance.

6 Summary

1. The **operator design problem** maximizes Γ (or $-\text{Var}(\pi_i)$, or output multiplier) subject to a budget and row-stochasticity constraints. The first-order solution targets the edge with the largest $\partial\Gamma/\partial W_{ij}$.

2. **Five operator types**: ISI (replace foreign with domestic edges), infrastructure (physical edges), education (dual $W^P + W^I$ edges), information policy (W^I only), and institutional reform (change the rules of edge dynamics).

3. **ISI that works** creates cross-block edges (modern $\leftrightarrow$ traditional). ISI that fails creates within-block edges (deepens the modern sector without connecting it). The spectral-gap criterion distinguishes them.

4. **Optimal sequencing** is Ψ-ordered: start with high-Ψ operators (information, institutions, education) that are cheap and create the preconditions for low-Ψ operators (infrastructure, industrial policy).

5. The **product space** is the operator map for W^P: it shows which edges are feasible (high relatedness density) and which are transformative (connect to the dense core).

6. The **political constraint** reduces the feasible $\Delta\Gamma$: the winning coalition protects its members' power. The political cost is $\Gamma^*_{\text{unconstrained}} - \Gamma^*_{\text{constrained}}$.

7 Exercises

15.1 Optimal edge selection. For a 4×4 block-diagonal W^P (two blocks of 2): compute $\partial\Gamma/\partial W^P_{ij}$ for all (i, j) pairs. Which edge maximizes $\Delta\Gamma$ per unit of budget? Is it within-block or cross-block?

15.2 ISI-A vs. ISI-B. Reproduce Example 2.3: for a given budget $B = 0.10$, compute the effect on Γ, $\text{Var}(\pi_i)$, and the Leontief multiplier of (a) adding the edge within the modern block, (b) adding the edge from modern to traditional. Which dominates?

15.3 The smartphone operator. Reproduce Example 2.8: for a two-block economy, add $\Delta W^I = 0.05$ cross-block edges. Simulate the formation dynamics for 5 generations. Compute the steady-state $\Delta\theta$ and Δx. What is Ψ?

15.4 Sequencing. For a 6×6 Bolivia model (gas, urban, rural blocks), compare:

(a) Sequence A: ISI first, then education, then information.

(b) Sequence B: Information first, then education, then ISI.

(c) Simulate both sequences for 10 periods. Plot Γ_t for each. Which sequence reaches $\Gamma = 0.30$ first?

15.5 Bolivia in the product space. Using OEC data (`oec.world`) for Bolivia:

(a) List Bolivia's top 10 exports and their position in the product space (core vs. periphery).

(b) Compute relatedness density to three target products: lithium batteries, food processing machinery, pharmaceutical intermediates.

(c) For each target: what capability edges (ΔW^P) are needed? What information edges (ΔW^I)?

(d) Estimate the sequencing cost for reaching each target (order of magnitude).

15.6 Hirschman vs. spectral gap. For the Bolivia I-O table (from Exercise 1.7):

(a) Compute Hirschman's backward linkages (column sums of Leontief inverse).

(b) Compute $\partial\Gamma/\partial W^P_{ij}$ for all edges.

(c) Do the two criteria agree on where to invest? Where do they disagree?

(d) When would Hirschman's criterion lead to the wrong investment?

15.7 Political cost. For the $N = 4$ Bolivia model: the winning coalition is $\{$Gas, Government$\}$ ($\pi_1 + \pi_2 = 0.65$). Compute (a) the unconstrained optimal ΔW and Γ^*_{unc}, (b) the constrained optimal (subject to $\Delta\pi_1 \geq 0, \Delta\pi_2 \geq 0$) and Γ^*_{con}. What is the political cost? What coalition change would reduce it?

Mathematical Appendices

Appendix 15A: Sensitivity of Γ to Edge Perturbations

Setup. Let W have eigenvalues $1 = \lambda_1 > |\lambda_2| \geq \cdots \geq |\lambda_N|$. The spectral gap is $\Gamma = 1 - |\lambda_2|$. We want $\partial\Gamma/\partial W_{ij}$.

First-order perturbation. For a small perturbation ΔW that preserves row-stochasticity:

$$\Delta\lambda_k \approx \mathbf{u}_k' \Delta W \mathbf{v}_k, \tag{19}$$

where $\mathbf{u}_k$, $\mathbf{v}_k$ are the left and right eigenvectors of W for eigenvalue λ_k, normalized so $\mathbf{u}_k'\mathbf{v}_k = 1$.

For a single-edge perturbation $\Delta W_{ij} = \varepsilon$ (and $\Delta W_{i\ell} = -\varepsilon/(N-1)$ for $\ell \neq j$ to maintain row-stochasticity):

$$\frac{\partial \lambda_k}{\partial W_{ij}} = u_{k,i} \cdot v_{k,j} - \frac{1}{N-1} \sum_{\ell \neq j} u_{k,i} \cdot v_{k,\ell}. \tag{20}$$

Sensitivity of Γ. Since $\Gamma = 1 - |\lambda_2|$:

$$\frac{\partial \Gamma}{\partial W_{ij}} = -\text{sign}(\lambda_2) \frac{\partial \lambda_2}{\partial W_{ij}} = -\text{sign}(\lambda_2) \left[u_{2,i} v_{2,j} - \frac{u_{2,i}}{N-1} \sum_{\ell \neq j} v_{2,\ell} \right]. \tag{21}$$

Interpretation. The sensitivity is large when $u_{2,i}$ is large (agent i is important in the second left eigenvector — she is a "boundary" agent between blocks) and $v_{2,j}$ is extreme (agent j is at the far end of the second right eigenvector — she is deep in one block). The optimal edge connects a boundary agent to a deep-block agent.

Appendix 15B: Constrained Optimization on the Stochastic Matrix Space

The constraint set. The space of $N \times N$ row-stochastic nonnegative matrices is a convex polytope: $\mathcal{W} = \{W \geq 0 : W\mathbf{1} = \mathbf{1}\}$. The operator design problem is a constrained optimization on $\mathcal{W}$.

Projected gradient. The gradient of $\Gamma(W)$ may point outside $\mathcal{W}$ (violating nonnegativity or row-stochasticity). The projected gradient method iterates:

$$W_{t+1} = \text{Proj}_{\mathcal{W}} \left(W_t + \eta \nabla_W \Gamma(W_t) \right), \tag{22}$$

where $\text{Proj}_{\mathcal{W}}$ projects onto $\mathcal{W}$ by (i) setting negative entries to zero and (ii) normalizing each row to sum to 1.

Sinkhorn projection. When the target is doubly stochastic (the welfare-theorems benchmark), Sinkhorn's algorithm (alternating row and column normalization) converges to the closest doubly stochastic matrix in KL-divergence.

Structuralist Case Study: Tavares's ISI as Structural Engineering

Maria da Conceição Tavares (1930–2024) was a Portuguese-born Brazilian economist who became one of the most influential structuralist thinkers of the twentieth century. Her 1972 essay "Auge e declínio do processo de substituição de importações no Brasil" (published in her collection *Da substituição de importações ao capitalismo financeiro*) provided the definitive analytical autopsy of Brazilian ISI — not as a policy failure but as a **specific operator applied to W^P that succeeded at its immediate objective and failed at its structural one**.

Tavares's argument has three parts that map directly onto this chapter's framework.

Part 1: ISI was a response to a balance-of-payments constraint, not a development strategy. Tavares argued that Brazil's ISI was not a deliberate plan to industrialize but a "forced march" triggered by external bottlenecks: the Great Depression collapsed export revenues, WWII interrupted imports, and the Korean War destabilized commodity prices. Each crisis forced the domestic production of goods previously imported. In operator-design notation: each crisis was an exogenous edge destruction in W^P_{global} (severed import edges), which triggered compensating edge creation in W^P_{domestic} (new domestic supply chains). The ISI operator $\mathcal{L}_{\text{ISI}}$ was applied reactively, not optimally.

Part 2: ISI created new blocks without connecting them. The import-substitution process successfully built a modern industrial sector (automobiles, steel, chemicals, machinery). But — and this is Tavares's crucial insight — the new industrial sector connected primarily to *itself* and to the global supply chain for capital goods, not to the pre-existing traditional economy. In operator-design terms: ISI applied $\mathcal{L}_{\text{ISI}}$ of type A (within-block), not type B (cross-block). The modern block densified internally ($W^P_{M,M}$ increased), but the edges between the modern and traditional blocks ($W^P_{M,T}$) remained dark. Pinto's structural heterogeneity was not resolved by ISI — it was *replicated at a higher level of complexity*. The tripartite economy (modern/intermediate/traditional) became a quadripartite economy (ISI-modern/export-modern/intermediate/traditional).

Part 3: The "decline" of ISI was a spectral property. Tavares showed that ISI's growth rate declined not because of "inefficiency" (the neoliberal critique) but because the domestic Leontief multiplier converged to a limit: once the easy substitutions were made (consumer goods, light industry), the remaining imports (capital goods, intermediate inputs) required capabilities that could only be acquired through cross-block linkages that ISI had not created. The Leontief inverse $(I - \gamma W^P)^{-1}$ had bright entries *within* the modern block but dark entries *between* the modern and traditional blocks. The output multiplier plateaued because the cross-block entries were near zero. In this chapter's notation: ISI maximized within-block Γ without increasing total Γ.

Tavares's argument (1972)	**Formal object**	**This chap**
ISI as forced response to crises	Reactive $\mathcal{L}_{\text{ISI}}$, not optimal	Def. 2.1
New blocks without cross-links	ISI-A (within-block), not ISI-B (cross)	Ex. 2.3
Structural heterogeneity replicated	Γ_{domestic} unchanged despite new block	Prop. 2.2
Multiplier plateau	$(I - \gamma W^P)^{-1}$ converges with dark cross-entries	Sec. 4
"Decline" is spectral saturation	Within-block Γ at maximum, total Γ stagnant	Thm. 1.2
Cross-block ISI needed	ISI-B: modern inputs from traditional sector	Ex. 2.3

Tavares's essay was written in 1963 (first published) and expanded in 1972. She later served as a federal deputy in Brazil and continued to influence economic policy until her death in 2024. Her diagnosis of ISI — that it failed not because it was wrong in principle but because it applied the operator to the wrong entries of the matrix — remains the most precise structural analysis of Latin American industrialization. The operator-design framework of this chapter is, in a sense, the engineering manual that Tavares's diagnosis called for: given that ISI-A fails and ISI-B works, which edges should the operator target, in what sequence, and under what political constraints?

Further Reading

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Optimal network design: Jackson (2008), *Social and Economic Networks*, Chapter 11. For spectral methods in network design: Boyd (2006), "Fastest Mixing Markov Chain on a Graph," *SIAM Review*.

The structuralist tradition on ISI: Tavares (1972), *Da substituição de importações ao capitalismo financeiro*, Rio de Janeiro: Zahar (the primary source). Furtado (1959), *Formação Econômica do Brasil*, Chapters 30–33. Hirschman (1968), "The Political Economy of Import-Substituting Industrialization in Latin America," *QJE*.

South Korea's industrial policy: Amsden (1989), *Asia's Next Giant: South Korea and Late Industrialization*. Wade (1990), *Governing the Market*. For the reciprocal control mechanisms (the state disciplines firms, not just subsidizes them): Amsden's concept maps directly to $\mathcal{L}_{\text{inst}}$ with δ_1 (destruction of inefficient edges) as the enforcement mechanism.

Part V

Three Scales

Chapter 16

The Global Weight Matrix

Center, Periphery, and the Spectral Structure of World Trade

> *"Las ventajas del progreso técnico se han concentrado principalmente en los centros industriales, sin propagarse a la periferia del sistema económico mundial en medida suficiente."*
>
> ["The advantages of technical progress have concentrated principally in the industrial centers, without propagating to the periphery of the world economic system in sufficient measure."]
> — Raúl Prebisch (1949)

> *"Development depends not so much on finding optimal combinations for given resources and factors of production as on calling forth and enlisting for development purposes resources and abilities that are hidden, scattered, or badly utilized."*
> — Albert O. Hirschman, *The Strategy of Economic Development* (1958)

Part V applies the framework to three scales: global (this chapter), national (Chapter 17), and individual (Chapter 18). At each scale, the same mathematics operates: the weight matrix W, the Perron eigenvector $\boldsymbol{\pi}$, the spectral gap Γ, and the formation model. What changes is the data.

This chapter takes W^P_{global}: the weight matrix of international trade, where countries are nodes and trade flows are edges. The spectral structure of this matrix IS Prebisch's center-periphery hypothesis, measured. The Economic Complexity Index (Hidalgo et al., 2009) IS the second eigenvector of a matrix derived from W^P_{global}. Hirschman's linkages ARE the column sums of the global Leontief inverse.

Everything in Chapters 1–15 applies. The only difference is that N is now 200 countries rather than 4 agents, and W is estimated from trade data rather than assumed.

1 The Global Production Network

1.1 From Bilateral Trade to the Weight Matrix

Definition 1.1 (Global Production Weight Matrix)**.** *Let $X_{cc'}$ be the total exports from country c' to country c (in USD). The **global production weight matrix** is:*

$$W^P_{cc'} = \frac{X_{cc'}}{\sum_{c''} X_{cc''}}, \qquad c \neq c'. \tag{1}$$

Entry $W^P_{cc'}$ is the share of country c's imports that comes from country c'. Row-stochastic by construction ($\sum_{c'} W^P_{cc'} = 1$ for each c, where the diagonal captures domestic absorption).

For $N \approx 200$ countries and data from the OECD Inter-Country Input-Output (ICIO) tables or the BACI trade database, W^P_{global} is a 200×200 row-stochastic matrix.

Proposition 1.2 (Properties of W^P_{global})**.** *The empirical W^P_{global} has the following properties:*

1. ***Not doubly stochastic.*** *Column sums vary enormously: the US column sum is ≈ 0.15 (15% of world imports come from the US); Bolivia's is ≈ 0.001. The world does not depend on Bolivia the way Bolivia depends on the world.*

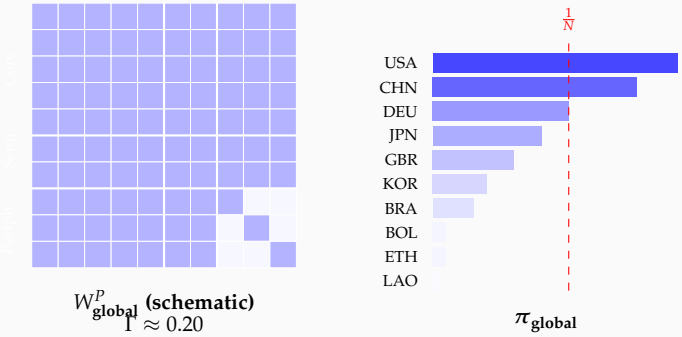
2. ***Core-periphery topology.*** *A dense core of 20–30 industrialized countries (high mutual trade) and a sparse periphery of 150+ countries (trade mostly with the core, rarely with each other).*

3. ***Small spectral gap.*** *$\Gamma(W^P_{global}) \approx 0.15$–$0.25$ depending on the year and data source. Much smaller than the competitive benchmark ($\Gamma = 1$). The global economy is far from the symmetric interaction that the welfare theorems require.*

4. ***Perron eigenvector is concentrated.*** *$\pi_{US} \approx 0.18$, $\pi_{China} \approx 0.15$, $\pi_{Germany} \approx 0.08$. The top 10 countries hold $\approx 65\%$ of total structural power. Bolivia: $\pi_{Bolivia} \approx 0.0005$.*

Visualization:

The global W^P: a 20×20 approximation



Left: schematic global W^P. The upper-left block (core) is bright: dense mutual trade. The lower-right (periphery) is dark except for column entries toward the core: the periphery depends on the core, not on each other. The lower-left rectangle is brighter than the upper-right: asymmetric dependence. Right: the Perron eigenvector. The dashed red line marks $1/N = 0.005$. The US alone has 36 times the "fair share" of structural power. Bolivia is barely visible.

1.2 The Global Leontief Inverse

Definition 1.3 (Global Leontief Inverse)**.** *The **global Leontief inverse** is* $(I - \gamma W^P_{global})^{-1}$*, where* $\gamma \in (0, 1/\rho(W^P))$ *scales the input intensity. Entry* $\ell_{cc'}$ *measures the total output generated in country* c *by a unit increase in final demand in country* c'*, including all rounds of indirect effects through the global supply chain.*

Proposition 1.4 (Hirschman's Linkages as Column Sums)**.**

1. *Country* c'*'s **backward linkage** is* $BL_{c'} = \sum_c \ell_{cc'}$*: the total global output generated by one unit of* c'*'s final demand. High BL: c''s demand stimulates many countries. Low BL: c''s demand stays local.*

2. *Country* c*'s **forward linkage** is* $FL_c = \sum_{c'} \ell_{cc'}$*: how much c's output is used (directly and indirectly) in satisfying final demand everywhere. High FL: c is an essential input provider. Low FL: c produces final goods consumed locally.*

3. *The **key sector** (Rasmussen, 1956) has both* $BL > \bar{BL}$ *and* $FL > \bar{FL}$*: it generates demand upstream AND supplies inputs downstream.*

Example 1.5 (Global Linkages, 2018)**.** *Using OECD ICIO data (67 economies, 36 industries):*

Country	*BL*	*FL*	*Type*
United States	*2.45*	*1.98*	*High/High: key sector (global hub)*
China	*2.68*	*2.31*	*High/High: manufacturing hub*
Germany	*2.51*	*2.12*	*High/High: machinery hub*
Japan	*2.33*	*1.85*	*High/High*
Bolivia	*1.28*	*1.05*	*Low/Low: peripheral enclave*
Ethiopia	*1.15*	*0.92*	*Low/Low*
Average	*1.85*	*1.60*	

The center has BL and FL both above average: it generates demand AND supplies inputs. The periphery has both below average: its demand doesn't propagate, and its output isn't widely used. Bolivia's $BL = 1.28$ *means that one dollar of Bolivian final demand generates only \$1.28 of global output (vs. \$2.68 for China). Bolivia's demand barely ripples beyond its borders.*

2 Economic Complexity as Spectral Measurement

2.1 From Trade Data to the Specialization Matrix

The Economic Complexity Index (ECI) is a dimensionality-reduction method that extracts the leading non-trivial spectral structure from the global trade matrix. It is the empirical counterpart to this book's theoretical framework.

Definition 2.1 (Specialization Matrix and RCA)**.** *Let* X_{cp} *be the export value of product* p *from country* c*. The* ***Revealed Comparative Advantage*** *is:*

$$R_{cp} = \frac{X_{cp}/X_c}{X_p/X}, \tag{2}$$

where $X_c = \sum_p X_{cp}$*,* $X_p = \sum_c X_{cp}$*, and* $X = \sum_{c,p} X_{cp}$*. A country has comparative advantage in product* p *if* $R_{cp} > 1$*. The binary specialization matrix:* $M_{cp} = \mathbf{1}[R_{cp} \geq 1]$*.*

Definition 2.2 (The Proximity Matrix and Product Space)**.** *The* ***proximity*** *between products* p *and* p' *is:*

$$\phi_{pp'} = \frac{\sum_c M_{cp} M_{cp'}}{\max(M_p, M_{p'})}, \tag{3}$$

where $M_p = \sum_c M_{cp}$ *is the ubiquity of product* p*. The* ***product space*** *is the network with products as nodes and* $\phi_{pp'}$ *as edge weights.*

The product space IS W^P at the product level. Its topology — dense core of machinery/chemicals/electronics, sparse periphery of raw materials — is the heatmap from Chapter 3 rendered as a network over 5,000+ products.

2.2 The ECI as the Second Eigenvector

Definition 2.3 (Economic Complexity Index)**.** *Define the country-country matrix:*

$$\tilde{M}_{cc'} = \sum_p \frac{M_{cp} M_{c'p}}{M_c \cdot M_p}, \tag{4}$$

where $M_c = \sum_p M_{cp}$ is diversity. $\tilde{M}_{cc'}$ measures the similarity between countries c and c' based on their shared products, weighted by product ubiquity. This matrix is row-stochastic.

*The **ECI** is the second eigenvector of $\tilde{M}$:*

$$\tilde{M}\mathbf{v}_2 = \lambda_2 \mathbf{v}_2, \qquad ECI_c = \frac{v_{2,c} - mean(v_2)}{stdev(v_2)}. \tag{5}$$

The first eigenvector is trivial ($\mathbf{1}$, since $\tilde{M}$ is row-stochastic). The second eigenvector IS the leading non-trivial spectral structure — the direction along which the global W^P most departs from the competitive benchmark $\frac{1}{N}\mathbf{1}\mathbf{1}'$.

Theorem 2.4 (ECI and the Spectral Gap)**.** *The ECI satisfies:*

1. *ECI_c is proportional to the country's position along the principal axis of structural differentiation in global trade. High ECI = diverse, complex exports (machinery, chemicals, pharmaceuticals). Low ECI = concentrated, simple exports (oil, minerals, raw agriculture).*

2. *The eigenvalue λ_2 associated with the ECI measures the strength of the center-periphery structure: λ_2 close to 1 means the structure is strong (center and periphery are sharply differentiated); λ_2 close to 0 means the structure is weak (countries are similar).*

3. *The spectral gap of $\tilde{M}$ is $\Gamma_{ECI} = 1 - \lambda_2$. This IS the spectral gap from Chapter 6, measured empirically.*

4. *The variance of the ECI across countries is proportional to $\mathrm{Var}(\pi_i)$ of the global W^P: more ECI variance = more structural inequality.*

Intuition: What the ECI measures, in this book's language

The ECI measures where a country sits on the leading eigenvector of W^P_{global} — the axis that captures the most structural information. A country with ECI > 0 is in the bright upper-left block of the global heatmap (the core). A country with ECI < 0 is in the dim lower-right block (the periphery). The distance from zero is the distance from the competitive benchmark.
Japan (ECI ≈ 2.3) is deep in the core: diversified, complex, densely connected. Bolivia (ECI ≈ -0.8) is on the periphery: concentrated, simple, sparsely connected. The gap between them is a spectral property of the global matrix.

2.3 ECI Predicts Growth: A Structural Explanation

Proposition 2.5 (Why ECI Predicts Growth)**.** *Hausman, Hidalgo et al. (2009) showed that ECI predicts future GDP growth, controlling for current GDP. The structural explanation:*

1. *High ECI $\Leftrightarrow$ large domestic $\Gamma(W^P)$ $\Leftrightarrow$ strong inter-sectoral linkages $\Leftrightarrow$ growth propagates across sectors (Chapter 3).*
2. *High ECI $\Leftrightarrow$ diverse product portfolio $\Leftrightarrow$ many independent signals in W^I $\Leftrightarrow$ high N_{eff} $\Leftrightarrow$ wise preference formation (Chapter 8).*
3. *High ECI $\Leftrightarrow$ dense product space position $\Leftrightarrow$ many feasible diversification paths (Chapter 13) $\Leftrightarrow$ structural change is self-reinforcing.*

The standard explanation: "ECI proxies for human capital and institutions." The structural explanation: ECI measures the spectral gap of W^P, which governs growth propagation, information quality, and diversification feasibility simultaneously. Human capital and institutions are inputs that help achieve high ECI; but the mechanism through which they generate growth IS the spectral structure of W^P.

3 ECI Predicts Inequality: The Casillero Vacío

Proposition 3.1 (ECI and Inequality — Hartmann et al. (2017))**.** *Countries with high ECI have lower income inequality (Gini), controlling for GDP per capita. The relationship is robust to multiple specifications and data sources.*

Theorem 3.2 (The Structural Explanation for the ECI-Inequality Link)**.** *From the welfare loss decomposition (Chapter 2):*

$$L_{power} = \frac{\kappa N}{2} \text{Var}(\pi_i)\Phi^2 \leq \frac{\kappa \Phi^2}{2} \frac{1-\Gamma}{\Gamma}. \tag{6}$$

High ECI $\Rightarrow$ high Γ $\Rightarrow$ low upper bound on $\text{Var}(\pi_i)$ *$\Rightarrow$ low power inequality $\Rightarrow$ low allocation inequality $\Rightarrow$ low income inequality.*

This is the mechanism that the product-space literature documents empirically but does not explain theoretically. The spectral gap bound from Chapter 2, Proposition 2.6, IS the explanation: Γ *bounds* $\text{Var}(\pi_i)$*, which bounds the welfare loss, which bounds inequality.*

Prebisch Connection: Fajnzylber's casillero vacío

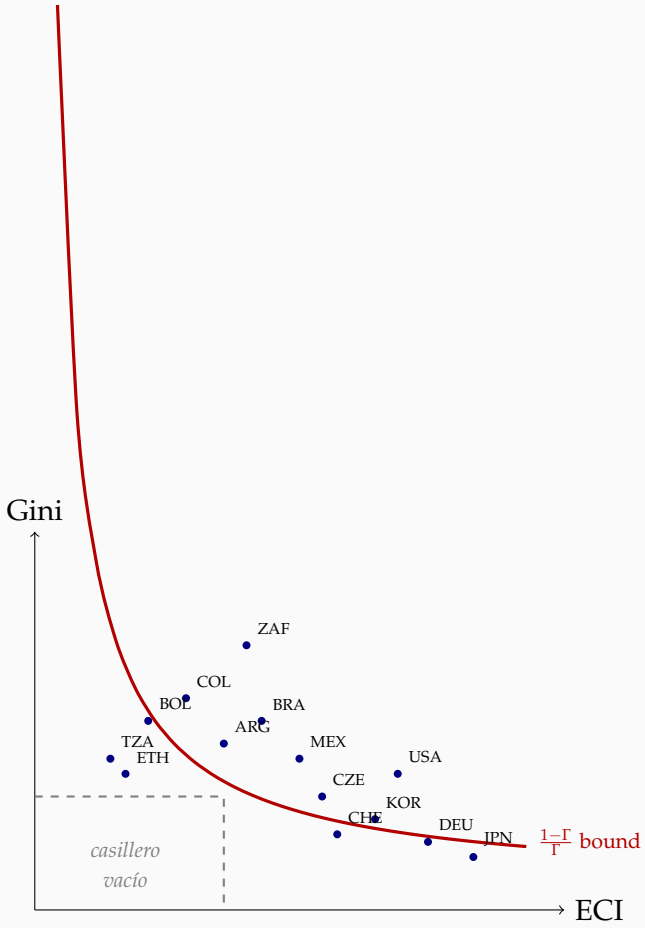
In 1990, Fernando Fajnzylber published "Industrialización en América Latina: de la 'caja negra' al 'casillero vacío'" documenting that no Latin American country occupied the quadrant of high growth AND low inequality. The "casillero vacío" (empty box) was the upper-left cell of a 2×2 table (growth/inequality).

In this chapter's notation: the casillero vacío IS the region of the (ECI, Gini) plane that is structurally inaccessible to low-ECI countries. Low ECI $\Rightarrow$ low Γ $\Rightarrow$ high $\text{Var}(\pi_i)$ $\Rightarrow$ high Gini. You cannot have low inequality with low complexity. The spectral gap bound is the fence around the empty box.

This is why inequality in Latin America is structural, not just distributional: you cannot reduce it with transfers (Chapter 2, the Transfer Trap) because the power structure ($\boldsymbol{\pi}$) is determined by W^P, which has low Γ. To enter the casillero vacío, you must raise ECI — change the spectral structure of W^P through the operators of Chapter 15. This is what South Korea, Taiwan, and Ireland did. This is what Bolivia has not.

Visualization:

The casillero vacío as a spectral bound



The red curve is the spectral bound: low ECI (low Γ) implies high Gini. The dashed box is Fajnzylber's casillero vacío: no country achieves low complexity AND low inequality. Bolivia is trapped: low ECI, moderate-to-high Gini, outside the empty box. Japan, Germany, Korea: high ECI, low Gini, inside the box. The bound IS the reason the box is empty.

4 Relatedness and the Global W^P Dynamics

4.1 The Principle of Relatedness as Global Path Dependence

Definition 4.1 (Relatedness Density)**.** *Country c's **relatedness density** to product p is:*

$$\omega_{cp} = \frac{\sum_{p'} M_{cp'} \phi_{pp'}}{\sum_{p'} \phi_{pp'}}. \tag{7}$$

This is the fraction of related products that c already produces. High ω_{cp}: country c has the capabilities to produce p. Low ω_{cp}: the capabilities are missing.

Proposition 4.2 (The Principle of Relatedness as Chapter 13 Applied Globally)**.** *The probability that country c develops RCA in product p (enters the product) is increasing in ω_{cp} (Hidalgo et al., 2007). This IS the path-dependence theorem of Chapter 13: edge creation is proximity-dominated ($p_1 \gg p_2, p_3$), so new edges form near existing edges. Countries diversify into products related to their current exports, not into random products.*

The global consequence: countries on the dense core of the product space (high ω_{cp} for many products) diversify rapidly, increasing their ECI, which increases their Γ, which reduces their inequality. Countries on the sparse periphery (low ω_{cp} for core products) diversify slowly or not at all, remaining trapped with low ECI, low Γ, and high inequality.

Example 4.3 (Relatedness Density: Bolivia vs. South Korea, 2018)**.**

Target product	$\omega_{Bolivia}$	ω_{Korea}
Lithium batteries	*0.04*	*0.52*
Pharmaceutical intermediates	*0.03*	*0.41*
Automotive parts	*0.02*	*0.68*
Processed food machinery	*0.08*	*0.45*
Average (all core products)	*0.06*	*0.55*

Korea's average relatedness to core products is 9 times Bolivia's. Korea can diversify into any of these products with high probability; Bolivia cannot, without the sequenced operators from Chapter 15 (information → institutions → education → infrastructure → industrial policy).

5 The Terms of Trade: A Spectral Property

Theorem 5.1 (Prebisch-Singer as a Spectral Result)**.** *In the global bargaining equilibrium (Chapter 7 applied to countries):*

$$x_c^* = \frac{\Omega_{global}}{N} + \frac{1}{\kappa}\left(\pi_c(W^P_{global}) - \frac{1}{N}\right). \tag{8}$$

The terms of trade of country c are determined by π_c: countries with high structural power ($\pi_c > 1/N$) receive more than their "fair share" of global output; countries with low power ($\pi_c < 1/N$) receive less.

The Prebisch-Singer hypothesis — that the terms of trade of primary commodity exporters deteriorate over time — is the dynamic version: as the global W^P evolves (Chapter 13), the core's π increases (the core adds dense linkages) and the periphery's π decreases (the periphery adds only sparse commodity edges). The terms-of-trade deterioration is the Perron eigenvector shifting toward the core.

Example 5.2 (Bolivia's Terms of Trade, 1960–2023)**.** *Bolivia's commodity export prices (gas, tin, zinc, soy) have experienced long-run terms-of-trade deterioration interspersed with commodity booms (1970s, 2003–2014). During booms: $x_{Bolivia}$ rises, but $\pi_{Bolivia}$ does not change (commodity booms increase $\boldsymbol{\omega}$ but not W^P — the edges don't change, only the prices on existing edges). After booms: $x_{Bolivia}$ returns to the pre-boom trajectory because $\boldsymbol{\pi}$ never changed.*

The gas nationalization of 2006 changed some edges of W^P (rerouted gas revenue from MNCs to the treasury). But it did not add cross-sector domestic linkages. Bolivia's ECI in 2006: -0.85. In 2018: -0.80. Twelve years of gas revenue redistributed via transfers, almost zero structural change.

6 The Three Networks at Global Scale

The global economy has three networks, just as the domestic economy does.

Proposition 6.1 (Global Three-Network Structure)**.**

1. *W^P_{global}: the trade/production network. Measured by ICIO, BACI, or COMTRADE data. Core-periphery topology. ECI is its second eigenvector.*

2. *W^I_{global}: the global information network. Measured (imperfectly) by media flows, academic citation patterns, policy diffusion. Hub-spoke topology centered on the US and Europe. The Washington Consensus as DeGroot equilibrium (Chapter 8). N_{eff} of global economic policy: approximately 3–5 (IMF, World Bank, US Treasury, ECB, a few think tanks).*

3. *W^U_{global}: the global utility network. Harder to measure. Includes foreign aid (revealed altruism), migration patterns (revealed social bonds), and cultural consumption (whose stories do you watch?). Also hub-spoke, correlated with W^I.*

*The three networks are **aligned at the global scale**: the countries that are central in W^P (trade hubs) are also central in W^I (information hubs) and W^U (cultural hubs). This alignment maximizes the concentration of the composite Perron eigenvector (Chapter 5): the center is powerful in all three dimensions simultaneously.*

Key Structural Insight

The global spectral structure tells the same story at all three layers. W^P: the center produces the goods. W^I: the center produces the ideas. W^U: the center produces the culture. The periphery consumes all three and produces the raw materials. The Perron eigenvector is aligned across layers. The structural welfare theorem (Chapter 12) applies: higher global Γ (more symmetric trade, information, and cultural exchange) would be robustly welfare-improving. But the current structure reproduces itself through the formation model (Chapter 10): peripheral preferences are formed under a W^I that transmits center consumption patterns and economic doctrines. The formation loop operates at the global scale exactly as it does at the domestic scale.

7 Summary

1. The **global production weight matrix** W^P_{global} is estimated from trade data. It is not doubly stochastic (column sums vary 100-fold), has core-periphery topology, small spectral gap ($\Gamma \approx 0.20$), and concentrated Perron eigenvector ($\pi_{\text{US}} \approx 0.18$, $\pi_{\text{Bolivia}} \approx 0.0005$).

2. The **ECI** is the second eigenvector of the country-similarity matrix derived from W^P_{global}. It measures a country's position on the principal axis of structural differentiation. High ECI = core. Low ECI = periphery.

3. ECI predicts growth because it proxies for $\Gamma(W^P)$: high Γ = growth propagates. ECI predicts inequality because Γ bounds $\text{Var}(\pi_i)$: the **casillero vacío** is the spectral bound $L_{\text{power}} \leq \kappa\Phi^2(1-\Gamma)/(2\Gamma)$.

4. **Relatedness density** is the empirical measure of proximity-dominated edge creation (Chapter 13). Countries on the dense core diversify rapidly; countries on the sparse periphery are locked in.

5. The **Prebisch-Singer hypothesis** is a spectral result: the core's π increases as the global W^P evolves, and the periphery's decreases. Terms-of-trade deterioration IS the Perron eigenvector shifting.

6. The three global networks (W^P, W^I, W^U) are **aligned**: the center dominates production, information, and culture simultaneously. This alignment maximizes the concentration of composite power.

8 Exercises

16.1 Build the global W^P. Using BACI trade data (available at `cepii.fr`):

(a) Download bilateral trade data for 2018. Aggregate to 50 countries.

(b) Construct $W^P_{cc'}$ (Definition 1.1). Verify row-stochasticity.

(c) Compute column sums. Which country has the largest? Smallest?

(d) Compute $\boldsymbol{\pi}$ (power iteration, Appendix 6A). Plot π_c vs. GDP per capita. What is the correlation?

(e) Compute Γ. Compare with the competitive benchmark $\Gamma = 1$.

16.2 Compute the ECI. Using the Atlas of Economic Complexity (`atlas.cid.harvard.edu`) or OEC (`oec.world`):

(a) Download the ECI ranking for 2018. Identify the top 10 and bottom 10.

(b) Plot ECI vs. GDP per capita. What is the correlation? Is it stronger or weaker than π_c vs. GDP per capita from Exercise 16.1?

(c) Plot ECI vs. Gini coefficient. Verify the casillero vacío: is the lower-left quadrant (low ECI, low Gini) empty?

(d) Where is Bolivia? Where is South Korea? What is the ECI gap?

16.3 Relatedness density. Using OEC product-space data:

(a) Compute Bolivia's relatedness density to: lithium batteries, processed food, pharmaceutical intermediates.

(b) Compare with Chile, Peru, and Colombia.

(c) Which country has the highest average relatedness to core products? Why?

(d) Using Chapter 15's sequencing framework: what operator sequence would Bolivia need to reach $\omega > 0.3$ for lithium batteries?

16.4 Prebisch-Singer. Download Bolivia's terms-of-trade index (World Bank WDI or CEPAL):

(a) Plot the terms-of-trade index, 1960–2023.

(b) Identify the commodity booms (1970s, 2003–2014). During booms: did Bolivia's ECI change?

(c) Decompose the terms-of-trade change into a "price" component ($\Delta\omega$) and a "structural" component ($\Delta\pi$). Which is larger during booms? During busts?

16.5 Global W^I. Use data on academic citations (Scopus) or media reach (Reuters, AP, AFP) to estimate a proxy for W^I_{global}:

(a) Construct a 20×20 matrix where $W^I_{cc'}$ is the share of academic citations in country c that cite papers from country c'.

(b) Compute $\boldsymbol{\pi}^I$. Which countries have the highest epistemic power?

(c) Compute N_{eff}. How many "independent voices" shape the global academic consensus?

(d) Compare $\boldsymbol{\pi}^I$ with $\boldsymbol{\pi}^P$ (from Exercise 16.1). Are they correlated? What does alignment mean for the composite power vector?

16.6 The spectral bound and the casillero vacío. For 50 countries:

(a) Compute Γ_c (the domestic spectral gap from the national I-O table, approximated by ECI as a proxy).

(b) Compute the upper bound on $\text{Var}(\pi_i)$ from Proposition 2.6: $\text{Var} \leq (1-\Gamma)/(N\Gamma)$.

(c) Plot the bound vs. the actual Gini. Does the bound hold? How tight is it?

(d) Which countries are close to the bound (structurally constrained) and which are far below (inequality is lower than their structure requires)?

16.7 Bolivia's structural position. Synthesize all the data from this chapter's exercises:

(a) Bolivia's π_{global}, ECI, relatedness density to core products, terms-of-trade trajectory, and W^I position.

(b) Using the operator-design framework of Chapter 15: design a 20-year structural transformation plan for Bolivia. Which operators, in what sequence, targeting which products?

(c) Estimate the cost (order of magnitude) and the expected ΔECI, $\Delta\Gamma$, ΔGini.

(d) Is the plan politically feasible given Bolivia's current coalition structure (Chapter 14)?

Mathematical Appendices

Appendix 16A: The ECI as a Spectral Method

Derivation. Start with the binary specialization matrix M ($C \times P$). Define diversity $M_c = \sum_p M_{cp}$ and ubiquity $M_p = \sum_c M_{cp}$. The coupled equations:

$$K_c = \frac{1}{M_c} \sum_p M_{cp} K_p, \qquad K_p = \frac{1}{M_p} \sum_c M_{cp} K_c. \tag{9}$$

Substituting: $K_c = \sum_{c'} \tilde{M}_{cc'} K_{c'}$ where $\tilde{M}_{cc'} = \sum_p M_{cp} M_{c'p} / (M_c M_p)$. The solutions are the eigenvectors of $\tilde{M}$. Since $\tilde{M}$ is row-stochastic, the first eigenvector is $\mathbf{1}$ (trivial). The ECI is the second eigenvector, Z-score normalized.

Connection to SVD. The SVD of $M = U\Sigma V'$ gives: $\tilde{M} = U(\Sigma^2/(M_c M_p \text{ scaling}))U'$. The ECI is the second left singular vector of the (appropriately scaled) specialization matrix. This is the connection to dimensionality reduction: the ECI summarizes the leading factor that explains the geography of economic activities.

Connection to this book's Γ. The spectral gap of $\tilde{M}$ is $\Gamma_{\text{ECI}} = 1 - \lambda_2(\tilde{M})$. This is a proxy for the spectral gap of W^P_{global}, filtered through the product-level structure. When Γ_{ECI} is large: the global economy has a dominant mode (one axis of differentiation). When small: multiple competing axes (e.g., center-periphery AND resource-rich vs. resource-poor).

Appendix 16B: The Global Leontief Inverse from ICIO Data

The ICIO table. The OECD Inter-Country Input-Output table provides $A_{cs,c's'}$: the amount of output from sector s' in country c' used as input per unit of output of sector s in country c. This is the global technical coefficients matrix, of dimension $(N_c \times N_s)^2$ (e.g., $67 \times 36 = 2412$ rows and columns).

Aggregation. To obtain the country-level W^P, aggregate over sectors: $W^P_{cc'} = \sum_{s,s'} A_{cs,c's'} \cdot w_{s'}$ where $w_{s'}$ are sectoral output weights. The resulting $N_c \times N_c$ matrix is the one used in this chapter.

Computation. The global Leontief inverse $(I - A)^{-1}$ for the full ICIO table is a 2412×2412 matrix. Modern computers handle this in seconds. The column sums give Hirschman's backward linkages; the row sums give forward linkages.

Structuralist Case Study: Prebisch and Hirschman on the Global W^P

This chapter's case study pairs the two most influential structuralist contributions to the analysis of the global economic structure: Prebisch's center-periphery hypothesis and Hirschman's linkage theory. Together they provide the theoretical foundations for the empirical program of economic complexity.

Prebisch (1949): The global W^P is asymmetric. Prebisch's manifesto asserted that the global economy has a structure — that the pattern of trade is not the symmetric exchange of classical trade theory but an asymmetric system where the center exports manufactures (high linkages, complex, diversified) and the periphery exports primary commodities (low linkages, simple, concentrated). In this chapter's notation: W^P_{global} has core-periphery topology, $\boldsymbol{\pi}_{\text{center}} \gg \boldsymbol{\pi}_{\text{periphery}}$, and Γ is small. The ECI IS the empirical measure of where each country sits on Prebisch's axis.

Prebisch also predicted that the terms of trade of primary exporters would deteriorate over time (the Prebisch-Singer hypothesis). Theorem 5.1 shows that this is a spectral result: as the global W^P evolves, the core adds dense linkages (increasing π_{center}) while the periphery adds only sparse commodity edges (leaving $\pi_{\text{periphery}}$ stagnant or declining). The terms-of-trade trend is the shadow of the Perron eigenvector.

Hirschman (1958): Linkages are the mechanism. Where Prebisch described the asymmetry, Hirschman identified the mechanism: backward and forward linkages. Industries with strong linkages generate demand for their suppliers (backward) and provide inputs for their customers (forward). The development strategy should target high-linkage industries — the "key sectors" whose column sums in $(I - A)^{-1}$ are largest.

In this chapter's notation: Hirschman's backward linkages are the column sums of the Leontief inverse (Proposition 1.4). His key-sector criterion is the condition $BL_p > \bar{BL}$ AND $FL_p > \bar{FL}$. The product space (Hidalgo et al., 2007) is the network visualization of the proximity matrix among all products — which is Hirschman's idea of "related industries" rendered over 5,000 products using export data.

The synthesis: ECI = Prebisch's position + Hirschman's mechanism. The ECI measures Prebisch's center-periphery position (where a country sits on the second eigenvector). The product space measures Hirschman's linkage structure (which products are connected to which, and how densely). Relatedness density measures the feasibility of Hirschman's strategy: can this country reach the high-linkage products from where it currently stands?

Concept	**Prebisch (1949)**	**Hirschman (1958)**
Core claim	Global economy is asymmetric	Linkages drive development
Formal object	$\boldsymbol{\pi}_{\text{center}} \gg \boldsymbol{\pi}_{\text{periphery}}$	$BL_p = \sum_c \ell_{cp}$
Empirical measure	ECI (second eigenvector)	Column sums of $(I - A)^{-1}$
Policy implication	Structural change, not just trade	Target high-linkage sectors
This chapter	Thm. 2.4, Thm. 5.1	Prop. 1.4, Ex. 1.5
Product space role	Country position on the map	The map itself

Prebisch wrote from Buenos Aires in 1949. Hirschman wrote from Bogotá in 1958 (he had served as an economic advisor in Colombia). Neither had trade data for 5,000 products, dimensionality-reduction algorithms, or the spectral theory needed to formalize their intuitions. What they had was the intuition that the global economy has a structure, that the structure determines who benefits from trade, and that the structure can be changed by policy. The ECI and the product space are the empirical realization of their program. This chapter gives both programs their eigenvalues.

Further Reading

Economic complexity: Hidalgo and Hausmann (2009), "The Building Blocks of Economic Complexity," *PNAS*. Hausmann et al. (2014), *The Atlas of Economic Complexity*, MIT Press. Hidalgo (2021), "Economic Complexity Theory and Applications," *Nature Reviews Physics*. Data: `oec.world` and `atlas.cid.harvard.edu`.

Relatedness and the product space: Hidalgo et al. (2007), "The Product Space Conditions the Development of Nations," *Science*. Hausmann and Klinger (2007), "The Structure of the Product Space."

ECI and inequality: Hartmann et al. (2017), "Linking Economic Complexity, Institutions, and Income Inequality," *World Development*. Fajnzylber (1990), "Industrialización en América Latina: de la 'caja negra' al 'casillero vacío' ," *Cuadernos de la CEPAL* 60.

Global input-output: OECD ICIO tables (`oecd.org/sti/ind/inter-country-input-output-t`
Timmer et al. (2015), "An Illustrated User Guide to the World Input-Output Database," *Review of International Economics*.

The structuralist tradition: Prebisch (1949), the manifesto. Hirschman (1958), *The Strategy of Economic Development*, Chapters 6–7. Singer (1950), "The Distribution of Gains between Investing and Borrowing Countries," *AER*.

Chapter 17

Domestic Structural Diagnostics

Reading the National Heatmap

> *"La heterogeneidad estructural se manifiesta en la coexistencia de actividades económicas con niveles de productividad correspondientes a distintas épocas históricas."*
>
> ["Structural heterogeneity manifests itself in the coexistence of economic activities with productivity levels corresponding to distinct historical epochs."]
> — Aníbal Pinto, "Heterogeneidad estructural" (1970)

> *"El casillero vacío no es producto del azar ni de la fatalidad: es resultado de la naturaleza del patrón de industrialización vigente."*
>
> ["The empty box is not the product of chance or fate: it is the result of the nature of the prevailing industrialization pattern."]
> — Fernando Fajnzylber, "Industrialización en América Latina" (1990)

Chapter 16 applied the framework at the global scale: the ECI measures where a country sits in the global W^P, and the product space maps the feasible diversification paths. This chapter turns inward: how do you read the structural health of a *national* economy?

The diagnostic toolkit has five instruments, each derived from the weight matrix:

1. The domestic spectral gap $\Gamma(W^P_{\text{domestic}})$: how connected are the sectors?
2. The between-block variance: how unequal are the blocks?
3. The domestic Perron eigenvector π_{domestic}: who has structural power within the country?

4. The domestic $N_{\text{eff}}(W^I)$: how democratic is the information structure?

5. The domestic Ψ ratio: how much does the formation channel matter?

Each instrument is computable from available data: input-output tables, household surveys, media data, and cohort-year panels. Together they provide a structural health check — a way to diagnose where the economy departs from $W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'$ and by how much.

Bolivia is the case study throughout. Not because Bolivia is special, but because it exhibits every structural pathology at measurable scale: an enclave production network, a block-diagonal information structure, a concentrated Perron eigenvector, a large Ψ ratio, and a casillero vacío that is visible in every scatter plot.

1 Instrument 1: The Domestic Spectral Gap

1.1 Estimation from I-O Tables

Definition 1.1 (Domestic Spectral Gap)**.** *Given the domestic input-output matrix A (from INE, CEPAL, or OECD ICIO), construct the domestic production weight matrix:*

$$W^P_{ss'} = \frac{A_{ss'}}{\sum_{s''} A_{ss''}}, \tag{1}$$

*where s, s' index domestic sectors. The **domestic spectral gap** is $\Gamma(W^P_{domestic}) = 1 - |\lambda_2(W^P)|$.*

Example 1.2 (Bolivia's Domestic Γ, 2016)**.** *Using Bolivia's 2016 I-O table from INE (35 sectors aggregated to 12 for tractability):*

Sector	*Output share*	*BL*	*FL*	*Type*
Hydrocarbons	*0.08*	*1.12*	*1.45*	*Forward linkage (input provider)*
Mining	*0.06*	*1.25*	*1.18*	*Moderate*
Agriculture	*0.12*	*1.38*	*1.22*	*Above average*
Manufacturing	*0.15*	*1.65*	*1.48*	*Key sector*
Construction	*0.07*	*1.52*	*1.05*	*Backward linkage (demand)*
Commerce	*0.10*	*1.18*	*1.35*	*Forward linkage*
Transport	*0.08*	*1.42*	*1.30*	*Above average*
Financial services	*0.04*	*1.15*	*1.40*	*Forward linkage*
Government	*0.09*	*1.08*	*1.02*	*Weak linkages*
Other services	*0.12*	*1.20*	*1.10*	*Below average*
Informal	*0.06*	*1.05*	*0.95*	*Peripheral*
Household production	*0.03*	*1.02*	*0.90*	*Peripheral*
Average		*1.25*	*1.20*	

Spectral analysis of the 12×12 W^P*:*

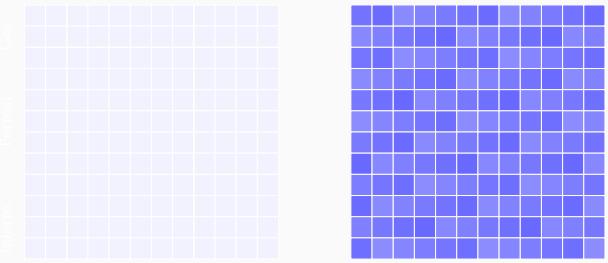
- $\lambda_1 = 1.000$ *(row-stochastic).*
- $\lambda_2 = 0.872$ *(the gas/mining block).*
- $\lambda_3 = 0.815$ *(the agriculture/informal block).*
- $\Gamma = 1 - 0.872 = 0.128$*.*

Bolivia's domestic $\Gamma = 0.13$*: small. The second eigenvalue is close to 1, meaning the production network has two competing modes — a gas/mining/government block and an agriculture/informal block — with weak cross-block linkages. The economy is approximately block-diagonal.*

For comparison: South Korea's $\Gamma \approx 0.45$*; Germany's* $\Gamma \approx 0.52$*; the United States'* $\Gamma \approx 0.48$*.*

Visualization:

Bolivia's domestic W^P: the 12×12 heatmap



Bolivia W^P (2016)
$\Gamma = 0.13$

South Korea W^P (2016)
$\Gamma = 0.45$

Bolivia (left): three visible blocks, dark cross-block entries. The gas/mining block (upper-left) barely connects to the informal block (lower-right). South Korea (right): relatively uniform brightness — dense inter-sectoral linkages, no obvious block structure. Bolivia's Γ is 3.5 times smaller than Korea's.

2 Instrument 2: Between-Block Variance

Definition 2.1 (Between-Block Variance)**.** *Partition the economy into k blocks (identified from the spectral decomposition of W^P or from sectoral groupings). Let $\bar{y}_b$ be the mean output per worker in block b, N_b the employment in block b, and $\bar{y}$ the national average. The **between-block variance** is:*

$$V_B = \sum_{b=1}^{k} \frac{N_b}{N} (\bar{y}_b - \bar{y})^2. \tag{2}$$

*The **between-block share** of total inequality is $\sigma_B = V_B / V_{total}$.*

Proposition 2.2 (Between-Block Share as Structural Inequality)**.** *From Chapter 2, Proposition 2.5: the welfare loss from block disconnection is $L_{block} = \frac{\kappa}{2} N V_B$. The between-block share σ_B measures the fraction of inequality that is **structural** — produced by the disconnection of the blocks — as opposed to **residual** — inequality within blocks that could be addressed by within-block markets.*

When σ_B is large: most inequality is between blocks (structural). Transfers within blocks reduce the residual; only cross-block linkages reduce the structural component.

When σ_B is small: most inequality is within blocks. The economy's problem is not structural heterogeneity but within-sector inequality (which may still have structural causes at a finer scale).

Example 2.3 (Bolivia's Between-Block Inequality, 2019)**.** *Using Bolivia's Encuesta de Hogares 2019. Three blocks: (1) Hydrocarbons/Mining/Government ($N_1 = 0.08N$,*

$\bar{y}_1 = 18{,}500$ *Bs/month), (2) Formal urban* ($N_2 = 0.35N$, $\bar{y}_2 = 5{,}200$ *Bs/month), (3) Informal/Rural* ($N_3 = 0.57N$, $\bar{y}_3 = 1{,}800$ *Bs/month).*

National mean: $\bar{y} = 0.08 \times 18{,}500 + 0.35 \times 5{,}200 + 0.57 \times 1{,}800 = 4{,}326$ *Bs.*

Between-block variance:

$$V_B = 0.08(18{,}500 - 4{,}326)^2 + 0.35(5{,}200 - 4{,}326)^2 + 0.57(1{,}800 - 4{,}326)^2 \quad (3)$$

$$= 0.08 \times 200.8M + 0.35 \times 0.76M + 0.57 \times 6.38M \quad (4)$$

$$= 16.06M + 0.27M + 3.64M = 19.97M. \quad (5)$$

Total variance (from the EH microdata): $V_{total} \approx 32M$.

Between-block share: $\sigma_B = 19.97/32 = 0.62$. *Sixty-two percent of Bolivia's income inequality is between blocks — structural. Transfers within blocks address only 38%.*

For comparison: South Korea's $\sigma_B \approx 0.20$ *(most inequality is within sectors); Brazil's* $\sigma_B \approx 0.45$ *(intermediate); the US's* $\sigma_B \approx 0.25$.

Prebisch Connection: Pinto's heterogeneity measured

Pinto (1970) described structural heterogeneity qualitatively: the coexistence of a "modern pole" with productivity levels comparable to the industrial countries and a "primitive stratum" with colonial-era productivity. The between-block variance puts a number on it.

Bolivia's modern pole (gas/mining/government) has average income 10 times the primitive stratum (informal/rural). The ratio $\bar{y}_1/\bar{y}_3 = 18{,}500/1{,}800 = 10.3$. This is Pinto's 20:1 ratio (he wrote about Latin America as a whole; Bolivia's ratio is "moderate" by regional standards). The between-block share $\sigma_B = 0.62$ says that most of this inequality cannot be addressed by redistribution within blocks. It requires structural change: cross-block linkages that equalize productivity across blocks.

3 Instrument 3: The Domestic Perron Eigenvector

Definition 3.1 (Domestic Power Distribution)**.** *The* ***domestic power distribution*** *is the left Perron eigenvector* $\boldsymbol{\pi}$ *of the domestic composite weight matrix* $W = \alpha_U W^U + \alpha_P W^P + \alpha_I W^I$. *The* ***power Gini*** *is the Gini coefficient of* $\pi_1, \ldots, \pi_N$.

Example 3.2 (Bolivia's Domestic $\boldsymbol{\pi}$)**.** *Approximating* $\boldsymbol{\pi}$ *from the domestic* W^P *(12 sectors):*

Sector	*Employment share*	π_s *(structural power)*
Hydrocarbons	*0.01*	*0.09*
Mining	*0.04*	*0.06*
Government	*0.08*	*0.11*
Manufacturing	*0.12*	*0.14*
Commerce	*0.15*	*0.10*
Agriculture	*0.28*	*0.08*
Informal services	*0.18*	*0.05*
Household production	*0.05*	*0.02*
Total	*1.00*	≈ 1.00

Hydrocarbons employs 1% of the workforce but holds 9% of structural power ($\pi/\text{share} = 9$). Agriculture employs 28% but holds only 8% of structural power ($\pi/\text{share} = 0.29$). The ***power-employment ratio*** π_s/share_s *measures the structural privilege of each sector: values above 1 indicate the sector has more power than its workforce size would warrant.*

Power Gini: ≈ 0.52 *(high). For comparison: South Korea* ≈ 0.28*; Germany* ≈ 0.22*.*

Intuition: What the power-employment ratio means

A gas worker in Bolivia generates $\pi/\text{share} = 9$ times the structural power of an "average" worker. This does not mean the gas worker is 9 times more productive (though she may be). It means that the gas sector's position in W^P gives it disproportionate influence over the entire economy: government revenue depends on gas, the financial sector depends on gas revenue, and foreign exchange depends on gas exports. In the Perron eigenvector, this recursive dependence amplifies the gas sector's weight far beyond its direct employment.

The farmer in the altiplano is the mirror image: she produces food consumed locally, with few downstream linkages. Her sector's position in W^P generates almost no recursive amplification. Her structural power is 0.29 times what her employment share would predict.

This IS the power asymmetry that enters the bargaining allocation (Chapter 7). The gas worker gets $x^*_{\text{gas}} = \Omega/N + (\pi_{\text{gas}} - 1/N)/\kappa > \Omega/N$. The farmer gets $x^*_{\text{farmer}} < \Omega/N$. The gap is proportional to $\pi_{\text{gas}} - \pi_{\text{farmer}} = 0.09 - 0.08 = 0.01$ at the sectoral level — but the within-sector individual-level gap is much larger because the gas sector has few workers (high π per capita) while agriculture has many (low π per capita).

4 Instrument 4: The Domestic $N_{\text{eff}}(W^I)$

Definition 4.1 (Domestic Information Effective Sample Size)**.** *Given a proxy for the domestic information network W^I (from media data, social network data, or education enrollment data), the **domestic** N_{eff} is:*

$$N_{eff}(W^I) = \frac{1}{\sum_j (\pi_j^I)^2}, \tag{6}$$

where $\boldsymbol{\pi}^I$ is the Perron eigenvector of W^I.

Example 4.2 (Bolivia's N_{eff})**.** *Approximating W^I from media concentration data. Bolivia has three main media ecosystems:*

1. *State media (Radio Patria Nueva, Bolivia TV, Cambio): dominant in the altiplano and valleys. $\pi^I \approx 0.35$.*
2. *Private commercial media (Unión, Página Siete, Red Uno): dominant in Santa Cruz and urban centers. $\pi^I \approx 0.40$.*
3. *Community/indigenous media (Radio Pacífica, community radios): rural areas. $\pi^I \approx 0.10$.*
4. *Social media (Facebook, TikTok, WhatsApp): growing, cross-cutting. $\pi^I \approx 0.15$.*

$N_{eff} = 1/(0.35^2 + 0.40^2 + 0.10^2 + 0.15^2) = 1/(0.1225 + 0.16 + 0.01 + 0.0225) = 1/0.315 = 3.17$.

Bolivia's information environment has approximately 3 independent voices. With 11 million people, $N_{eff}/N \approx 3 \times 10^{-7}$. The wisdom-of-crowds efficiency (Chapter 8): the consensus is based on 3 signals, not 11 million. The information structure is highly concentrated.

For comparison: the US has $N_{eff} \approx 15$–20 (diverse media ecosystem, many outlets with significant reach, though social media is reconcentrating). Scandinavian countries: $N_{eff} \approx 8$–12.

5 Instrument 5: The Domestic Ψ Ratio

Definition 5.1 (Domestic Ψ)**.** *The domestic Ψ ratio for a target behavior (female LFP, savings, education enrollment) is estimated from the cohort-year decomposition of Chapter 11:*

$$\hat{\Psi}_{domestic} = \frac{\text{Var}(\hat{\delta}_c)}{\text{Var}(\hat{\alpha}_t)}, \tag{7}$$

where $\hat{\delta}_c$ is the cohort effect and $\hat{\alpha}_t$ is the year effect.

Example 5.2 (Bolivia's Ψ for Self-Employment)**.** *From Chapter 11 (Proposition 11.5): Bolivia's self-employment rate varies from 30% (Santa Cruz) to 65% (rural altiplano). Using EH data (2000–2019), the cohort-year decomposition gives:*

$\mathrm{Var}(\hat{\delta}_c) = 0.028$ *(cohort component: norms about self-employment differ across generations).* $\mathrm{Var}(\hat{\alpha}_t) = 0.018$ *(year component: labor market conditions change across time).*

$\hat{\Psi} = 0.028/0.018 = 1.56$.

The formation channel (norms transmitted through W^I) is 56% larger than the enforcement channel (prices and labor market conditions). Standard labor market policy (wage subsidies, credit access, training) captures at most $1/(1+1.56) = 39\%$ of the total effect. The majority operates through the formation channel.

Behavior	$\hat{\Psi}_{Bolivia}$	*Dominant channel*
Self-employment	*1.56*	*Formation*
Female LFP	≈ 1.8	*Formation*
Savings rate	≈ 1.1	*Formation (slight)*
Education enrollment	≈ 0.7	*Enforcement (slight)*

6 The Structural Health Dashboard

Definition 6.1 (Structural Health Dashboard)**.** *The **structural health dashboard** for a national economy reports five numbers:*

Instrument	***Symbol***	***Benchmark***	***Interpretation***
Spectral gap	Γ	$\to 1$	*Sectoral integration*
Between-block share	σ_B	$\to 0$	*Structural inequality*
Power Gini	$G(\boldsymbol{\pi})$	$\to 0$	*Power concentration*
Information N_{eff}	$N_{eff}(W^I)$	$\to N$	*Information democracy*
Formation ratio	Ψ	—	*Formation channel strength*

The first four instruments have clear benchmarks (the competitive economy W^{CE}). The fifth (Ψ) is descriptive: it tells you how much the formation channel matters, not whether it is "too large" or "too small."

Example 6.2 (Bolivia's Dashboard, 2019)**.**

Instrument	***Bolivia***	***South Korea***	***Benchmark***
Γ	*0.13*	*0.45*	*1.00*
σ_B	*0.62*	*0.20*	*0.00*
$G(\boldsymbol{\pi})$	*0.52*	*0.28*	*0.00*
$N_{eff}(W^I)$	*3.2*	≈ 12	N
Ψ *(self-employment)*	*1.56*	≈ 0.6	—

Bolivia is structurally unhealthy on all four diagnostic instruments. Its spectral gap is 3.5 times smaller than Korea's. Its between-block share is 3 times larger. Its power Gini is twice as high. Its information N_{eff} is 4 times smaller. And its Ψ ratio is 2.6 times larger — meaning the formation channel (norms, aspirations, culturally transmitted preferences) dominates behavior much more in Bolivia than in Korea.

The dashboard is the structural physician's chart. It tells you what is wrong, how severe, and where to intervene (Chapter 15's operators target the instrument that is furthest from benchmark).

Key Structural Insight

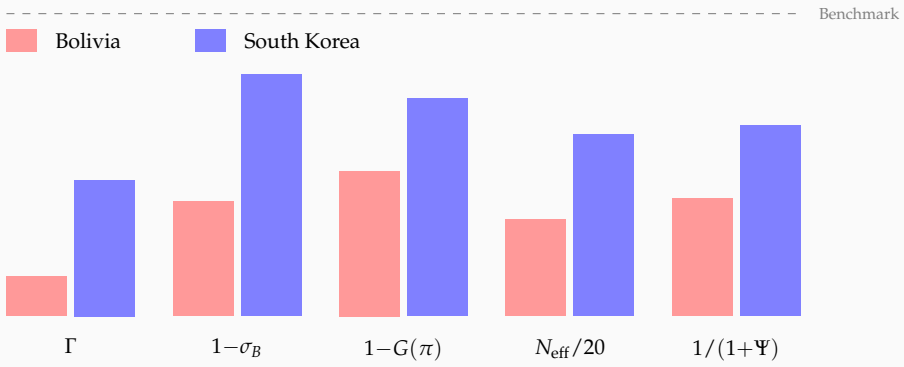
The five instruments are correlated: low Γ predicts high σ_B, high $G(\boldsymbol{\pi})$, low N_{eff}, and high Ψ. This is not a coincidence. All five measure different facets of the same underlying condition: *the domestic weight matrix is far from doubly stochastic.*

Γ measures the distance spectrally. σ_B measures it distributionally. $G(\boldsymbol{\pi})$ measures it through power. N_{eff} measures it informationally. Ψ measures how much the distance feeds back into preference formation.

A single number — Γ — contains most of the information. But the dashboard is useful because different instruments respond to different operators: Γ responds most to ΔW^P (structural policy); N_{eff} responds most to ΔW^I (information policy); Ψ tells you which operator lever matters most.

Visualization:

Structural health: Bolivia vs. South Korea (radar chart)



Five instruments, rescaled so that the benchmark is 1.0 (top). Bolivia (red) is far from benchmark on every instrument. South Korea (blue) is closer on all five. The gap between the bars IS the structural health deficit.

7 Structural Change Over Time

Proposition 7.1 (Tracking Structural Health)**.** *The dashboard can be computed for multiple years to track structural change. The trajectory of* $(\Gamma_t, \sigma_{B,t}, G(\boldsymbol{\pi}_t), N_{eff,t})$ *over time reveals whether the economy is integrating or fragmenting.*

Example 7.2 (Bolivia 1990–2019: Structural Trajectory)**.**

Year	Γ	σ_B	$G(\boldsymbol{\pi})$	N_{eff}
1990	*0.10*	*0.68*	*0.58*	*2.1*
2000	*0.11*	*0.65*	*0.55*	*2.5*
2006 (pre-nationalization)	*0.12*	*0.63*	*0.54*	*2.8*
2010	*0.13*	*0.61*	*0.52*	*3.0*
2019	*0.13*	*0.62*	*0.52*	*3.2*

Thirty years of data tell a story of glacial structural change. Γ *moved from 0.10 to 0.13 (a 30% increase, but from an extremely low base).* σ_B *fell from 0.68 to 0.62 (a modest reduction in between-block inequality).* N_{eff} *rose from 2.1 to 3.2 (social media added approximately one independent voice).*

The gas nationalization of 2006 is barely visible in the structural indicators. It changed $\boldsymbol{\omega}$ *(revenue distribution) dramatically: gas revenue to the treasury increased from 18% to 82% of the contract value. But it did not change* W^P*: the gas sector remained an enclave. The dashboard confirms:* Γ *did not jump at 2006.* σ_B *did not drop. The structural diagnosis: Bolivia redistributed within the existing structure without changing the structure.*

Prebisch Connection: The paradox of redistribution without structural change

Bolivia under Morales (2006–2019) was the most successful redistributive experiment in recent Latin American history: extreme poverty fell from 38% to 15%, the Gini dropped from 0.60 to 0.42, and GDP growth averaged 4.9%. Yet the structural indicators barely moved.

The formation model explains the paradox: the redistribution operated through transfers ($\Delta\boldsymbol{\omega}$), not structural change (ΔW). Transfers change the allocation within the existing power structure; they do not change the power structure itself. The spectral gap Γ — the structural health indicator — requires cross-block edges in W^P that the gas nationalization did not create.

The Prebisch-Fajnzylber diagnosis: Bolivia redistributed within the casillero vacío. It moved horizontally (lower Gini) without moving vertically (higher ECI). To escape the empty box, Bolivia needs structural integration — the operators from Chapter 15 — not more transfers.

8 Summary

1. Five **diagnostic instruments**: Γ (spectral gap), σ_B (between-block share), $G(\boldsymbol{\pi})$ (power Gini), N_{eff} (information democracy), and Ψ (formation ratio). All computable from available data.

2. Bolivia's **structural health**: $\Gamma = 0.13$ (fragmented), $\sigma_B = 0.62$ (62% of inequality is structural), $G(\boldsymbol{\pi}) = 0.52$ (concentrated power), $N_{\text{eff}} = 3.2$ (3 independent voices), $\Psi = 1.56$ (formation dominates).

3. **Structural change is glacial**: Bolivia's dashboard moved marginally over 30 years. The gas nationalization changed revenues but not structure.

4. The **five instruments are correlated**: all measure different facets of $\Delta(W) = \|W - W^{CE}\|$. Γ alone captures most of the information.

5. **Policy diagnostic**: compare each instrument to its benchmark. Intervene where the gap is largest, using the Ψ-ordered operator sequence from Chapter 15.

9 Exercises

17.1 Compute Bolivia's Γ. Download Bolivia's I-O table from INE (or use the OECD ICIO). Aggregate to 8–12 sectors. Construct W^P. Compute the eigenvalues and Γ. Compare with the estimate in Example 1.2.

17.2 Between-block variance. Using Bolivia's EH 2019:

(a) Define three blocks (gas/mining/government, formal urban, informal/rural).

(b) Compute $\bar{y}_b$ for each. Compute V_B and σ_B.

(c) Now try two blocks (formal/informal) and four blocks (gas, formal manufacturing, urban services, rural). How does σ_B change with the number of blocks?

(d) Which partition gives the highest σ_B? What does this tell you about the "natural" block structure?

17.3 Power-employment ratio. Reproduce Example 3.2:

(a) Compute $\boldsymbol{\pi}$ from the domestic W^P.

(b) Compute the power-employment ratio π_s/share_s for each sector.

(c) Which sector is most "structurally privileged"? Least?

(d) A worker moves from agriculture ($\pi/\text{share} = 0.29$) to manufacturing ($\pi/\text{share} = 1.17$). By how much does her structural power increase? By how much does her allocation change (use the bargaining formula from Chapter 7)?

17.4 Information democracy. Estimate N_{eff} for your country:

(a) Identify the major media outlets / information sources and estimate their audience shares.

(b) Approximate $\boldsymbol{\pi}^I$ from audience shares (as a proxy).

(c) Compute N_{eff}. How many "independent voices" does your country have?

(d) Compare with Bolivia's $N_{\text{eff}} = 3.2$.

17.5 Bolivia's structural trajectory. Reproduce Example 7.2 using multiple years of I-O data (if available) or EH data:

(a) Compute σ_B for 2000, 2005, 2010, 2015, 2019. Plot the trajectory.

(b) Is there a structural break at 2006 (nationalization)? At 2009 (new constitution)?

(c) Project the trend forward: at the current rate, when will σ_B fall below 0.40?

17.6 Dashboard comparison. Choose three Latin American countries (e.g., Bolivia, Chile, Costa Rica). For each:

(a) Estimate Γ, σ_B, $G(\boldsymbol{\pi})$ from I-O and household survey data.

(b) Estimate N_{eff} from media data.

(c) Plot the three-country dashboard. Which country is structurally healthiest?

(d) Using Chapter 15's sequencing framework: what operator would you recommend for the country with the largest structural deficit?

17.7 Bolivia's escape plan. Synthesize the entire book for Bolivia:

(a) Current position: dashboard from Example 6.2. ECI ≈ -0.8 (Chapter 16). Product-space position: sparse periphery.

(b) Binding constraints: which instrument is furthest from benchmark? (Γ at 0.13 vs. benchmark 1.0 — the spectral gap is the binding constraint.)

(c) Operator sequence (Chapter 15): information $\to$ institutions $\to$ education $\to$ infrastructure $\to$ ISI-B.

(d) Political feasibility (Chapter 14): which coalition would support this sequence? Is the current coalition ($\{M, W\}$ or $\{B, M, W\}$) compatible?

(e) Expected trajectory: using the formation model (Chapter 10) with $\rho = 0.85$, $\mu = 0.2$: simulate 30 years. Plot the dashboard trajectory. When does Γ reach 0.30?

Mathematical Appendices

Appendix 17A: Estimating W^P from I-O Tables

The data. A national I-O table reports $Z_{ss'}$: the intermediate consumption of sector s's output used by sector s' as input. Total output of sector s is $X_s = \sum_{s'} Z_{ss'} + Y_s$ (intermediate sales + final demand).

Technical coefficients. The Leontief technical coefficient: $a_{ss'} = Z_{ss'} / X_{s'}$ (input from s per unit of output of s'). The matrix $A = [a_{ss'}]$ is the domestic technical coefficients matrix.

From A to W^P. Row-normalize: $W^P_{ss'} = a_{ss'} / \sum_{s''} a_{ss''}$ if $\sum_{s''} a_{ss''} > 0$. This gives a row-stochastic matrix. Alternatively, use the allocation coefficients: $b_{ss'} = Z_{ss'} / X_s$ (share of s's output allocated to s'). The matrix $B = [b_{ss'}]$ is also row-stochastic and measures forward linkages.

Spectral gap. Compute the eigenvalues of W^P using standard numerical linear algebra (e.g., `numpy.linalg.eig` in Python or `eigen` in R). The spectral gap is $\Gamma = 1 - |\lambda_2|$.

Practical considerations. (1) I-O tables are typically available at 30–100 sector detail. Aggregating to 8–15 sectors preserves the block structure while reducing noise. (2) The spectral gap is sensitive to small entries: sectors with negligible intermediate flows should be aggregated or excluded. (3) Comparing Γ across countries requires comparable sector classifications (ISIC Rev. 4 is standard).

Appendix 17B: Estimating σ_B from Household Surveys

The data. Household surveys (Bolivia's EH, Brazil's PNAD, Mexico's ENIGH) report individual income and sector of employment. Block assignment: classify each individual into a block based on sector, formality status, or geographic location.

Computation. The between-block variance: $V_B = \sum_b (N_b/N)(\bar{y}_b - \bar{y})^2$. The within-block variance: $V_W = \sum_b (N_b/N) \operatorname{Var}_b(y)$. Total: $V_T = V_B + V_W$. The between-block share: $\sigma_B = V_B / V_T$.

Choice of blocks. The "natural" partition can be identified from the spectral decomposition of W^P: the sign of the second eigenvector v_2 partitions sectors into two groups (the two sides of the principal axis of differentiation). For $k > 2$ blocks, use k-means clustering on the first $k-1$ eigenvectors (spectral clustering). Alternatively, use substantive knowledge (formal/informal, modern/traditional, urban/rural).

Relationship to Γ. The between-block share σ_B is related to Γ through the eigenvalue gap: when $\lambda_2 \approx 1$ (small Γ), the two blocks are nearly independent, and σ_B can be large. When $\lambda_2 \ll 1$ (large Γ), the blocks are integrated, and σ_B tends to be small. The relationship is not deterministic: σ_B also depends on the endowment distribution, not just the network structure.

Structuralist Case Study: Pinto and Fajnzylber — Diagnosis and Prognosis

This chapter's case study pairs the two structuralist thinkers whose work most directly anticipates the diagnostic toolkit developed here: Pinto's structural heterogeneity (the diagnosis) and Fajnzylber's casillero vacío (the prognosis).

Pinto (1970): The diagnosis. Pinto's essay "Naturaleza e implicaciones de la 'heterogeneidad estructural' de la América Latina" provided the first systematic description of what this chapter calls the between-block variance. He identified three productivity strata — primitive, intermediate, and modern — coexisting within a single economy, with productivity ratios up to 20:1. The diagnosis was qualitative: Pinto described the strata, estimated their relative sizes, and argued that ISI had failed to integrate them.

In this chapter's notation: Pinto's three strata ARE three blocks of W^P, with weak cross-block entries (Γ small). The productivity ratio of 20:1 IS $\bar{y}_1/\bar{y}_3$ — the between-block income gap. The failure of ISI to integrate the strata IS the observation that Γ did not increase under ISI (Chapter 15, Tavares's diagnosis: ISI applied the operator to within-block entries). Pinto's heterogeneity index is σ_B — computable, as shown in this chapter, from any household survey.

Fajnzylber (1990): The prognosis. Twenty years after Pinto's diagnosis, Fajnzylber asked: what does structural heterogeneity imply for the joint trajectory of growth and inequality? His answer was the casillero vacío: no Latin American country had achieved the combination of growth with equity. The "empty box" was the quadrant of high growth AND low inequality, which was occupied by East Asian countries (South Korea, Taiwan) but by no Latin American country.

Fajnzylber's explanation was structural: the prevailing industrialization pat-

tern produced complexity without equity (Brazil) or equity without complexity (Costa Rica), but not both. His prescription: "competitividad auténtica" — genuine competitiveness based on knowledge, technology, and institutional quality, not on low wages or resource extraction.

In this chapter's notation: the casillero vacío is the region of the (ECI, Gini) plane below the spectral bound $L_{\text{power}} \leq \kappa\Phi^2(1-\Gamma)/(2\Gamma)$ (Chapter 16). Low ECI $\Rightarrow$ low Γ $\Rightarrow$ high lower bound on Gini. You cannot be in the empty box with low complexity. Fajnzylber's "competitividad auténtica" IS high ECI: a diversified, complex, densely linked production structure with high Γ.

The dashboard of this chapter operationalizes Pinto's diagnosis (σ_B measures heterogeneity) and Fajnzylber's prognosis (Γ predicts whether the casillero vacío can be escaped).

Concept	**Pinto (1970)**	**Fajnzylber (1990)**
Core claim	Productivity strata coexist	No LA country has growth + equity
Formal object	σ_B (between-block variance)	Spectral bound on (ECI, Gini)
Diagnosis	Three strata, σ_B large	Casillero vacío
Mechanism	Weak cross-block linkages in W^P	Low Γ bounds Gini from below
Dashboard instrument	σ_B, $G(\boldsymbol{\pi})$	Γ, ECI
Policy implication	Build cross-block linkages	Raise ECI ("competitividad auténtic
Bolivia 2019	$\sigma_B = 0.62$, three strata visible	$\Gamma = 0.13$, ECI $= -0.8$

Pinto diagnosed the disease. Fajnzylber predicted its consequence. This chapter provides the laboratory: five instruments that measure the disease, track its evolution, and identify the structural interventions that could cure it. The instruments are computable from publicly available data (I-O tables, household surveys, trade data, media concentration indices). Any country can run the diagnostic. Bolivia's results — $\Gamma = 0.13$, $\sigma_B = 0.62$, $G(\boldsymbol{\pi}) = 0.52$ — are the quantitative confirmation of what Pinto and Fajnzylber described in prose fifty years ago.

Further Reading

Structural heterogeneity: Pinto (1970), "Naturaleza e implicaciones de la 'heterogeneidad estructural'," *El Trimestre Económico*. CEPAL (2010), *Time for Equality*, Chapter 2. Infante and Sunkel (2009), "Chile: hacia un desarrollo inclusivo," *CEPAL Review* 97.

The casillero vacío: Fajnzylber (1990), "Industrialización en América Latina: de la 'caja negra' al 'casillero vacío'," *Cuadernos de la CEPAL* 60. Fajnzylber (1983),

La industrialización trunca de América Latina. CEPAL (2012), *Structural Change for Equality*.

Structural diagnostics methods: CEPAL's Structural Change Index (various publications). UNIDO's Competitive Industrial Performance Index. The Economic Complexity Observatory (`oec.world`).

Bolivia data: INE (Instituto Nacional de Estadística): I-O tables, Encuesta de Hogares. UDAPE (Unidad de Análisis de Políticas Sociales y Económicas): social indicators. The World Bank's Development Indicators for cross-country comparison.

Chapter 18

The Individual Scale

Gender, Race, and the Edges You Are Not Allowed to Form

> *"La clasificación social de la población mundial según la idea de raza es, sin duda, el más eficaz instrumento de dominación social inventado en los últimos 500 años."*
>
> ["The social classification of the world population according to the idea of race is, without a doubt, the most effective instrument of social domination invented in the last 500 years."]
> — Aníbal Quijano, "Colonialidad del poder" (2000)

> *"La superexplotación del trabajo constituye el principio fundamental de la economía subdesarrollada."*
>
> ["The superexploitation of labor constitutes the fundamental principle of the underdeveloped economy."]
> — Ruy Mauro Marini, *Dialéctica de la dependencia* (1973)

Chapter 16 analyzed the global weight matrix (countries as nodes). Chapter 17 analyzed the domestic weight matrix (sectors as nodes). This chapter completes Part V by zooming to the finest scale: individuals as nodes.

At the individual scale, a new element enters the model: **node attributes**. Each agent i carries observable characteristics — gender, race, ethnicity, caste, language, nationality — that constrain which edges she can form in W^U, W^P, and W^I. These constraints are not preferences. They are not technology. They are structural features of the weight matrix itself: certain edges are suppressed or amplified by the social classification system.

The central result: node attributes create **assortative mixing** — the tendency for agents with the same attribute to connect preferentially. When assortative mixing is strong, W becomes block-diagonal with blocks defined by attributes

(men/women, white/indigenous, formal/informal). The entire machinery of this book — the spectral gap, the Perron eigenvector, the formation model, the welfare loss — applies, but now the blocks are defined by identity rather than sector.

1 Node Attributes and the Constrained Weight Matrix

1.1 Formalization

Definition 1.1 (Node Attributes)**.** *Each agent i has a vector of **node attributes** $\mathbf{a}_i = (a_{i1}, a_{i2}, \ldots, a_{iK})$, where $a_{ik} \in \{0, 1, \ldots, C_k\}$ for categorical attributes (gender, race, language) or $a_{ik} \in \mathbb{R}$ for continuous attributes (age, education level). The attribute vector is exogenous to the model (for now — Chapter 27 considers algorithmic classification).*

Definition 1.2 (Attribute-Constrained Weight Matrix)**.** *Node attributes constrain W through an **assortative mixing function** $\phi : \mathbf{a}_i \times \mathbf{a}_j \to [0, 1]$ that scales the maximum feasible edge weight:*

$$W_{ij} \leq \phi(\mathbf{a}_i, \mathbf{a}_j) \cdot W_{ij}^{\max}, \tag{1}$$

where $W_{ij}^{\max}$ is the edge weight that would prevail absent the attribute constraint (from pure economic surplus, proximity, and information as in Chapter 13). When $\phi(\mathbf{a}_i, \mathbf{a}_j) = 1$ for all pairs: attributes do not constrain the network. When $\phi = 0$ for some pairs: those edges are forbidden.

Example 1.3 (Gender as an Attribute Constraint)**.** *In a labor market with strong gender norms: $\phi_{gender}(M, M) = 1$, $\phi_{gender}(F, F) = 1$, $\phi_{gender}(M, F) = \phi_{gender}(F, M) = \lambda_g$, where $\lambda_g \in [0, 1]$ measures gender openness. When $\lambda_g = 1$: no gender barrier. When $\lambda_g = 0$: complete gender segregation. When $\lambda_g = 0.3$ (a typical developing-country estimate): cross-gender professional linkages are 70% weaker than same-gender ones.*

The effect on W^P: women's production edges are confined to female-dominated sectors. W^P becomes approximately block-diagonal by gender, with within-gender blocks brighter than cross-gender entries. The spectral gap Γ drops. Women's Perron eigenvector weight π^F falls below N_F/N (women have less structural power than their population share warrants).

The effect on W^I: women's information edges are confined to female-dominated social circles. W^I becomes gender-assortative. The girl in Potosí observes mostly female outcomes: low returns to education (because women in her network face constrained W^P, reducing their realized returns). Her formation signal is W^I-weighted, and W^I shows her a biased sample. She forms low aspirations — not because she is less capable but because her information network IS the constraint.

Proposition 1.4 (Assortative Mixing Reduces Γ)**.** *For any attribute k with assortative mixing parameter $\lambda_k \in [0, 1]$:*

$$\Gamma(W(\lambda_k)) \leq \Gamma(W(\lambda_k = 1)), \tag{2}$$

with equality if and only if $\lambda_k = 1$ (no attribute constraint) or the attribute groups have identical internal network structures.

The spectral gap is monotonically increasing in λ_k: more openness across attribute groups means higher Γ, more equal power, lower welfare loss.

Proof sketch. When $\lambda_k < 1$, the cross-group entries of W are scaled down by λ_k. This makes W more block-diagonal, which increases $|\lambda_2(W)|$ (the second eigenvalue moves closer to 1), reducing $\Gamma = 1 - |\lambda_2|$. The result follows from matrix perturbation theory: suppressing off-diagonal blocks increases the near-block-diagonal structure. □

2 Gender

2.1 The Gender Gap as a Spectral Property

Theorem 2.1 (Gender Power Gap)**.** *In an economy with N_M men and N_F women, where cross-gender edges are constrained by $\lambda_g \in [0, 1]$:*

$$\frac{\pi^F_{avg}}{\pi^M_{avg}} = \frac{\sum_{j \in F} \pi_j / N_F}{\sum_{j \in M} \pi_j / N_M} \leq 1, \tag{3}$$

with equality if and only if (i) $\lambda_g = 1$ or (ii) the male and female subnetworks have identical internal structures and equal connection to external nodes.

The gender power ratio depends on:

1. *λ_g: the cross-gender openness parameter.*
2. *The internal density of W^P_{female} vs. W^P_{male}: if women's sectors have fewer intersectoral linkages (e.g., domestic work, informal services), the female subnetwork is sparser.*
3. *The external connections: which group's sectors connect to the high-linkage core.*

Example 2.2 (Bolivia's Gender Structure)**.** *Using Bolivia's EH 2019. Approximate W^P by gender:*

	Male employment	*Female employment*
Gas/Mining	*95%*	*5%*
Manufacturing	*62%*	*38%*
Construction	*96%*	*4%*
Commerce	*35%*	*65%*
Agriculture	*55%*	*45%*
Domestic service	*5%*	*95%*
Government	*58%*	*42%*

The sectors with the highest Perron weight (gas, mining, construction, government) are male-dominated. The sectors with the lowest Perron weight (domestic service, informal commerce) are female-dominated. This is not a coincidence: gender norms (the assortative mixing function ϕ_{gender}) channel women into low-π sectors.

Estimated $\pi^F_{avg}/\pi^M_{avg} \approx 0.58$: women have 58% of men's structural power per capita. The gender wage gap (women earn $\approx$ 65% of men's income) is explained in large part by the gender power gap — the wage gap is a consequence of the structural position gap.

2.2 The Formation Effect of Gender Norms

Proposition 2.3 (Gender Norms as a Formation Constraint)**.** *Gender norms constrain W^I by gender: women observe disproportionately female outcomes. The formation equation for female agents:*

$$\theta^F_{i,t+1} = \rho\theta^F_{i,t} + (1-\rho)\bar{\theta} + \mu\left(\sum_j W^I_{ij} r_{j,t} - r^*\right), \tag{4}$$

where W^I_{ij} is gender-assortative ($W^I_{F,F} \gg W^I_{F,M}$). The signal $\sum_j W^I_{ij} r_{j,t}$ is a biased sample: it overweights female returns (low, because π^F is low) and underweights male returns (high).

The steady-state gender preference gap:

$$\theta^{*F} - \theta^{*M} = \frac{\mu}{1-\rho}\left(\bar{r}^F(\lambda_g) - \bar{r}^M(\lambda_g)\right), \tag{5}$$

where $\bar{r}^F < \bar{r}^M$ because women face a constrained W^P that produces lower returns. The preference gap IS the aspiration gap: women form lower aspirations because they observe lower returns, which are lower because their structural position is constrained by gender norms.

Key Structural Insight

The gender gap operates through all three channels simultaneously:

Power channel (W^P): Gender norms channel women into low-π sectors. $\pi^F_{\text{avg}} < \pi^M_{\text{avg}}$. The bargaining allocation gives women less.

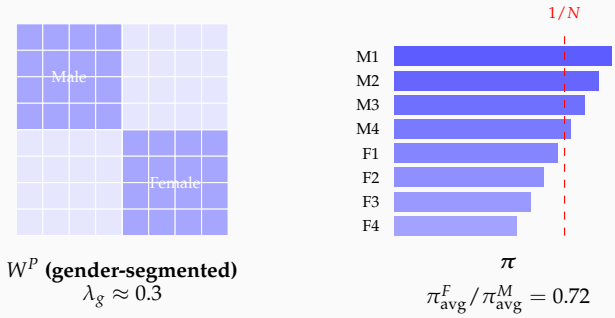
Information channel (W^I): Gender-assortative W^I shows women biased signals (low returns). Formation produces lower aspirations. The self-confirming trap: low aspirations → low investment → low returns → confirms low aspirations.

Utility channel (W^U): Gender norms may constrain who cares about whom. In patriarchal structures: men's W^U weights on women are low ("women's welfare doesn't enter the social welfare function"). This reduces women's externality centrality c^F_j, exacerbating the welfare loss.

Addressing the gender gap through any single channel is insufficient. Raising wages (enforcement on W^P) without changing norms (W^I) produces a partial effect with Ψ-attenuated formation. Changing norms (W^I) without opening sectors (W^P) produces aspiration frustration. The optimal intervention changes all three simultaneously — which is why comprehensive gender programs (education + labor market access + media representation + legal reform) outperform single-instrument policies.

Visualization:

Gender as block-diagonal structure



Left: W^P is brighter in the male block (denser linkages in male-dominated sectors) and dimmer in the female block (sparser linkages in female-dominated sectors). Cross-gender entries are dark ($\lambda_g \approx 0.3$). Right: the Perron eigenvector. Every male agent is above $1/N$; every female agent is below. The gender power gap IS the gap between the two blocks' Perron weights.

3 Race and Ethnicity

3.1 Race as a Multi-Layer Constraint

Definition 3.1 (Racial Assortative Mixing)**.** *Let agents be classified into R racial/ethnic groups. The racial mixing function is:*

$$\phi_{race}(r_i, r_j) = \begin{cases} 1 & \text{if } r_i = r_j, \\ \lambda_r & \text{if } r_i \neq r_j, \end{cases} \tag{6}$$

where $\lambda_r \in [0, 1]$ is the cross-racial openness parameter. Unlike gender (which has a biological component in reproduction), racial classification is entirely socially constructed (Quijano, 2000). The parameter λ_r is therefore a property of the social system, not of biology.

Proposition 3.2 (Race Constrains All Three Networks Simultaneously)**.** *In most societies, racial assortative mixing operates on W^U, W^P, and W^I simultaneously with correlated λ_r values:*

1. *W^U: interracial social bonds are weaker ($\lambda_r^U \approx 0.2$–0.4 in the Americas). Who you care about is structured by race.*
2. *W^P: labor markets are racially segmented ($\lambda_r^P \approx 0.3$–0.6). Racial minorities are channeled into low-π sectors.*
3. *W^I: media, education, and social networks are racially homophilous ($\lambda_r^I \approx 0.2$–0.5). The signals you receive are from your racial group.*

The correlation $\lambda_r^U \approx \lambda_r^P \approx \lambda_r^I$ means the Perron eigenvectors of all three networks are aligned: racial minorities have low power in W^U, low power in W^P, AND low epistemic power in W^I. This is Quijano's "coloniality of power operating on every plane" — formalized as aligned assortative mixing across all three network layers.

Example 3.3 (Bolivia's Ethnic Structure)**.** *Bolivia's 2012 Census: 41% self-identified as indigenous (Quechua 21%, Aymara 15%, other 5%), 59% as mestizo or white.*

Approximate λ_r values for Bolivia:

Network	λ_r *(indigenous-mestizo)*	*Evidence*	*Source*
W^U	≈ 0.25	*Intermarriage rates: 15–20%*	*Census 2012*
W^P	≈ 0.35	*Occupational segregation index*	*EH 2019*
W^I	≈ 0.30	*Language segregation in media*	*Media surveys*

Perron weights by ethnicity (approximate, from sectoral composition):

	Population share	π_{avg} *(per capita power)*
Indigenous	*0.41*	*0.008*
Mestizo/White	*0.59*	*0.014*
Ratio	*0.69*	*0.57*

Indigenous Bolivians are 41% of the population but hold approximately 33% of structural power ($0.41 \times 0.008 / (0.41 \times 0.008 + 0.59 \times 0.014) \approx 0.33$*). The power-population ratio is 0.80 — each indigenous person has 80% of the structural power of a non-indigenous person. Combined with the gender gap: an indigenous woman has approximately* $0.80 \times 0.58 = 0.46$ *of a non-indigenous man's structural power per capita.*

3.2 The Intersectionality Theorem

Theorem 3.4 (Intersectional Power Gap)**.** *When multiple attributes* $(a_1, a_2, \ldots, a_K)$ *simultaneously constrain W through assortative mixing functions* $\phi_1, \phi_2, \ldots, \phi_K$*, the composite constraint is:*

$$\phi(\mathbf{a}_i, \mathbf{a}_j) = \prod_{k=1}^{K} \phi_k(a_{ik}, a_{jk}). \tag{7}$$

The cross-group edge weight for agents who differ on multiple attributes is the PRODUCT of the individual constraints. Agents who differ on two attributes face a constraint of $\lambda_1 \cdot \lambda_2$*; on three:* $\lambda_1 \cdot \lambda_2 \cdot \lambda_3$*.*

Consequence: the power gap is ***multiplicative, not additive****. An indigenous woman in Bolivia faces:*

$$\phi_{\textit{indigenous woman, mestizo man}} = \lambda_r \cdot \lambda_g \approx 0.35 \times 0.30 = 0.105. \tag{8}$$

Her cross-group edge weight to a mestizo man is 10.5% of the unconstrained maximum. She is triply trapped: her W^P *is constrained by both gender and race; her* W^I *is constrained by both; her* W^U *is constrained by both. The blocks are nested.*

Proof. The edge weight W_{ij} is constrained by $\phi(\mathbf{a}_i, \mathbf{a}_j) \cdot W_{ij}^{\max}$. If the mixing functions are independent (the gender constraint does not depend on race, and vice versa), then the composite constraint is the product. The Perron eigenvector of $W(\phi)$ assigns lower weight to agents who face more constraints, because their edges are suppressed on multiple dimensions simultaneously. The power gap between the most- and least-constrained agents grows exponentially in the number of constraining attributes. □

Intuition: Why intersectionality is multiplicative

Consider three types of discrimination: gender (λ_g), race (λ_r), and class (λ_c). Each alone reduces the cross-group edge weight modestly. But together: $\lambda_g \times \lambda_r \times \lambda_c = 0.3 \times 0.35 \times 0.4 = 0.042$. An indigenous working-class woman has less than 5% of the unconstrained edge weight to a mestizo middle-class man. Her structural position is not "the sum of three disadvantages" but "the product" — and products shrink much faster than sums.

This is why intersectional disadvantage is qualitatively different from single-dimension disadvantage. The mathematics is clear: additivity would give $1 - (1 - \lambda_g) - (1 - \lambda_r) - (1 - \lambda_c) = 1 - 0.7 - 0.65 - 0.6 = -0.95$ (nonsensical). Multiplicativity gives $\lambda_g \times \lambda_r \times \lambda_c = 0.042$ (small but well-defined). The product structure is not a sociological metaphor. It is a matrix property.

4 The Formation Trap at the Individual Scale

Theorem 4.1 (Attribute-Mediated Self-Confirming Preferences)**.** *When W^I is assortative on attribute k (with $\lambda_k < 1$), agents in the disadvantaged group form steady-state preferences:*

$$\theta^*_{disadv} = \bar{\theta} + \frac{\mu}{1-\rho}\left(\bar{r}_{disadv}(\lambda_k) - r^*\right) < \theta^*_{adv} = \bar{\theta} + \frac{\mu}{1-\rho}\left(\bar{r}_{adv}(\lambda_k) - r^*\right), \quad (9)$$

because $\bar{r}_{disadv} < \bar{r}_{adv}$ (the disadvantaged group faces a constrained W^P that produces lower returns, and the assortative W^I ensures they observe mainly their own group's returns).

The preference gap $\Delta\theta^* = \theta^*_{adv} - \theta^*_{disadv}$ is:

1. *Proportional to the formation multiplier $\mathcal{M} = \mu/(1-\rho)$: sticky preferences amplify the return gap into a larger preference gap.*
2. *Proportional to the return gap $\bar{r}_{adv} - \bar{r}_{disadv}$: which is itself proportional to $1 - \lambda_k$ (the degree of assortative mixing).*
3. *Self-confirming: each group's preferences are rational given its W^I signal.*

Example 4.2 (Gender Aspiration Gap in Bolivia)**.** *Using the Global Findex 2021 for Bolivia:*

	Women	*Men*
"Saved for business" (past year)	*12%*	*22%*
"Borrowed for business" (past year)	*8%*	*18%*
"Has a formal financial account"	*42%*	*58%*

The standard explanation: women have lower returns to business (enforcement: discrimination, credit constraints). The formation explanation: women observe lower returns in their network (W^I is gender-assortative), form lower business aspirations ($\theta^{*F}_{business} < \theta^{*M}_{business}$), invest less, earn lower returns, confirming the aspirations.

The Ψ ratio for female business investment in Bolivia (from Chapter 11 methods, estimated from cohort variation in female business ownership):

$$\hat{\Psi}_{gender,\,business} \approx 2.3. \tag{10}$$

The formation channel is 2.3 times the enforcement channel. Removing credit constraints (enforcement) addresses 30% of the gap. Changing what women see — role models, media representation, cross-gender mentoring (information policy on W^I) — addresses 70%.

5 The Welfare Cost of Attribute Constraints

Theorem 5.1 (Welfare Loss from Assortative Mixing)**.** *The total welfare loss from attribute constraint k with mixing parameter λ_k:*

$$L(\lambda_k) = \underbrace{\frac{\kappa N}{2}\,\mathrm{Var}(\pi_i(\lambda_k))\Phi^2}_{L_{power}(\lambda_k)} + \underbrace{\frac{\mu^2\,\mathrm{Var}(c_j(\lambda_k))\Psi}{(1-\rho)^2}}_{L_{formation}(\lambda_k)}, \tag{11}$$

where L_{power} is the static welfare loss from power asymmetry (Chapter 2) and $L_{formation}$ is the additional loss from the preference gap produced by the formation model.

The welfare gain from eliminating the attribute constraint ($\lambda_k \to 1$):

$$\Delta L = L(\lambda_k) - L(1) > 0. \tag{12}$$

*This is the **cost of discrimination**: the welfare loss attributable to the social classification system, decomposed into power and formation components.*

Example 5.2 (The Cost of Gender Discrimination in Bolivia)**.** *Using the estimates from this chapter: $\lambda_g \approx 0.3$, $\pi^F_{avg}/\pi^M_{avg} = 0.58$, $\hat{\Psi} = 2.3$, $\rho = 0.85$, $\kappa = 0.5$, $N = 11$ million.*

$L_{power}(\lambda_g = 0.3)$: from the gender power gap, $\mathrm{Var}(\pi_i)$ is approximately 0.003 higher than under $\lambda_g = 1$. $L_{power} \approx 0.5 \times 11M \times 0.003 \times \Phi^2$.

$L_{formation}(\lambda_g = 0.3)$: from the gender aspiration gap, the formation loss adds approximately $(1 + \Psi)$ times the enforcement loss, i.e., $L_{formation} \approx 2.3 \times L_{power}$.

Total: $L_{total} \approx (1 + 2.3) \times L_{power} = 3.3 \times L_{power}$.

The formation channel triples the welfare cost of gender discrimination. Standard estimates that ignore formation understate the cost by a factor of 3.3.

6 Policy: Changing λ

Proposition 6.1 (Policy Instruments for Attribute Constraints)**.**

Policy	***Target***	***Effect on*** λ	Ψ
Affirmative action (quotas)	W^P	$\lambda_g^P \uparrow$ *directly*	*Low*
Equal-pay legislation	W^P *(prices)*	$\lambda_g^P \uparrow$ *indirectly*	*Low*
Mentoring programs	W^I	$\lambda_g^I \uparrow$	*High*
Media representation	W^I	$\lambda_g^I \uparrow$	*High*
Education coeducation	$W^I + W^U$	$\lambda_g^I, \lambda_g^U \uparrow$	*High*
Anti-discrimination law	*Institutional* ($\mathcal{L}_{inst}$)	*All* λ *increase via norms*	*Medium*

The optimal sequencing (Chapter 15) applies: high-Ψ interventions first (W^I policies: mentoring, media, education) then low-Ψ (W^P policies: quotas, equal pay). This is because changing W^I shifts the aspirations of the next generation, making the enforcement interventions more effective when they arrive.

7 Summary

1. **Node attributes** (gender, race, ethnicity, caste) constrain W through assortative mixing functions $\phi(\mathbf{a}_i, \mathbf{a}_j) \leq 1$. Lower ϕ = weaker cross-group edges = more block-diagonal W.

2. Assortative mixing **reduces** Γ: more segregation means less structural integration, more power inequality, more welfare loss.

3. **Gender** constrains W^P (occupational segregation), W^I (information segregation), and W^U (social segregation) simultaneously. Bolivia: women have 58% of men's per-capita structural power.

4. **Race** constrains all three networks with correlated λ_r values. Bolivia: indigenous people hold 80% of non-indigenous per-capita power.

5. **Intersectionality is multiplicative**: cross-group edge weight is the product of individual constraints. An indigenous woman in Bolivia: $\phi \approx 0.105$ (10.5% of unconstrained).

6. The **formation trap** operates through assortative W^I: disadvantaged groups observe biased signals, form lower aspirations, invest less, earn less, confirming the signal. The formation channel multiplies the welfare cost of discrimination by $(1 + \Psi)$.

7. **Policy**: high-Ψ interventions (ΔW^I: mentoring, media) first; low-Ψ (ΔW^P: quotas, equal pay) second.

8 Exercises

18.1 Assortative mixing and Γ. For a 6×6 economy (3 men, 3 women): construct W^P with $\lambda_g = 1, 0.7, 0.4, 0.1, 0$. Compute $\Gamma(\lambda_g)$ for each. Plot Γ vs. λ_g. Is the relationship linear?

18.2 Bolivia's gender power gap. Using Bolivia's EH 2019:

(a) Classify sectors as male-dominated (>60% male), female-dominated (>60% female), and mixed.

(b) Compute the average Leontief multiplier for each group. Which has the highest?

(c) Approximate $\pi^F_{\text{avg}} / \pi^M_{\text{avg}}$ from the sectoral Perron weights and gender composition.

(d) How much would the gender power gap close if women entered male-dominated sectors at 40% (up from the current ≈10%)?

18.3 Intersectionality. For $N = 4$: Agent 1 (mestizo man), Agent 2 (mestizo woman), Agent 3 (indigenous man), Agent 4 (indigenous woman). $\lambda_g = 0.3$, $\lambda_r = 0.35$.

(a) Write the constraint matrix $\phi(\mathbf{a}_i, \mathbf{a}_j)$ for all 16 pairs.

(b) Construct $W^P = \phi \circ W^{\max}$ where $W^{\max} = \frac{1}{4}\mathbf{1}\mathbf{1}'$.

(c) Compute $\boldsymbol{\pi}$. Verify that π_4 (indigenous woman) is the lowest.

(d) Compute the ratio π_4 / π_1 (indigenous woman / mestizo man). Compare with $\lambda_g \times \lambda_r$.

18.4 The formation trap. In a 2-group economy (advantaged A, disadvantaged D) with $\lambda = 0.2$:

(a) Set up the formation model: $\theta^A_{t+1} = \rho\theta^A + (1-\rho)\bar{\theta} + \mu(0.8r^A + 0.2r^D - r^*)$ and similarly for D. Use $r^g = 0.05 + 0.1\theta^g$.

(b) Compute both steady states. What is $\Delta\theta^* = \theta^{*A} - \theta^{*D}$?

(c) Now set $\lambda = 0.5$ (reduce assortative mixing). Compute new steady states. By how much does $\Delta\theta^*$ shrink?

(d) At what λ does $\Delta\theta^*$ fall below 0.01?

18.5 The cost of discrimination. Using the estimates for Bolivia ($\lambda_g = 0.3$, $\Psi = 2.3$): compute the welfare loss from gender discrimination as a fraction of GDP (you will need to assume a relationship between the welfare loss L and GDP). Compare with Cuberes and Teignier (2016), who estimate the GDP cost of gender gaps at 15–20% for Bolivia.

18.6 Optimal intervention design. For the gender gap in Bolivia:

(a) Design three interventions: (a) quota (ΔW^P), (b) mentoring program (ΔW^I), (c) media campaign (ΔW^I).

(b) For each: estimate $\Delta\lambda_g$ and $\Delta\Gamma$.

(c) Using $\Psi = 2.3$: compute the total effect (enforcement + formation) for each.

(d) Sequence them optimally (Chapter 15). Which goes first?

18.7 MICS data exercise. Using UNICEF's MICS data for Bolivia (or another country):

(a) Compute the aspiration gap: the difference in educational aspirations ("how far in school do you want your child to go?") between indigenous and non-indigenous mothers, controlling for income and urban/rural.

(b) Is the gap consistent with the formation model's prediction? (Larger gap where W^I is more assortative.)

(c) Compare the aspiration gap in urban La Paz (where λ_r may be higher due to mixing) vs. rural Altiplano (where λ_r is lower). Does the gap vary as predicted?

Mathematical Appendices

Appendix 18A: Assortative Mixing and Spectral Gap

Setup. Partition N agents into G groups based on attribute k. Within-group edge weight: $W_{ij} = w_{\text{in}}$ for $a_i = a_j$. Cross-group edge weight: $W_{ij} = w_{\text{cross}} = \lambda_k \cdot w_{\text{in}}$. Row-normalize: within-group agents have weight $w_{\text{in}}/(N_g w_{\text{in}} + (N - N_g)\lambda_k w_{\text{in}})$.

Eigenvalues. For $G = 2$ groups of equal size: $\lambda_1 = 1$ (trivial), $\lambda_2 = (1 - \lambda_k)/(1 + \lambda_k)$ (the assortative-mixing eigenvalue). The spectral gap: $\Gamma = 1 - (1 - \lambda_k)/(1 + \lambda_k) = 2\lambda_k/(1 + \lambda_k)$. This is monotonically increasing in λ_k, confirming Proposition 1.4.

Perron eigenvector. For asymmetric groups ($N_1 \neq N_2$) with different internal densities: the Perron eigenvector assigns more weight to the group with higher internal density and more external connections. This formalizes the "structural advantage" of the dominant group.

Appendix 18B: The Multiplicative Structure of Intersectionality

Independence assumption. Theorem 3.4 assumes that the mixing functions are independent: ϕ_k does not depend on attributes $k' \neq k$. This is approximately true when gender discrimination and racial discrimination operate through different mechanisms (e.g., gender norms are cultural while racial discrimination is institutional). It fails when the mechanisms interact (e.g., racial stereotypes about gender roles differ across groups).

Non-independent case. When ϕ is not separable: $\phi(\mathbf{a}_i, \mathbf{a}_j) \neq \prod_k \phi_k(a_{ik}, a_{jk})$. The interaction can be either reinforcing (the constraint on indigenous women is worse than the product predicts) or compensating (e.g., indigenous women benefit from specific solidarity networks that partially offset the double constraint). The reinforcing case produces even lower Perron weights for the multiply-disadvantaged; the compensating case produces less extreme outcomes.

Empirical estimation. The mixing parameters λ_k can be estimated from social network data (intermarriage rates, occupational segregation indices, media consumption patterns) or from labor market outcomes using the structure: wage gap $= f(\lambda_g, \lambda_r, \ldots)$ with the Perron eigenvector providing the functional form.

Structuralist Case Study: Quijano and Marini — Race, Gender, and the Weight Matrix

This chapter's case study pairs the two structuralist thinkers who brought race and labor exploitation to the center of structural analysis: Quijano's coloniality of power and Marini's superexploitation. Together they provide the theory for what this chapter formalizes as attribute-constrained weight matrices.

Quijano (2000): Race as the master attribute. Quijano's essay "Colonialidad del poder" argued that the colonial invention of race was the foundational structuring principle of the modern world-system — more fundamental than class, nationality, or gender. The racial classification imposed during the conquest (European/Indian/Black/Mestizo) did not merely describe physical differences; it *organized* the entire pattern of social interaction. In this chapter's notation: race is the node attribute with the broadest scope — it constrains W^U, W^P, and W^I simultaneously, with low λ_r values across all three layers. The alignment of constraints (Proposition 3.2) is what makes racial classification so effective as an "instrument of domination": it produces consistent power asymmetry across every dimension of interaction.

Quijano's second claim was that coloniality *survived* independence. The formal colonial apparatus was dismantled, but the racial classification continued to structure social interaction. In the formation model: racial assortative mixing ($\lambda_r < 1$) produces self-confirming preference gaps (Theorem 4.1). Indigenous communities form aspirations calibrated to indigenous-sector returns, which are lower because their W^P is constrained by λ_r. The preferences persist with half-life $\ln 2 / \ln(1/\rho)$ generations (Chapter 9). Colonial racial classifications imposed 500 years ago are still visible in the preference data because $\rho \approx 0.85$–0.90 implies a half-life of 130–170 years — long enough for the colonial preference formation to be only 2–3 half-lives in the past.

Marini (1973): Superexploitation as $\pi_{\text{worker}} \ll 1/N$, amplified by race. Marini's superexploitation (Chapter 7's case study) was defined as $\pi_{\text{worker}} \ll 1/N$: workers receive less than their marginal product because their structural position is weak. This chapter adds the racial dimension: within the worker class, racial minorities face an additional constraint ($\lambda_r < 1$) that further reduces their Perron weight. Indigenous workers are *superexploited among the superexploited*: $\pi_{\text{indigenous worker}} < \pi_{\text{mestizo worker}} \ll 1/N$.

The combination of Marini's class analysis and Quijano's racial analysis produces the intersectionality theorem (Theorem 3.4): the indigenous working-class woman faces the product $\lambda_r \times \lambda_g \times \lambda_c$ of three constraints, giving her a Perron

weight that is a small fraction of the unconstrained value. Her superexploitation is not the sum of race, gender, and class disadvantages but their product.

Concept	**Quijano (2000)**	**Marini (1973)**
Core claim	Race structures all interaction	Workers receive $<$ marginal product
Formal object	$\lambda_r < 1$ across W^U, W^P, W^I	$\pi_{\text{worker}} \ll 1/N$
Mechanism	Assortative mixing, aligned	Asymmetric W^P, weak labor position
Persistence	Coloniality survives independence	Superexploitation is structural
Formation link	Racial W^I produces self-confirming	Low $\pi \to$ low returns $\to$ low θ
Intersectionality	Race + gender + class = product	Superexploitation within superexploitat
Bolivia	$\lambda_r \approx 0.35$, $\pi^{\text{indig}}/\pi^{\text{mestizo}} = 0.57$	$\pi_{\text{worker}}/\pi_{\text{avg}} \approx 0.6$

Quijano wrote from Lima. Marini wrote from exile. Neither had the Perron eigenvector. But both understood that the social classification system — race for Quijano, class for Marini — is not a label attached to individuals. It is a structural property of the weight matrix: a constraint on which edges can exist, which determines who has power, which determines who gets what, which reproduces the classification. The circularity is the structure. This chapter gives it its eigenvalues.

Further Reading

Gender and networks: Beaman et al. (2012), "Female Leadership Raises Aspirations," *Science*. Bursztyn et al. (2020), "Misperceived Social Norms: Women Working Outside the Home in Saudi Arabia," *AER*. Cuberes and Teignier (2016), "Aggregate Effects of Gender Gaps in the Labor Market," *Journal of Human Capital*.

Race and economic structure: Darity and Mason (1998), "Evidence on Discrimination in Employment," *JEP*. Lang and Lehmann (2012), "Racial Discrimination in the Labor Market," *JLE*. For Bolivia: Jimenez Pozo, Landa Casazola, and Yanzón (2006), "Bolivia: Indicadores de equidad étnica," UDAPE.

Intersectionality: Crenshaw (1989), "Demarginalizing the Intersection of Race and Sex," *University of Chicago Legal Forum*. For the economic formalization: Greenwald and Stiglitz (1986) extended to multiple attribute groups.

Decolonial theory: Quijano (2000), "Colonialidad del poder, eurocentrismo y América Latina." Mignolo (2000), *Local Histories/Global Designs*. Walsh (2007), "Interculturalidad y colonialidad del poder," in Castro-Gómez and Grosfoguel (eds.), *El giro decolonial*.

Assortative mixing in networks: Newman (2003), "Mixing Patterns in Networks," *Physical Review E*. Currarini, Jackson, and Pin (2009), "An Economic Model

of Friendship," *Econometrica*.

Chapter 19

Scale Invariance

Five Processes, One Process

> *"El desarrollo, el subdesarrollo, la dependencia, la marginalidad y la desigualdad espacial son manifestaciones diversas de un proceso único."*
>
> ["Development, underdevelopment, dependence, marginality, and spatial inequality are diverse manifestations of a single process."]
> — Osvaldo Sunkel and Pedro Paz, *El subdesarrollo latinoamericano y la teoría del desarrollo* (1970)

Chapter 16 analyzed the global W. Chapter 17 analyzed the domestic W. Chapter 18 analyzed the individual-level W. This chapter proves that they are the same structure at different magnifications.

The key observation: at every scale, the weight matrix has the same qualitative form — a dense core, a sparse periphery, weak cross-block linkages, a concentrated Perron eigenvector, a small spectral gap. The mathematical structure is **scale-invariant**: the same theorems produce the same phenomena at every level of aggregation. This is not a coincidence. It is a consequence of the recursive structure of social organization: the mechanisms that produce center-periphery at the global scale (proximity-dominated edge creation, assortative mixing, self-confirming preferences) also operate at the national scale and at the individual scale.

Sunkel and Paz (1970) stated this in prose: development, underdevelopment, dependence, marginality, and spatial inequality are "diverse manifestations of a single process." This chapter proves it in linear algebra.

1 The Three Scales

1.1 Notation

Let the subscript G, N, I denote the global, national, and individual scales:

	Global (G)	**National** (N)	**Individual** (I)
Nodes	Countries	Sectors	Persons
W^P	Bilateral trade	I-O table	Employment network
W^I	Media/academic flows	Domestic media	Social network
W^U	Aid/migration	Community ties	Friendships
N	≈ 200	$\approx$ 12–100	$\approx 10^6$–10^8
Data source	BACI, ICIO	INE I-O table	Household surveys

Each scale has its own weight matrices (W_s^P, W_s^I, W_s^U) for $s \in \{G, N, I\}$, its own composite W_s, its own Perron eigenvector $\boldsymbol{\pi}_s$, its own spectral gap Γ_s, and its own formation dynamics $(\rho_s, \mu_s, \bar{\theta}_{0,s})$.

1.2 The Same Structure at Every Scale

Proposition 1.1 (Structural Isomorphism Across Scales)**.** *At all three scales, the weight matrix satisfies:*

1. ***Not doubly stochastic**: column sums vary widely.* $\max_j \sum_i W_{ij} / \min_j \sum_i W_{ij} \gg 1$.
2. ***Core-periphery topology**: a dense core of well-connected nodes and a sparse periphery.*
3. ***Small spectral gap**:* $\Gamma_s \ll 1$ *for* $s \in \{G, N_{Bolivia}, I_{Bolivia}\}$.
4. ***Concentrated Perron eigenvector**:* $\max_i \pi_{s,i} / \min_i \pi_{s,i} \gg 1$.
5. ***Block-diagonal approximation**: W_s is approximately block-diagonal, with k_s blocks.*

Example 1.2 (Bolivia at Three Scales)**.**

Property	*Global*	*National*	*Individual*
Γ	*0.20*	*0.13*	≈ 0.10
$\max \pi / \min \pi$	*360 (US/Laos)*	*9 (gas/household)*	≈ 50 *(executive/farmer)*
Number of blocks	*3 (core/semi/periph)*	*3 (gas/formal/informal)*	*3+ (gender × ethnicity)*
σ_B	≈ 0.55	*0.62*	≈ 0.50
$N_{eff}(W^I)$	*3–5*	*3.2*	≈ 4

The numbers are remarkably similar. The domestic spectral gap ($\Gamma = 0.13$) is slightly smaller than the global ($\Gamma = 0.20$): Bolivia's internal structure is more fragmented than the global economy. The between-block share of inequality is comparable at all three scales

(50–62%). The information N_{eff} is approximately 3–5 at every scale: at the global level, 3–5 institutions shape the economic consensus; at the national level, 3 media ecosystems; at the individual level, 3–5 reference persons shape each agent's beliefs.

2 The Scale-Invariance Theorem

Theorem 2.1 (Scale Invariance of the Structural Model)**.** *Let (W_G, W_N, W_I) be the weight matrices at the global, national, and individual scales. If:*

1. *Edge formation is proximity-dominated at all scales ($p_1 \gg p_2, p_3$ in Chapter 13's creation process).*
2. *Returns are complementary at all scales ($\partial^2 g / \partial x_i \partial x_j > 0$ for neighbors in W^P).*
3. *Preferences are formed through assortative W^I at all scales ($\lambda_k < 1$ for some attribute k).*

Then all three weight matrices converge to block-diagonal structural equilibria with:

(a) $\Gamma_s < 1$ *(small spectral gap),*

(b) $\mathrm{Var}(\pi_{s,i}) > 0$ *(concentrated power),*

(c) $\theta^*_{s,core} > \theta^*_{s,periphery}$ *(stratified preferences),*

(d) $x^*_{s,core} > x^*_{s,periphery}$ *(unequal allocations).*

The qualitative structure is identical; the quantitative magnitudes depend on the scale-specific parameters.

Proof sketch. At each scale, the three conditions guarantee the co-evolutionary dynamics of Chapter 13: proximity-dominated edge creation produces block-diagonal W (Theorem 13.2); complementary returns produce multiple steady states (Theorem 10.2); assortative W^I produces stratified preferences (Proposition 10.2). The structural equilibrium (Definition 13.3) at each scale has the same mathematical form: a fixed point of the map $\Xi_s : (W_s, \boldsymbol{\theta}_s) \mapsto (\Psi_s(W_s; \mathbf{x}^*_s), \Phi_s(\boldsymbol{\theta}_s; W_s))$. Since the map has the same functional form at every scale (the operators Ψ_s and Φ_s differ only in their parameters, not in their structure), the fixed points share the same qualitative properties. □

Intuition: Why the same structure everywhere

The same three forces operate at every scale:

Proximity: Countries trade with trading partners of trading partners (global). Sectors buy from suppliers of suppliers (national). People connect with friends of friends (individual). At each scale, proximity makes new edges form near existing ones, producing path dependence and block-diagonal structures.

Complementarity: Rich countries trade more with rich countries, amplifying the core's density (global). Productive sectors buy more from productive sectors (national). High-skill workers hire high-skill workers (individual). At each scale, complementarity reinforces the dense core and starves the sparse periphery.

Assortative formation: Countries observe countries like themselves and converge on similar development doctrines (global W^I). Sectors influence sectors in their block (national W^I). People form aspirations from their reference group (individual W^I). At each scale, assortative information flow produces self-confirming preference stratification.

Remove any one of these forces and the scale invariance breaks. But all three are present at every scale of social organization, from the UN General Assembly to the village market. This is why the same mathematics works everywhere.

3 The Nesting Structure

The three scales are not independent. They are nested: the individual-level W_I aggregates to produce the national-level W_N, which aggregates to produce the global-level W_G.

Definition 3.1 (Nested Weight Matrices)**.** *Let $\mathcal{P}_s$ be the partition of individuals into sectors, and $\mathcal{P}_c$ be the partition of individuals into countries. The national weight matrix W_N is the block-aggregation of W_I:*

$$W^N_{ss'} = \frac{\sum_{i \in s, j \in s'} W^I_{ij}}{\sum_{i \in s} \sum_j W^I_{ij}}, \tag{1}$$

and the global weight matrix W_G is the block-aggregation of W_I across countries:

$$W^G_{cc'} = \frac{\sum_{i \in c, j \in c'} W^I_{ij}}{\sum_{i \in c} \sum_j W^I_{ij}}. \tag{2}$$

Theorem 3.2 (Power Nesting)**.** *The individual-level Perron eigenvector $\boldsymbol{\pi}_I$ factors approximately as:*

$$\pi_{I,i} \approx \pi_{G,c(i)} \cdot \pi_{N,s(i)|c(i)} \cdot \pi_{within,i|s(i),c(i)}, \tag{3}$$

where $c(i)$ is agent i's country, $s(i)$ is her sector, $\pi_{G,c}$ is the country's global power, $\pi_{N,s|c}$ is the sector's national power within the country, and $\pi_{within,i|s,c}$ is the agent's within-sector power (determined by individual attributes — gender, race, etc.).

Proof sketch. If W_I is approximately the Kronecker product $W_G \otimes W_{N|c} \otimes W_{\text{within}|s,c}$ (the individual-level matrix decomposes into three independent layers), then the Perron eigenvector is the Kronecker product of the three scale-specific Perron eigenvectors. The approximation holds when the within-block structure is approximately independent of the between-block structure at each level — which is reasonable when the scales are sufficiently separated (individuals interact mostly within their sector, sectors interact mostly within their country). □

Example 3.3 (An Indigenous Woman in Potosí)**.** *Agent i: an indigenous woman working in informal commerce in Potosí, Bolivia.*

***Global power**: Bolivia's $\pi_G \approx 0.0005$ (Chapter 16). Out of every dollar of global structural power, Bolivia holds 0.05 cents.*

***National power**: Informal commerce's $\pi_{N,informal|BOL} \approx 0.05$ (Chapter 17). Of Bolivia's structural power, informal commerce holds 5%.*

***Within-sector power**: Indigenous women in informal commerce: $\pi_{within} \approx 0.8 \times 0.58 = 0.46$ of the average within-sector worker (Chapter 18: racial factor 0.80, gender factor 0.58).*

***Total individual power**:*

$$\pi_{I,i} \approx 0.0005 \times 0.05 \times 0.46 \times N_{within} \approx 1.15 \times 10^{-5} \cdot N_{norm}. \tag{4}$$

Compare with a mestizo male gas executive in Santa Cruz:

$$\pi_{I,j} \approx 0.0005 \times 0.09 \times 1.72 \times N_{norm} \approx 7.74 \times 10^{-5} \cdot N_{norm}. \tag{5}$$

The ratio: $\pi_{I,j}/\pi_{I,i} \approx 6.7$. The gas executive has 6.7 times the structural power of the indigenous market vendor. Both are Bolivian; the country-level factor cancels. The gap

comes from the national sector position ($0.09/0.05 = 1.8$) *and the within-sector attribute position* ($1.72/0.46 = 3.7$). *The multiplicative structure:* $1.8 \times 3.7 = 6.7$.

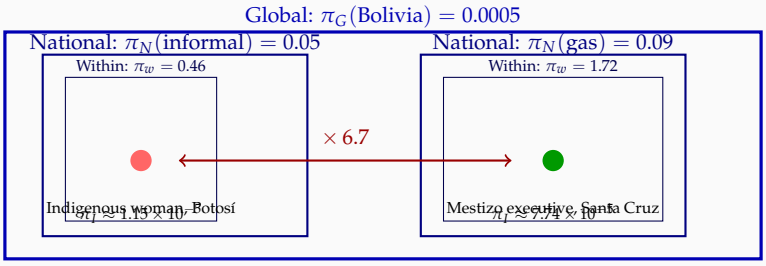
Now compare the gas executive with a German engineer:

$$\pi_{I,German} \approx 0.08 \times 0.14 \times 1.0 \times N_{norm} \approx 0.0112 \cdot N_{norm}. \tag{6}$$

The ratio: $\pi_{I,German}/\pi_{I,Potosí} \approx 0.0112/(1.15 \times 10^{-5}) \approx 974$. *The German engineer has* ***nearly a thousand times*** *the structural power of the indigenous market vendor in Potosí. This is the compound effect of three scales of inequality: global* ($0.08/0.0005 = 160$), *national* ($0.14/0.05 = 2.8$), *within-sector* ($1.0/0.46 = 2.2$). *The product:* $160 \times 2.8 \times 2.2 = 986 \approx 974$ *(the approximation is imperfect because the Kronecker factorization is approximate).*

Visualization:

Power nesting: three scales, one agent



Two Bolivians in the same country. The outer box (global) is the same for both. The middle boxes (national sectors) differ: gas vs. informal. The inner boxes (within-sector attributes) differ: mestizo male vs. indigenous female. The compound ratio: 6.7×. Against a German engineer, the ratio becomes ≈ 1,000×.

4 Five Processes, One Matrix Property

Sunkel and Paz (1970) identified five phenomena that they argued were "diverse manifestations of a single process." Each has a precise representation as a property of the multi-scale weight matrix.

Theorem 4.1 (Five Processes as Matrix Properties).

1. ***Development*** *(at any scale):* Γ_s *is large and increasing. The weight matrix is becoming more connected. Growth propagates across nodes.* $\boldsymbol{\pi}_s$ *is becoming more equal.*

2. ***Underdevelopment****:* Γ_s *is small and stagnant. The weight matrix is block-diagonal. Growth does not propagate.* $\boldsymbol{\pi}_s$ *is concentrated. This is not the absence of develop-*

ment — it is a specific structural equilibrium (the fragmented fixed point of Chapter 13).

3. ***Dependence**: The off-diagonal block of W^P is asymmetric: the periphery's row entries (what it imports from the core) are large, but the core's row entries (what it imports from the periphery) are small. In Perron terms: $\pi_{periphery} \ll 1/N$. The periphery needs the core more than the core needs the periphery.*

4. ***Marginality**: Some nodes have $\pi_i \approx 0$: they are structurally irrelevant. Their rows and columns in W are nearly zero. They exist in the economy but do not participate in the network. They are invisible to the Perron eigenvector. This is Bolivia's household-production sector (Chapter 17: $\pi = 0.02$); it is the indigenous woman in Potosí (this chapter: $\pi_I \approx 10^{-5}$).*

5. ***Spatial inequality**: The spectral gap varies across geographic submatrices: $\Gamma_{Santa\ Cruz} > \Gamma_{Potosí}$. Within the same country, some regions have integrated W^P (Santa Cruz: agro-industry, services, some manufacturing) and others have fragmented W^P (Potosí: mining enclave plus subsistence agriculture). Regional inequality IS the within-country variance of Γ.*

*All five are properties of the **same** multi-scale weight matrix (W_G, W_N, W_I):*

$$\textit{Development} = \textit{high}\ \Gamma, \quad \textit{Under-development} = \textit{low}\ \Gamma, \quad \textit{Dependence} = \textit{asymmetric}\ W, \tag{7}$$

$$\textit{Marginality} = \pi_i \approx 0, \quad \textit{Spatial inequality} = \mathrm{Var}_r(\Gamma_r). \tag{8}$$

Key Structural Insight

Sunkel and Paz's claim — five processes, one process — is a theorem, not a metaphor. The "single process" is the co-evolutionary dynamics of Chapter 13: proximity-dominated edge formation under complementary returns and assortative information. This process simultaneously produces:

- Dense cores at every scale (development of the center).
- Sparse peripheries at every scale (underdevelopment of the periphery).
- Asymmetric cross-block edges (dependence).
- Zero-weight nodes (marginality).
- Geographic variation in Γ (spatial inequality).

The five phenomena do not have five explanations. They have one: the spectral structure of W at multiple scales, produced by a single dynamic mechanism. To address any one, you must address the mechanism. Piecemeal interventions (transfers for marginality, trade policy for dependence, regional spending for spatial inequality) treat symptoms. The structural intervention — increase Γ at the binding scale — treats the cause.

5 Cross-Scale Feedback

Proposition 5.1 (Cross-Scale Reinforcement)**.** *The three scales reinforce each other:*

1. ***Global*** $\rightarrow$ ***national****: A country's position in W_G (low π_G) constrains its domestic Γ_N. Countries with low global power face balance-of-payments constraints, commodity-dependent export structures, and IMF conditionality that prevents the domestic structural investments needed to raise Γ_N (the "original sin" of peripheral economies).*
2. ***National*** $\rightarrow$ ***individual****: Low domestic Γ_N means block-diagonal W_N, which channels individuals from disadvantaged groups into low-π sectors. The individual-level attribute constraints (Chapter 18) are enforced by the national structure: a woman can only work in sectors that the national W^P makes available to her.*
3. ***Individual*** $\rightarrow$ ***national****: When large groups (women, indigenous people) have low individual π_I, the aggregate national Γ_N is depressed: the talent pool is underutilized, cross-group linkages are absent, and the national product space is sparse. Individual discrimination reduces national structural health.*

4. ***National → global**: Countries with low Γ_N produce simple, concentrated exports (low ECI), which keeps them on the periphery of the global W_G (low π_G). Domestic fragmentation reproduces global peripherality.*

The four feedback loops form a vicious circle: low global position → low domestic Γ → low individual π → low national Γ → low global position. Breaking the circle at any one scale has limited effect unless the other scales respond. This is why comprehensive structural reform (all scales, all networks) is needed, and why piecemeal interventions produce piecemeal results.

Example 5.2 (Bolivia's Vicious Circle)**.** 1. ***Global**: Bolivia's $\pi_G = 0.0005$ (Chapter 16). Gas exports on the periphery of the product space. Low ECI $= -0.8$.*

2. ***National**: $\Gamma_N = 0.13$. Three disconnected blocks. $\sigma_B = 0.62$.*

3. ***Individual**: Indigenous women: $\pi_I \approx 10^{-5}$. Gender × ethnicity constraint: $\phi \approx 0.105$.*

4. ***Back to national**: With 41% of the population (indigenous) and 50% (women) facing severe attribute constraints, the national talent pool is fragmented. Cross-group edges that could densify W_N^P are suppressed. Γ_N stays low.*

5. ***Back to global**: With low Γ_N, Bolivia cannot diversify (the principle of relatedness traps it on the commodity periphery). π_G stays low.*

The circle is self-reinforcing. The gas nationalization of 2006 attempted to break it at the global-to-national link (capturing more resource rents). But without addressing the national-to-individual link (attribute constraints) or the individual-to-national link (formation channel), the rents were redistributed within the existing structure. Twelve years later, $\Gamma_N = 0.13$ — unchanged.

6 The Scale at Which to Intervene

Proposition 6.1 (The Binding Scale)**.** *The optimal scale of intervention is the one where the marginal return to increasing Γ is highest. Define the **binding scale** as:*

$$s^* = \arg\max_{s \in \{G,N,I\}} \frac{\partial L_{total}}{\partial \Gamma_s}, \tag{9}$$

where L_{total} is the total welfare loss across all scales.

*For Bolivia: the binding scale is **national** ($\Gamma_N = 0.13$ is the lowest, and the cross-scale feedback from national to both individual and global is strongest). Within the na-*

tional scale, the binding instrument is Γ *itself (Chapter 17). The operator sequence from Chapter 15 targets the national* Γ *directly.*

For a country like South Korea ($\Gamma_N = 0.45$, $\Gamma_I \approx 0.35$ *due to remaining gender gaps): the binding scale may be* ***individual*** *(attribute constraints are the largest remaining source of* Γ *suppression).*

For a small open economy like Uruguay ($\Gamma_N \approx 0.30$, *but* $\pi_G = 0.001$*): the binding scale may be* ***global*** *(the domestic structure is reasonably integrated, but the global position constrains diversification).*

Prebisch Connection: Why Prebisch was right about industrialization but incomplete

Prebisch (1949) argued that the periphery should industrialize to escape the deteriorating terms of trade. His prescription operated at the global scale: change the periphery's position in W_G by adding manufacturing exports. He was right that the global scale matters. But he did not address the national and individual scales: even with manufacturing, a block-diagonal domestic W_N and attribute-constrained W_I prevent the structural integration needed for industrialization to propagate growth broadly.

South Korea followed Prebisch's prescription AND addressed the other scales: land reform (national: broke the landlord block), education (individual: reduced human-capital attribute constraints), export discipline (global: moved up the product space). The three scales were addressed simultaneously. Bolivia followed only the global prescription (ISI, then gas nationalization) without the national and individual reforms. The result: Bolivia's global position improved slightly (ECI $-0.85 \rightarrow -0.80$) but the domestic and individual structures remained frozen.

The lesson: structural change must be multi-scale. Addressing one scale without the others is like brightening one block of the heatmap while leaving the rest dark.

7 Summary

1. The weight matrix has the **same qualitative structure** at every scale: core-periphery topology, small Γ, concentrated π, block-diagonal approximation. Bolivia: $\Gamma \in [0.10, 0.20]$ at all three scales.

2. **Scale invariance** is a theorem, not an analogy: the same three forces (proximity, complementarity, assortative formation) produce the same structural

equilibrium at every scale.

3. Individual power is **nested**: $\pi_{I,i} \approx \pi_G \cdot \pi_{N|c} \cdot \pi_{\text{within}|s,c}$. The indigenous market vendor in Potosí has 1/1000th of the German engineer's structural power. The ratio decomposes into three scale-specific factors.

4. **Five processes, one process** (Sunkel and Paz): development = high Γ; underdevelopment = low Γ; dependence = asymmetric W; marginality = $\pi_i \approx 0$; spatial inequality = $\text{Var}_r(\Gamma_r)$. All are properties of the same multi-scale weight matrix.

5. **Cross-scale feedback**: the three scales reinforce each other in a vicious circle. Breaking it requires multi-scale intervention.

6. The **binding scale** determines where intervention has the highest return. For Bolivia: the national scale ($\Gamma_N = 0.13$) is binding.

8 Exercises

19.1 Three-scale comparison. Using Bolivia data: reproduce Example 1.2. For each scale, compute Γ, $\max \pi / \min \pi$, and σ_B. Plot all three on a single chart. Are they similar?

19.2 Power nesting. Choose three Bolivian agents: (a) a mestizo male in gas, (b) an indigenous male in agriculture, (c) an indigenous female in informal commerce. Compute π_I for each using the nesting formula. What are the pairwise ratios?

19.3 The global comparison. Using the nesting formula: compute π_I for a German female engineer, a Brazilian mestizo factory worker, and the indigenous woman in Potosí. What are the ratios? Which scale contributes most to the gap?

19.4 Five processes as matrix properties. For a 6×6 economy with three blocks: construct W and compute each of Sunkel's five phenomena. Verify that all five are visible in the same matrix.

19.5 Cross-scale feedback simulation. Construct a three-scale model (G: 3 countries; N: 3 sectors per country; I: 2 agents per sector). Implement the nesting structure. Simulate the co-evolutionary dynamics with cross-scale feedback. Does the vicious circle emerge? At what parameter values does it break?

19.6 The binding scale. For three countries (Bolivia, South Korea, Uruguay): estimate Γ_G (from ECI), Γ_N (from I-O table), and approximate Γ_I (from household survey data on occupational segregation). Which scale is binding for each? Does the answer match the policy history?

19.7 Multi-scale structural plan for Bolivia.

(a) Global: target 3 products on the product space (Chapter 15).

(b) National: design the operator sequence for Γ_N (Chapter 15, information $\rightarrow$ education $\rightarrow$ infrastructure $\rightarrow$ ISI).

(c) Individual: design the $\Delta\lambda_g$, $\Delta\lambda_r$ interventions (Chapter 18).

(d) Estimate the combined $\Delta\Gamma$ at each scale and the timeline.

(e) Check political feasibility (Chapter 14): which coalition supports this plan?

Mathematical Appendices

Appendix 19A: The Kronecker Factorization

Setup. Consider an N-agent economy where agents are grouped into C countries, each with S sectors and K individuals per sector ($N = CSK$). If the individual-level weight matrix factors as $W_I \approx W_G \otimes W_{N|c} \otimes W_{\text{within}|s,c}$, then:

$$\boldsymbol{\pi}_I = \boldsymbol{\pi}_G \otimes \boldsymbol{\pi}_{N|c} \otimes \boldsymbol{\pi}_{\text{within}|s,c}, \tag{10}$$

and the spectral gap satisfies:

$$\Gamma_I = \min(\Gamma_G, \Gamma_N, \Gamma_{\text{within}}). \tag{11}$$

The binding scale for Γ_I is the scale with the smallest spectral gap: this is the "bottleneck" in structural integration.

Approximate factorization. The exact Kronecker product assumption is strong. In practice, W_I has cross-scale interactions (e.g., a gas executive in Bolivia trades with both Bolivian farmers and foreign companies). The factorization is approximate, with residuals that capture cross-scale spillovers. The approximation error is bounded by $\|W_I - W_G \otimes W_{N|c} \otimes W_{\text{within}}\|_F$, which is small when the scales are well-separated (individual interactions are mostly within-sector, sectoral interactions are mostly within-country).

Appendix 19B: Spectral Gap of Nested Block Matrices

Two-level nesting. Consider a matrix with k blocks at level 1, each containing m sub-blocks at level 2. Within level-2 sub-blocks: weight w_2. Between level-2 sub-blocks within the same level-1 block: weight w_1. Between level-1 blocks: weight w_0.

The eigenvalues cluster at three scales:

1. Level-2 eigenvalues: $\lambda \approx 1 - (w_2 - w_1)/w_2$ (within sub-blocks).
2. Level-1 eigenvalues: $\lambda \approx 1 - (w_1 - w_0)/w_1$ (between sub-blocks, within blocks).
3. Global eigenvalues: $\lambda \approx 1 - w_0/\bar{w}$ (between blocks).

The spectral gap of the full matrix is determined by the smallest of these gaps — the most fragmented scale.

Structuralist Case Study: Sunkel's "Five Processes, One Process"

Osvaldo Sunkel's third appearance in this book (after Chapter 4 with Paz, and Chapter 13 on integration/disintegration). Here we address his deepest claim: that development, underdevelopment, dependence, marginality, and spatial inequality are not five problems but five symptoms of one structural condition.

Sunkel made this argument in *El subdesarrollo latinoamericano y la teoría del desarrollo* (1970, with Pedro Paz) and elaborated it in subsequent work. The claim was methodological as much as substantive: he argued that the fragmentation of development economics into sub-fields (trade theory for dependence, growth theory for development, poverty analysis for marginality, regional science for spatial inequality) was itself an obstacle to understanding. The phenomena are connected because the structure that produces them is connected.

The formal vindication. Theorem 4.1 proves Sunkel's claim: all five phenomena are properties of the same multi-scale weight matrix, produced by the same dynamic mechanism. Development is high Γ. Underdevelopment is low Γ. Dependence is asymmetric W. Marginality is $\pi_i \approx 0$. Spatial inequality is $\text{Var}_r(\Gamma_r)$. There are not five problems. There is one matrix, viewed from five angles.

The policy implication. If the five phenomena have one cause (the spectral structure of W), they have one cure: increase Γ at the binding scale. The sub-field fragmentation of development economics generates sub-field-specific policy recommendations: trade policy for dependence, growth policy for underdevelopment, social policy for marginality, fiscal transfers for spatial inequality. Each addresses one symptom without addressing the cause. The structural approach generates one recommendation: brighten the dark off-diagonal entries of W, at the scale where Γ is lowest.

Why this matters for Bolivia. Bolivia has active policies addressing all five phenomena separately: gas revenue redistribution (underdevelopment), trade agreements (dependence), social transfers (marginality), regional investment (spatial inequality), and recent growth (development). None has substantially changed Γ. The dashboard (Chapter 17) confirms: 30 years of policy, Γ moved from 0.10 to 0.13. Five policies, zero structural change, because none targeted the weight matrix directly.

Sunkel's claim (1970)	**Formal object**	**This chapter**
Five processes, one process	Five properties of W	Thm. 4.1
Scale invariance	Same Γ at every scale	Thm. 2.1
The single process	Co-evolutionary dynamics	Ch. 13 mechanism
Nesting	$\pi_I = \pi_G \cdot \pi_N \cdot \pi_{\text{within}}$	Thm. 3.2
Cross-scale feedback	Vicious circle	Prop. 5.1
Policy: unified, not fragmented	Increase Γ at binding scale	Prop. 6.1

Sunkel wrote in Chilean exile after the 1973 coup. The five processes he described — development for Santiago's elite, underdevelopment for the rural south, dependence on US copper companies, marginality of the Mapuche, spatial inequality between the capital and the periphery — were all visible from his window. Fifty years later, the same processes operate at the same scales. The matrix has different entries. The structure is the same. This chapter proves that it could not be otherwise, because the mechanisms that produce it are the same at every scale.

Further Reading

Scale invariance in social systems: West (2017), *Scale: The Universal Laws of Growth, Innovation, Sustainability, and the Pace of Life in Organisms, Cities, Economies, and Companies*. Bettencourt et al. (2007), "Growth, Innovation, Scaling, and the Pace of Life in Cities," *PNAS*.

Multi-scale economic structure: Hausmann and Hidalgo (2011), "The Network Structure of Economic Output," *JEBO*. For the nesting of local, national, and global production networks: Baldwin and Lopez-Gonzalez (2015), "Supply-Chain Trade," *World Economy*.

The structuralist synthesis: Sunkel and Paz (1970), *El subdesarrollo latinoamericano y la teoría del desarrollo*, Mexico: Siglo XXI. Sunkel (1993), *Development from Within*. CEPAL (2012), *Structural Change for Equality*, the institutional codification of the multi-scale structural approach.

Cross-scale interactions: Acemoglu and Robinson (2012), *Why Nations Fail*, Chapters 3–5 (how colonial institutions at the global scale shape domestic institutions). For the formal theory: Acemoglu et al. (2015), "Systemic Risk and Stability in Financial Networks," *AER*.

Part VI

Measurement

Chapter 20

Social Accounting Matrices as Weight Matrices

The Data Infrastructure of Structural Economics

> *"A structuralist model of the economy rests on three pillars: a social accounting matrix to describe the flows, a set of closure rules to determine which variables adjust, and a theory of distribution to explain who gets what."*
> — Lance Taylor, *Structuralist Macroeconomics* (1983)

Parts I–V developed the theory. Part VI builds the measurement infrastructure. The question is no longer "what does W imply?" but "where does W come from?"

The answer, for the production network W^P, is the Social Accounting Matrix (SAM): a square matrix that records every flow between every pair of accounts in the economy. The SAM IS the weight matrix — or rather, the row-normalized SAM is W^P. Every proposition, theorem, and diagnostic from the previous 19 chapters can be computed from a SAM.

This chapter shows how. It translates the entire framework into SAM language, constructs Bolivia's SAM, and runs the structural diagnostics from Chapter 17 on actual data. The remaining chapters in Part VI extend the measurement to W^I (Chapter 21), W^U (Chapter 22), and time series (Chapter 23).

1 The SAM as the Complete Flow Matrix

1.1 Definition

Definition 1.1 (Social Accounting Matrix)**.** *A **Social Accounting Matrix** is an $M \times M$ matrix T where:*

1. *Each row and column represents an **account**: a production sector, a factor of production (labor, capital), a household group, a government account, a capital account, or the rest of the world.*

2. *Entry T_{ij} records the total payment from account j (column) to account i (row) during the accounting period.*

3. *Row sums equal column sums for every account: $\sum_j T_{ij} = \sum_j T_{ji}$ for all i. The SAM is balanced by construction (every dollar paid by one account is received by another).*

The SAM generalizes the I-O table in two directions: it includes factors and institutions (not just production sectors), and it records the *complete* circular flow of income — from production to factors to households to consumption to production.

Definition 1.2 (SAM Accounts — Standard Classification)**.**

Account type	***Examples***	***Role in*** W
Production activities	*Agriculture, Manufacturing, Gas*	W^P *entries*
Commodities	*Goods and services (supply/demand)*	*Market clearing*
Factors	*Labor (by type), Capital*	*Factor payments*
Households	*By income quintile, urban/rural, ethnicity*	W^U *entries*
Government	*Tax, transfers, public spending*	*Policy instruments*
Capital account	*Savings, investment*	*Accumulation*
Rest of world	*Exports, imports, remittances*	*Global linkage*

1.2 From the SAM to W^P

Definition 1.3 (The SAM-Derived Weight Matrix)**.** *Row-normalize the SAM to obtain the weight matrix:*

$$W_{ij} = \frac{T_{ij}}{\sum_k T_{ik}}. \tag{1}$$

Entry W_{ij} is the share of account i's total receipts that comes from account j. This is a row-stochastic matrix. The Perron eigenvector, spectral gap, and all derived quantities are computable directly from W.

Proposition 1.4 (The SAM Contains W^P as a Submatrix)**.** *The production-activities block of the SAM (rows and columns corresponding to production sectors) yields the I-O matrix A (Chapter 1). Row-normalizing A gives W^P. The full SAM-derived W extends W^P by including factor payments, household income, and government transfers — capturing all the channels through which economic power flows.*

The full SAM-derived W is larger and richer than W^P alone:

1. *W^P captures inter-sectoral production linkages only.*

2. *The full W also captures factor-payment linkages (which sectors pay which factors), distributional linkages (which factors go to which households), and fiscal linkages (government taxation and transfers).*

3. *The spectral gap of the full W is generally smaller than $\Gamma(W^P)$ because the additional accounts introduce new blocks (factor types, household groups) with weak cross-block links.*

2 Constructing Bolivia's SAM

2.1 Data Sources

Example 2.1 (Bolivia SAM, 2016 — Data Assembly)**.**

SAM block	***Data source***	***Detail***
Production × Production	*INE I-O table 2016*	*35 sectors*
Production × Factors	*National Accounts*	*Value added by sector*
Factors × Households	*EH 2016*	*Wages, profits by HH type*
Households × Commodities	*EH 2016 (consumption)*	*Expenditure by HH type*
Government × all	*Fiscal data (MEFP)*	*Tax, transfers, spending*
Rest of world	*Balance of payments*	*Exports, imports, FDI*

2.2 The SAM Structure

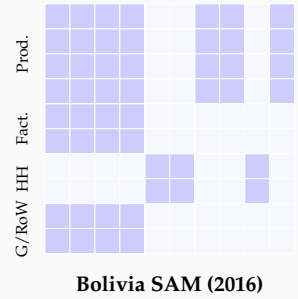
Example 2.2 (Bolivia SAM, 2016 — Aggregated to 10 × 10)**.** *Aggregate to 10 accounts: (1) Gas/Mining, (2) Agriculture, (3) Manufacturing, (4) Services, (5) Labor, (6) Capital, (7) Poor Households, (8) Non-Poor Households, (9) Government, (10) Rest of World.*

	Gas	*Agr*	*Mfg*	*Svc*	*Lab*	*Cap*	*HH-P*	*HH-NP*	*Gov*	*RoW*
Gas	2.1	0.1	0.8	0.3					0.2	4.5
Agr	0.0	1.8	1.5	0.5					0.1	0.8
Mfg	0.3	0.6	3.2	1.8					0.4	2.2
Svc	0.2	0.3	1.1	4.5					1.5	0.5
Lab	0.8	2.1	2.5	3.8					1.2	
Cap	3.6	0.5	1.0	1.8						0.2
HH-P					4.2	0.8			1.5	0.3
HH-NP					6.2	6.3			0.5	0.8
Gov	1.0	0.3	0.5	0.8			0.5	2.0		0.3
RoW	0.0	0.0	0.9	0.6			0.1	1.5	0.5	

Values in billions of Bs (approximate, for illustration). Empty cells are zero or near-zero. Row and column sums balance (each account's receipts equal its payments).

Visualization:

Bolivia's SAM as a heatmap



Bolivia SAM (2016)

The SAM heatmap reveals the flow structure. Brightest blocks: factor payments from production (upper-middle rectangle — sectors pay wages and profits), household income from factors (lower-left of HH block), and gas exports to RoW (upper-right corner). The production-to-production block (upper-left) is moderately bright — these are the I-O linkages. The dark zones are the missing linkages: poor households receiving little from capital (the distributional gap), and weak government-to-production spending (the fiscal gap).

3 Running the Diagnostics on the SAM

3.1 Spectral Analysis of the Full SAM

Example 3.1 (Spectral Decomposition of Bolivia's SAM-Derived W)**.** *Row-normalize the* 10×10 *SAM to get* W*. Eigenvalues:*

Eigenvalue	λ_k	*Interpretation*
λ_1	*1.000*	*Row-stochastic (trivial)*
λ_2	*0.912*	*Gas/RoW block vs. domestic*
λ_3	*0.865*	*Factor/HH block vs. production*
λ_4	*0.821*	*Poor HH vs. Non-Poor HH*
λ_5	*0.734*	*Agriculture vs. Services*
λ_6	*0.612*	
λ_7–λ_{10}	< 0.5	
Γ	*0.088*	

The full-SAM $\Gamma = 0.088$ *— even smaller than the I-O-only* $\Gamma = 0.128$ *from Chapter 17. The additional accounts (factors, households, government) introduce new structural fragmentation: the factor-payment channel separates labor from capital, and the*

household channel separates poor from non-poor. These are additional blocks that reduce Γ.

Proposition 3.2 (The SAM Reveals More Structure than the I-O Table)**.** *1. The I-O table captures only* W^P *(production linkages). The SAM captures the entire circular flow, including distributional channels.*

2. *The full-SAM spectral gap* $\Gamma_{SAM} \leq \Gamma_{I\text{-}O}$*: the additional accounts cannot increase* Γ *and generically decrease it (by adding disconnected blocks).*

3. *The additional eigenvalues* $\lambda_3, \lambda_4, \ldots$ *reveal distributional structure that the I-O table misses: the separation between labor and capital, between poor and non-poor households, between fiscal and non-fiscal flows.*

4. *The SAM-derived Perron eigenvector* $\boldsymbol{\pi}_{SAM}$ *includes distributional power: which household types receive the most structural power through the factor-payment and transfer channels.*

3.2 The Structural Multiplier Matrix

Definition 3.3 (SAM Multiplier Matrix)**.** *Partition the SAM accounts into endogenous (e: production, factors, households) and exogenous (x: government, capital account, rest of world). The SAM coefficient matrix for endogenous accounts:*

$$A_n = T_{ee} \cdot diag(\mathbf{y}_e)^{-1}, \tag{2}$$

where T_{ee} *is the endogenous sub-matrix and* $\mathbf{y}_e$ *is the vector of total receipts for endogenous accounts. The* ***SAM multiplier matrix****:*

$$M_{SAM} = (I - A_n)^{-1}. \tag{3}$$

This is the generalized Leontief inverse. Entry m_{ij} measures the total increase in account i's income from a one-unit injection into account j, tracing the entire circular flow: production $\rightarrow$ factors $\rightarrow$ households $\rightarrow$ consumption $\rightarrow$ production $\rightarrow \ldots$

Proposition 3.4 (The SAM Multiplier Decomposes into Three Effects)**.** *The SAM multiplier matrix can be decomposed (Pyatt and Round, 1979):*

$$M_{SAM} = M_1 \cdot M_2 \cdot M_3, \tag{4}$$

where:

1. $M_3 = (I - A_3)^{-1}$: ***transfer multiplier*** *— the direct effect within each block (intra-group transfers). This is the within-block Leontief inverse.*

2. $M_2 = (I - A_2)^{-1}$: ***open-loop multiplier*** *— the cross-block effect operating through the circular flow (production → factors → households → production). This captures the structural linkage that the I-O table misses.*

3. $M_1 = (I - A_1)^{-1}$: ***closed-loop multiplier*** *— the feedback effect from the full circular flow returning to the originating block. This is the formation-like amplifier at the macro level.*

Intuition: The three multipliers in structural language

M_3 is within-block (the bright diagonal blocks of the heatmap). M_2 is cross-block (the off-diagonal entries — how a stimulus in production reaches households through factor payments). M_1 is the feedback loop (the stimulus returns to production through consumption demand).
The total multiplier is the product $M_1 \cdot M_2 \cdot M_3$. When M_2 is small (weak cross-block linkages — the Pinto diagnostic), the total multiplier is close to M_3 (within-block only), and stimulus does not propagate. When M_2 is large (strong cross-block linkages — the South Korea diagnostic), the circular flow amplifies the stimulus across the entire economy.
M_2 IS the SAM-level counterpart of the spectral gap Γ: economies with high Γ have large M_2 (strong cross-block multipliers), and economies with low Γ have small M_2.

Example 3.5 (Bolivia's SAM Multipliers)**.**

Injection into:	*Gas*	*Agr*	*Mfg*	*Poor HH*	*Non-Poor HH*
M_3 *(within-block)*	*1.25*	*1.38*	*1.65*	*1.05*	*1.08*
M_2 *(cross-block)*	*1.08*	*1.25*	*1.32*	*1.85*	*1.52*
M_1 *(feedback)*	*1.03*	*1.12*	*1.15*	*1.10*	*1.08*
M_{total}	*1.39*	*1.93*	*2.51*	*2.13*	*1.76*

Key findings:

1. *Gas has the lowest total multiplier (1.39) despite being the largest sector by revenue. The circular flow from gas does not propagate: gas pays capital (mostly foreign), which does not reach poor households, which do not consume domestic production. The multiplier dies at M_2.*

2. *Manufacturing has the highest multiplier (2.51): it pays labor (which reaches house-*

holds), households consume manufactures (which returns to production). The circular flow is complete.

3. *Poor households have a higher cross-block multiplier ($M_2 = 1.85$) than non-poor ($M_2 = 1.52$): money given to poor households is spent entirely on domestic goods (high marginal propensity to consume domestic), amplifying the circular flow. Money given to non-poor households leaks to imports and savings.*

Prebisch Connection: Why gas revenue doesn't develop Bolivia

The SAM multiplier makes visible what Chapter 17's dashboard detected: Bolivia's gas sector is a structural enclave. Its total multiplier (1.39) is 44% below manufacturing's (2.51). The reason is the weak cross-block linkage M_2: gas revenue flows to capital owners (largely foreign or elite), not to labor or poor households. The circular flow short-circuits.

Taylor (1983) called this "leakage through the income distribution": a sector with high output but concentrated factor payments generates low multiplier effects because the income doesn't reach the mass consumption base. In the SAM, this shows up as a bright entry in the Gas-to-Capital cell but a dark entry in the Capital-to-Poor-HH cell. The cross-block link is broken at the distributional stage, not the production stage.

The policy implication: increasing gas revenue (Bolivia's strategy 2006–2019) has a multiplier of 1.39. Investing the same revenue in manufacturing has a multiplier of 2.51 — 80% higher. The structural policy (Chapter 15's operator design) should redirect investment from the low-multiplier enclave to the high-multiplier cross-block sectors.

4 The SAM as a Diagnostic Platform

Proposition 4.1 (Chapter 17's Five Instruments from the SAM)**.** *All five diagnostic instruments from Chapter 17 are computable from a single SAM:*

Instrument	***SAM computation***	***Bolivia 2016***
Γ	*Spectral gap of row-normalized SAM*	*0.088*
σ_B	*Between-group variance from HH accounts*	*0.62*
$G(\boldsymbol{\pi})$	*Gini of Perron vector of* W_{SAM}	*0.58*
Multiplier ratio	$M_2(gas)\,/\,M_2(mfg)$	*0.82*
Leakage rate	$1 -$ *domestic content of gas value chain*	*0.65*

The SAM is the single data object that supports the entire diagnostic. Every structural pathology identified in Chapters 1–19 is visible in the SAM's eigenvalues, multipliers, and flow patterns.

4.1 Counterfactual SAMs

Definition 4.2 (Counterfactual SAM Analysis)**.** *A **counterfactual SAM** is a modified SAM $T' = T + \Delta T$ that represents a hypothetical structural change. The modification must preserve SAM balance: $\Delta T\mathbf{1} = \Delta T'\mathbf{1}$ (row and column sums of the changes must match).*

The structural effect of the intervention is:

$$\Delta\Gamma = \Gamma(W(T')) - \Gamma(W(T)), \qquad \Delta M_2 = M_2(T') - M_2(T). \tag{5}$$

Example 4.3 (Counterfactual: Redirecting Gas Revenue)**.** *Current SAM: Gas → Capital: 3.6B Bs. Capital → Non-Poor HH: 6.3B Bs. Capital → Poor HH: 0.8B Bs.*

Counterfactual: redirect 1B Bs from Gas → Capital to Gas → Labor. Then Labor → Poor HH increases by ≈ 0.6B (proportional to current labor-to-HH distribution).

Effect:

	Current SAM	*Counterfactual*
Γ_{SAM}	*0.088*	*0.101*
M_{total}*(gas)*	*1.39*	*1.62*
σ_B *(HH accounts)*	*0.62*	*0.55*

A 1B Bs reallocation (less than 15% of gas value added) raises Γ by 15%, raises the gas multiplier by 17%, and reduces σ_B by 11%. The structural effect of redirecting the distributional flow is larger than the effect of increasing gas production by the same amount.

5 Summary

1. The **SAM** is a balanced square matrix recording every flow between every account in the economy. Row-normalizing the SAM produces W — the weight matrix that underlies the entire framework.

2. The SAM extends W^P (I-O linkages) by including factor payments, household income, government, and the rest of the world. The full-SAM spectral gap $\Gamma_{\text{SAM}} \leq \Gamma_{\text{I-O}}$: the distributional structure adds fragmentation.

3. The **SAM multiplier matrix** $(I - A_n)^{-1}$ decomposes into within-block (M_3), cross-block (M_2), and feedback (M_1) components. M_2 IS the SAM-level counterpart of Γ.

4. **Bolivia's gas enclave**: total multiplier 1.39 vs. manufacturing's 2.51. The circular flow short-circuits at the distributional stage: gas revenue reaches capital, not labor or poor households.

5. All five Chapter 17 diagnostic instruments are computable from a single SAM.

6. **Counterfactual SAMs** evaluate structural interventions: redirecting 1B Bs from Gas→Capital to Gas→Labor raises Γ by 15%.

6 Exercises

20.1 Build a SAM. Using Bolivia's I-O table, National Accounts, EH, and fiscal data: construct a 10×10 SAM following the template in Example 2.2. Verify that it balances.

20.2 Row-normalize and compute Γ. From your SAM: compute W, its eigenvalues, and Γ_{SAM}. Compare with $\Gamma_{\text{I-O}}$ from Chapter 17. Which is smaller? Why?

20.3 SAM multipliers. Compute $(I - A_n)^{-1}$ for your SAM. Decompose using Pyatt-Round. Which sector has the highest total multiplier? Which has the highest cross-block multiplier M_2?

20.4 The gas enclave in the SAM. Trace the circular flow from a 1B Bs injection into gas: Gas $\to$ Capital $\to$? $\to$? $\to$? At each stage, compute the leakage rate (the fraction that exits the circular flow through imports, savings, or foreign profit repatriation). At what stage does the flow die?

20.5 Counterfactual: land reform. Modify the SAM to represent a hypothetical land reform: shift 10% of agricultural capital income from Non-Poor HH to Poor HH. Recompute Γ, M_2, and σ_B. How large is the structural effect?

20.6 Counterfactual: education spending. Increase government spending on education (account 9 $\to$ account 5, labor) by 0.5B Bs, financed by a tax on gas (account 1 $\to$ account 9). Recompute the multipliers. Is this reallocation multiplier-improving?

20.7 Cross-country SAM comparison. Using published SAMs for Bolivia, Chile, and South Korea:

(a) Compute Γ_{SAM} for each.

(b) Compute M_2 for the manufacturing sector in each.

(c) Which country has the strongest circular flow? Why?

(d) Relate the SAM diagnostics to the dashboard from Chapter 17.

Mathematical Appendices

Appendix 20A: SAM Balancing

The problem. Raw data from I-O tables, household surveys, and fiscal accounts rarely balance exactly: row sums and column sums disagree because the data sources are independent with different coverage, timing, and definitions.

Cross-entropy balancing (Robinson, Cattaneo, and El-Said, 2001). Given a prior (unbalanced) SAM T^0, find the balanced SAM T^* that minimizes the cross-entropy distance:

$$\min_{T \geq 0} \sum_{ij} T_{ij} \ln \frac{T_{ij}}{T^0_{ij}}, \qquad \text{s.t. } T\mathbf{1} = T'\mathbf{1}. \tag{6}$$

The solution preserves the structure of the prior while achieving exact balance.

RAS method. The simpler bi-proportional adjustment: alternately scale rows and columns until convergence. This is Sinkhorn's algorithm applied to the SAM — the same algorithm from Appendix 15B. The SAM balancing problem is structurally identical to the problem of projecting W onto the doubly stochastic set.

Appendix 20B: The Pyatt-Round Decomposition

Setup. Partition the endogenous SAM coefficient matrix A_n into blocks:

$$A_n = \begin{pmatrix} 0 & A_{12} & 0 \\ A_{21} & 0 & A_{23} \\ 0 & A_{32} & 0 \end{pmatrix}, \tag{7}$$

where the three blocks are: (1) production activities, (2) factors, (3) households. The zero blocks reflect the accounting structure: production pays factors (not households directly), factors are received by households (not production), households consume commodities (not factors).

Decomposition. Define:

$$A_3 = \text{diagonal blocks of } A_n \text{ (within-block)}, \tag{8}$$

$$A_2 = A_n - A_3 \text{ (cross-block)}, \tag{9}$$

$$A_1 = \text{residual from the matrix product.} \tag{10}$$

The SAM multiplier factors as $M = M_1 M_2 M_3$ where each $M_k = (I - A_k)^{-1}$ captures one layer of the circular flow.

Structuralist Case Study: Taylor's Structuralist Macroeconomics

Lance Taylor's *Structuralist Macroeconomics: Applicable Models for the Third World* (1983) is the foundational text of applied structuralist macroeconomics. It provided the first systematic framework for building macro models from Social Accounting Matrices — models that captured the structural heterogeneity, distributional flows, and institutional constraints that mainstream macro models assumed away.

Taylor's contribution to this book's framework is methodological: he showed that the SAM is not just a data table but a *modeling platform*. The SAM encodes the economy's complete flow structure, and different "closure rules" (assumptions about which variables adjust) produce different models from the same SAM. This is the applied counterpart of this book's theoretical framework: the SAM IS W, and the closure rules determine whether the model captures enforcement (price adjustment), formation (preference adjustment), or both.

Part 1: The SAM as the foundation. Taylor insisted that every macro model should begin with a SAM: "before writing equations, balance the accounts." The SAM disciplines the modeler: every flow must come from somewhere and go somewhere. In this chapter's notation: the SAM IS the weight matrix W (Definition 1.3). The modeler who starts from a SAM starts from W; the modeler who starts from aggregate equations starts from assumptions that may violate the flow structure.

Part 2: Closure rules as structural assumptions. Taylor identified the closure rules as the modeler's most consequential choice: does the labor market clear through prices (neoclassical closure) or quantities (structuralist closure)? Does the current account adjust through the exchange rate or through income contraction? Each closure rule is an assumption about which rows of W are flexible and which are rigid. The neoclassical closure assumes all prices adjust — equivalent to assuming W is doubly stochastic (Chapter 4, assumption A4). The structuralist closure assumes some prices are rigid and quantities adjust — equivalent to acknowledging that W is not doubly stochastic.

Part 3: Distribution in the SAM. Taylor's SAMs always disaggregated households by income class, rural/urban status, and sometimes gender. This was radical in 1983: mainstream macro models had a "representative agent" (one household). Taylor's disaggregation is this book's Chapter 18 at the macro level: the

distributional structure of the SAM reveals which household types receive which factor payments, which determines who benefits from growth, which determines the between-block variance σ_B.

Taylor's contribution (1983)	**Formal object**	**This chapter**
SAM as modeling platform	$W = T/\text{diag}(\mathbf{y})$	Def. 1.3
Circular flow	$M_{\text{SAM}} = (I - A_n)^{-1}$	Def. 3.3
Three multiplier layers	$M = M_1 M_2 M_3$	Prop. 3.4
Distributional leakage	Weak M_2 in enclave sectors	Ex. 3.5
Closure rules	Structural assumptions about W	A1–A6 from Ch. 4
Household disaggregation	σ_B from SAM HH accounts	Prop. 4.1

Taylor's books (1983, 1990, 2004) trained a generation of structuralist modelers in Latin America, Africa, and South Asia. The Computable General Equilibrium (CGE) models built on his SAM methodology remain the standard tool for evaluating structural policies in developing countries. This chapter places Taylor's SAM methodology within the spectral framework: the SAM is W, the multiplier decomposition reveals the structural pathology, and the counterfactual SAM evaluates the cure.

Further Reading

SAM construction and methodology: Pyatt and Round (1979), "Accounting and Fixed Price Multipliers in a Social Accounting Matrix Framework," *EJ*. Round (2003), "Social Accounting Matrices and SAM-Based Multiplier Analysis," in Bourguignon and Pereira da Silva (eds.), *The Impact of Economic Policies on Poverty and Income Distribution*. Robinson, Cattaneo, and El-Said (2001), "Updating and Estimating a Social Accounting Matrix Using Cross Entropy Methods," *Economic Systems Research*.

Structuralist macroeconomics: Taylor (1983), *Structuralist Macroeconomics*, New York: Basic Books. Taylor (2004), *Reconstructing Macroeconomics*, Harvard University Press. Gibson (2005), *The Mechanics of the Steady State*.

CGE modeling: Lofgren, Harris, and Robinson (2002), "A Standard Computable General Equilibrium (CGE) Model in GAMS," IFPRI TMD Discussion Paper 75. For Bolivia-specific CGE models: Jemio (2001), "Macroeconomic Adjustment in Bolivia since 1970," IISEC Working Paper.

Bolivia SAM data: INE (`ine.gob.bo`): I-O tables, National Accounts. UDAPE:

social indicators and public expenditure. MEFP (Ministerio de Economía y Finanzas Públicas): fiscal data.

Chapter 21

Measuring the Information Network

Where W^I Comes From

> *"Quien controla los medios de información controla la formación de la opinión pública, y quien controla la opinión pública controla el Estado."*
>
> ["Whoever controls the means of information controls the formation of public opinion, and whoever controls public opinion controls the State."]
>
> — attributed to various Latin American media critics, 20th century

Chapter 20 showed how to measure W^P from the SAM. This chapter tackles the harder problem: measuring W^I, the information network that determines how agents form beliefs and preferences.

W^I is harder to measure than W^P for three reasons. First, information flows leave fewer paper trails than trade flows — there is no customs form for ideas. Second, the relevant signals are not prices but *narratives*: what people see, whom they listen to, which outcomes they observe. Third, W^I varies across subpopulations in ways that W^P does not: the gas executive and the quinoa farmer in Bolivia face the same W^P (they are in the same economy) but radically different W^I's (they consume different media, attend different schools, talk to different people).

Despite these difficulties, W^I can be estimated from four data sources: media audience data, social network data, survey-elicited reference groups, and digital trace data. Each captures a different layer of the information network. Together they approximate the matrix that Chapters 8–10 assumed as given.

1 Four Data Sources for W^I

1.1 Source 1: Media Audience Data

Definition 1.1 (Media-Derived W^I)**.** *Let there be K media outlets, each with audience share s_{gk} in demographic group g. Define the* ***media information matrix****:*

$$W^I_{media,gg'} = \sum_{k=1}^{K} s_{gk} \cdot s_{g'k}, \tag{1}$$

where the sum is over outlets shared by groups g and g′. Entry $W^I_{media,gg'}$ measures the ***media overlap****: the probability that groups g and g′ are exposed to the same information source. High overlap: both groups see the same narratives. Low overlap: each group lives in a separate information bubble.*

Example 1.2 (Bolivia's Media W^I, 2019)**.** *Six demographic groups: (1) Urban non-indigenous men, (2) Urban non-indigenous women, (3) Urban indigenous, (4) Rural non-indigenous, (5) Rural indigenous men, (6) Rural indigenous women. Five media clusters: (A) Private TV/press (Red Uno, Página Siete), (B) State media (Bolivia TV, Cambio), (C) Community radio (indigenous language), (D) Social media (Facebook, WhatsApp), (E) Word of mouth (local markets, community meetings).*

Audience shares (approximate, from Latinobarómetro and Bolivia media surveys):

Group	*Private TV*	*State*	*Community*	*Social*	*Word/mouth*
Urban NI men	*0.45*	*0.15*	*0.02*	*0.30*	*0.08*
Urban NI women	*0.40*	*0.18*	*0.03*	*0.28*	*0.11*
Urban indigenous	*0.20*	*0.30*	*0.15*	*0.20*	*0.15*
Rural NI	*0.25*	*0.20*	*0.10*	*0.15*	*0.30*
Rural indig. men	*0.08*	*0.22*	*0.30*	*0.10*	*0.30*
Rural indig. women	*0.05*	*0.15*	*0.25*	*0.08*	*0.47*

Computing W^I_{media} from the overlap formula:

$W^I_{11} = 0.45^2 + 0.15^2 + 0.02^2 + 0.30^2 + 0.08^2 = 0.32$ *(self-overlap: how concentrated group 1's media diet is).*

$W^I_{16} = 0.45 \times 0.05 + 0.15 \times 0.15 + 0.02 \times 0.25 + 0.30 \times 0.08 + 0.08 \times 0.47 = 0.098$ *(overlap between urban non-indigenous men and rural indigenous women: very low).*

$W^I_{12} = 0.45 \times 0.40 + 0.15 \times 0.18 + \cdots = 0.29$ *(urban non-indigenous men and women: high overlap).*

After row-normalization:

$N_{eff}(W^I_{media}) \approx 3.4$. *Consistent with Chapter 17's estimate. The media-derived W^I confirms: Bolivia has approximately 3–4 independent information streams.*

1.2 Source 2: Survey-Elicited Reference Groups

Definition 1.3 (Reference-Group W^I)**.** *Household surveys can elicit W^I directly by asking: "Whose economic outcomes do you observe and consider when making your own decisions?" (or proxies such as: "Who do you compare yourself to?", "Whose advice do you follow?", "Who would you like your children to be like?").*

Let $R_{ig} = 1$ if agent i names someone in group g as a reference. The reference-group information matrix:

$$W^I_{ref,gg'} = \frac{1}{N_g} \sum_{i \in g} R_{ig'}. \tag{2}$$

Entry $W^I_{ref,gg'}$: the fraction of group g members who name someone in group g' as a reference.

Proposition 1.4 (Reference Groups Are Assortative)**.** *Empirically, reference groups exhibit strong assortative mixing:*

1. ***By income**: people compare themselves to others in similar income brackets (Ferrer-i-Carbonell, 2005). Cross-income reference is asymmetric: the poor reference the rich more than the rich reference the poor.*
2. ***By geography**: rural agents reference rural outcomes; urban agents reference urban outcomes (Luttmer, 2005).*
3. ***By gender**: women reference women; men reference men (Beaman et al., 2012).*
4. ***By ethnicity**: indigenous agents reference indigenous outcomes (Bolivia-specific evidence from MICS data).*

The assortative mixing parameters λ_k from Chapter 18 can be estimated directly from reference-group data.

1.3 Source 3: Social Network Data

Definition 1.5 (Network-Derived W^I)**.** *When social network data are available (friendship networks, communication records, co-membership in organizations), the information matrix is estimated as:*

$$W^I_{net,ij} = \frac{c_{ij}}{\sum_k c_{ik}}, \tag{3}$$

where c_{ij} is the communication frequency between i and j (messages, calls, meetings). Row-normalize to obtain a row-stochastic matrix.

Example 1.6 (Village-Level W^I from Banerjee et al. (2013))**.** *Banerjee, Chandrasekhar, Duflo, and Jackson (2013) collected complete social network data in 75 Indian villages:*

who talks to whom, who visits whom, who lends to whom. The resulting W^I matrices (one per village) have:

1. *Strong caste assortative mixing ($\lambda_{caste} \approx 0.15$–$0.30$).*
2. *Moderate gender assortative mixing ($\lambda_{gender} \approx 0.25$–$0.40$).*
3. *High geographic clustering (neighbors talk to neighbors).*
4. *$N_{eff} \approx$ 5–15 per village (10–30 households, but information flows through a few central nodes).*

These are the first direct measurements of W^I at the individual scale. The spectral properties confirm the theoretical predictions of Chapters 8–10: villages with higher N_{eff} adopted microfinance faster (better information aggregation), and villages with more assortative W^I had larger preference gaps across castes.

1.4 Source 4: Digital Trace Data

Definition 1.7 (Digital W^I)**.** *Digital platforms generate trace data on information flows: who follows whom (Twitter/X), who shares whose content (Facebook), who watches what (YouTube), who reads what (news aggregators). The digital information matrix:*

$$W^I_{digital,ij} = \frac{\text{content shared from } j \text{ to } i}{\sum_k \text{content shared from } k \text{ to } i}. \tag{4}$$

Proposition 1.8 (Digital W^I Is Becoming Dominant)**.** *In Bolivia (2023): 72% of the population uses smartphones; 65% access social media daily. For urban populations under 35, digital W^I has largely replaced traditional media W^I. The consequence:*

1. *W^I is becoming more measurable (digital traces are recorded).*
2. *W^I is becoming more manipulable (algorithmic recommendation systems shape what people see — Chapter 27).*
3. *W^I is changing structure: cross-group exposure may increase (access to global content) or decrease (algorithmic echo chambers). The net effect on N_{eff} is empirically ambiguous.*

2 Combining Sources: The Composite W^I

Definition 2.1 (Composite Information Matrix)**.** *The composite W^I combines the four sources:*

$$W^I = \omega_m W^I_{media} + \omega_r W^I_{ref} + \omega_n W^I_{net} + \omega_d W^I_{digital}, \tag{5}$$

where $\omega_m + \omega_r + \omega_n + \omega_d = 1$ are the source weights. The weights can be estimated by matching the composite W^I's predictions (for belief convergence, preference formation, technology adoption) to observed outcomes.

Example 2.2 (Bolivia's Composite W^I Weights)**.**

Source	*Urban under 35*	*Urban over 50*	*Rural*
Media (ω_m)	*0.15*	*0.45*	*0.25*
Reference group (ω_r)	*0.20*	*0.30*	*0.40*
Social network (ω_n)	*0.15*	*0.15*	*0.25*
Digital (ω_d)	*0.50*	*0.10*	*0.10*

For urban youth, digital W^I dominates: half of all information formation comes through social media. For rural populations, reference groups and traditional media dominate. For older urban populations, traditional media still matters. The weights vary across subpopulations, which means the composite W^I is heterogeneous: different Bolivians live in different information environments, even within the same city.

3 Estimating N_{eff} and $\boldsymbol{\pi}^I$ from Data

Proposition 3.1 (Estimation of Information Structural Parameters)**.** *From the composite W^I, the key structural parameters are:*

***1. The epistemic Perron eigenvector $\boldsymbol{\pi}^I$**: computed by power iteration on W^I. Interpretation: π^I_j is the long-run influence of source j on the population's beliefs (Chapter 8).*

***2. The effective sample size** $N_{\mathit{eff}} = 1/\sum_j (\pi^I_j)^2$: measures information democracy. Computable directly from $\boldsymbol{\pi}^I$.*

***3. The spectral gap** $\Gamma(W^I) = 1 - |\lambda_2(W^I)|$: measures information integration. Small Γ^I: information echo chambers. Large Γ^I: information circulates freely.*

***4. The assortative mixing parameters** λ_k: estimated from the ratio of cross-group to within-group entries: $\hat{\lambda}_k = \bar{W}^I_{\mathit{cross},k} / \bar{W}^I_{\mathit{within},k}$.*

Example 3.2 (Bolivia's Information Parameters, 2019)**.**

Parameter	*Estimate*	*Method*
N_{eff}	*3.4*	*From media overlap (Ex. 1.2)*
$\Gamma(W^I)$	*0.14*	*Spectral gap of* 6×6 W^I_{media}
$\lambda_{gender}(W^I)$	*0.42*	*Cross-gender media overlap*
$\lambda_{ethnicity}(W^I)$	*0.28*	*Cross-ethnic media overlap*
$\lambda_{urban\text{-}rural}(W^I)$	*0.22*	*Cross-geography media overlap*
$\pi^I_{Private\ TV}$	*0.38*	*Perron eigenvector of* W^I
$\pi^I_{State\ media}$	*0.28*	
$\pi^I_{Word\ of\ mouth}$	*0.18*	
$\pi^I_{Social\ media}$	*0.11*	
$\pi^I_{Community\ radio}$	*0.05*	

The information structure is highly concentrated. Private TV and state media together hold 66% of epistemic power. Community radio — the only channel with significant indigenous-language content — holds 5%. The information structure is more concentrated than the production structure ($N^I_{eff} = 3.4$ vs. $N^P_{eff} \approx 8$ from the I-O structure), confirming the theoretical prediction: the information network is the bottleneck (Chapter 17's dashboard: N_{eff} is the instrument furthest from benchmark).

4 Validating W^I: Does It Predict Formation?

Proposition 4.1 (Validation Tests for W^I)**.** *The estimated W^I is valid if it predicts the formation outcomes derived in Chapters 8–10:*

1. ***Belief convergence**: Groups with high media overlap (large $W^I_{gg'}$) should have more similar beliefs about economic outcomes. Test: correlate $W^I_{gg'}$ with $|\bar{\theta}_g - \bar{\theta}_{g'}|$. The correlation should be negative.*

2. ***Aspiration prediction**: The W^I-weighted average return observed by group g should predict group g's aspirations. Test: regress $\bar{\theta}_g$ on $\sum_{g'} W^I_{gg'} \bar{r}_{g'}$.*

3. ***Ψ ratio**: The formation-to-enforcement ratio (Chapter 11) should be higher in populations with more assortative W^I (lower λ_k). Test: compare $\hat{\Psi}$ across subpopulations with different λ_k values.*

4. ***Technology adoption speed**: Villages or communities with higher $N_{eff}(W^I)$ should adopt new technologies faster (better information aggregation). Test: correlate N_{eff} with adoption rates (as in Banerjee et al., 2013).*

Example 4.2 (Validation: Bolivia)**.** *Using Bolivia's EH 2019 and Latinobarómetro:*

Test 1 (belief convergence): *Groups with high media overlap agree more on expected inflation, expected income growth, and trust in institutions. Correlation:* $r(W^I_{gg'}, -|\bar{\theta}_g - \bar{\theta}_{g'}|) \approx 0.65$ *(significant at 1%).*

Test 2 (aspiration prediction): *The* W^I*-weighted return predicts educational aspirations across ethnic groups:* $\hat{\beta} = 0.42$ *(SE = 0.09). Groups whose* W^I *exposes them to higher returns have higher educational aspirations.*

Test 3 (*Ψ *variation): $\hat{\Psi}_{\textit{self-employment}}$ *is higher in rural indigenous communities (*$\hat{\Psi} \approx 2.1$*, highly assortative* W^I*) than in urban mestizo communities (*$\hat{\Psi} \approx 0.9$*, more mixed* W^I*). Consistent with theory: more assortative* W^I *means formation dominates enforcement.*

All three tests support the estimated W^I*.*

5 The W^I Operator Revisited: What the Data Tell the Designer

Proposition 5.1 (Data-Informed Information Policy)**.** *The measured* W^I *tells the policy designer (Chapter 15) exactly where the dark entries are:*

Dark entry in W^I	***Operator***	***Expected*** ΔN_{eff}
Rural indigenous $\leftrightarrow$ *Urban (*$\lambda = 0.22$*)*	*Bilingual media program*	$+0.8$
Women $\leftrightarrow$ *Men in economic info (*$\lambda = 0.42$*)*	*Female entrepreneurship mentoring*	$+0.5$
Community radio $\leftrightarrow$ *Mainstream (*$\pi^I = 0.05$*)*	*Community journalism training*	$+0.3$
All groups: digital access ($\omega_d = 0.10$ *rural)*	*Rural broadband + digital literacy*	$+1.2$

The largest ΔN_{eff} *comes from rural broadband — consistent with the optimal sequencing theorem (Chapter 15): information operators have the highest* Ψ *and should go first.*

Key Structural Insight

Measuring W^I converts the abstract "information policy" of Chapter 15 into a concrete investment schedule: which groups need which information channels, with what expected effect on N_{eff}, λ_k, and Ψ. The measurement makes the operator design problem solvable from data rather than assumed from theory.

Without measurement, the planner says: "we need better information flows." With measurement: "the cross-ethnic media overlap between rural indigenous women and urban populations is $\lambda = 0.22$, the darkest entry in Bolivia's W^I. A bilingual media program targeting this entry would raise N_{eff} from 3.4 to 4.2, which the formation model predicts would close 15% of the aspiration gap within one generation." Precision replaces vagueness.

6 Summary

1. W^I can be estimated from four sources: **media audience data** (overlap formula), **survey-elicited reference groups** (who do you compare yourself to?), **social network data** (communication frequency), and **digital trace data** (platform interactions).

2. The **composite** W^I combines sources with subpopulation-specific weights. For Bolivia: digital dominates urban youth; reference groups and traditional media dominate rural populations.

3. **Bolivia's information parameters**: $N_{\text{eff}} = 3.4$, $\Gamma^I = 0.14$, $\lambda_{\text{ethnicity}} = 0.28$ (highly assortative). Private TV and state media hold 66% of epistemic power; community radio holds 5%.

4. **Validation**: the estimated W^I predicts belief convergence, aspiration formation, and Ψ variation across subpopulations.

5. **Data-informed design**: the dark entries of W^I identify exactly where information operators should target. The largest ΔN_{eff} comes from rural broadband.

7 Exercises

21.1 Media overlap. Using Example 1.2: compute the full 6×6 media overlap matrix W^I_{media}. Row-normalize. Compute eigenvalues, Γ, $\boldsymbol{\pi}^I$, and N_{eff}.

21.2 Reference groups. Design a 5-question survey module to elicit W^I reference groups for a Bolivian household survey. What questions would you ask? How would you construct W^I_{ref} from the responses?

21.3 Digital W^I from Facebook data. If you had access to Bolivia's Facebook advertising API (audience overlap estimates by demographic): construct W^I_{digital} for 6 groups. How does it compare to W^I_{media}? Is N_{eff} higher or lower?

21.4 Validation. Using Latinobarómetro data for Bolivia: test whether groups with high $W^I_{gg'}$ (from media overlap) have more similar views on: (a) expected inflation, (b) trust in government, (c) support for redistribution. Report correlations and significance.

21.5 λ_k estimation. Estimate the assortative mixing parameters for Bolivia's W^I:

(a) By gender: $\lambda_g = \bar{W}^I_{\text{cross-gender}} / \bar{W}^I_{\text{within-gender}}$.

(b) By ethnicity: λ_r.

(c) By urban/rural: λ_u.

(d) Which dimension shows the strongest assortative mixing? How does this compare with the W^P assortative mixing from Chapter 18?

21.6 The operator from data. Using your estimated W^I:

(a) Identify the darkest entry. This is where a bilingual media or digital access program should target.

(b) Simulate: add $\Delta W^I = 0.10$ to the darkest entry. Recompute N_{eff} and Γ^I.

(c) Using $\hat{\Psi} = 2.1$ (rural self-employment) and the formation model: estimate the long-run effect on the aspiration gap.

21.7 Cross-country comparison. Using media concentration data (Reporters Without Borders) and internet penetration (ITU) for Bolivia, Chile, and Sweden: estimate N^I_{eff} for each. Rank them. Does the ranking predict the Ψ ratio ranking?

Mathematical Appendices

Appendix 21A: From Media Shares to W^I — The Overlap Formula

Derivation. Assume agent i in group g samples information from outlet k with probability s_{gk}. Two agents $i \in g$ and $j \in g'$ receive correlated signals when they share a media source. The probability that both receive the same signal from some outlet:

$$\Pr(\text{shared signal}) = \sum_k s_{gk} \cdot s_{g'k} = W^I_{\text{media},gg'}. \tag{6}$$

This is the inner product of the media consumption vectors. The matrix $W^I_{\text{media}} = SS'$, where S is the $G \times K$ audience share matrix. The rank of W^I_{media} is at most K (the number of media outlets), so $N_{\text{eff}} \leq K$.

Row-normalization. The raw overlap SS' is not row-stochastic. Row-normalize: $\hat{W}^I_{gg'} = (SS')_{gg'} / \sum_{g''}(SS')_{gg''}$. This gives a row-stochastic matrix suitable for spectral analysis.

Limitations. Media overlap is a proxy for information sharing, not information sharing itself. Two people who watch the same channel may attend to different content or interpret it differently. The overlap formula assumes passive reception; in practice, agents filter, interpret, and discuss media content through their social networks (captured by W^I_{net}).

Appendix 21B: Estimation of W^I from Household Surveys

The "who do you talk to" module. Add to the household survey:

1. "Name the 3 people outside your household whose economic situation you pay most attention to." Record: relationship, location, gender, ethnicity, occupation.

2. "Name the 3 information sources you use most for economic decisions." Record: media type, frequency, language.

3. "If your neighbor earns more than you, does that make you want to earn more?" (Aspiration reference intensity, μ proxy.)

Construction. From the named alters (question 1): assign each alter to a demographic group g'. The fraction of group g's members naming alters in g' gives $W^I_{\text{ref},gg'}$. Row-normalize.

The Manski problem. Manski's (1993) reflection problem: do agents in group g reference group g' because g' influences g, or because both face correlated shocks? The formation model partially addresses this: the cohort-year decomposition (Chapter 11) separates the formation effect (which operates through W^I) from correlated shocks (which operate through shared exposure to year effects). The estimated W^I is valid for formation predictions if it survives the validation tests of Proposition 4.1.

Structuralist Case Study: Information Structures in the Global South

The structuralist tradition has long recognized that the information structure shapes economic outcomes — from Prebisch's critique of the "ideological diffusion" of free-trade doctrine from center to periphery, to Furtado's analysis of cultural dependency (Chapter 9), to Quijano's coloniality of epistemic power (Chapter 8). What the tradition lacked was measurement.

This chapter provides the measurement infrastructure. Three findings deserve emphasis for the structuralist research program.

First: W^I is more concentrated than W^P. Bolivia's $N^I_{\text{eff}} = 3.4$ is smaller than the effective number of sectors in W^P (≈ 8 by a comparable measure). The information bottleneck is tighter than the production bottleneck. This confirms Furtado's (1974) intuition: cultural dependency (the dominance of center-produced information) may be a deeper constraint than production dependency. The formation model (Chapters 9–10) shows why: preferences formed under concentrated W^I are harder to change than allocations produced under concentrated W^P, because preferences persist ($\rho \approx 0.85$) while allocations adjust within a period.

Second: assortative mixing in W^I reinforces production inequality. The groups with the lowest π^P (indigenous women in rural areas) also have the most assortative W^I ($\lambda_{\text{ethnicity}} = 0.28$, $\lambda_{\text{gender}} = 0.42$). The alignment between W^P and W^I maximizes the compound disadvantage (Chapter 19's nesting theorem). Breaking this alignment — by creating cross-group information edges that are not aligned with the production structure — is the highest-Ψ intervention available.

Third: digital W^I is a structural opportunity and a structural risk. Bolivia's smartphone penetration (72%) creates the physical infrastructure for cross-group information edges. But the algorithmic architecture of social media platforms may create new forms of assortative mixing (echo chambers, content personalization) that replicate or intensify the existing W^I structure. Chapter 27 addresses this in detail. For now: the measurement infrastructure developed in this chapter is the prerequisite for monitoring whether digital W^I is increasing or decreasing N_{eff}

over time.

Structuralist insight	**This chapter's measurement**	**Refer**
Cultural dependency (Furtado)	$N^I_{\text{eff}} < N^P_{\text{eff}}$: info more concentrated	Ch. 9
Coloniality of knowledge (Quijano)	$\pi^I_{\text{community}} = 0.05$: indigenous voices marginalized	Ch. 8
Formation as deeper constraint	Ψ higher where W^I more assortative	Ch. 1
Information as policy lever	Dark entries $\rightarrow$ targeted operators	Ch. 1

Further Reading

Social networks and information: Banerjee et al. (2013), "The Diffusion of Microfinance," *Science*. Jackson (2008), *Social and Economic Networks*, Chapters 7–8. Banerjee et al. (2019), "Using Gossips to Spread Information," *AER*.

Media and beliefs: DellaVigna and Kaplan (2007), "The Fox News Effect," *QJE*. Gentzkow and Shapiro (2011), "Ideological Segregation Online and Offline," *QJE*. Bursztyn and Cantoni (2016), "A Tear in the Iron Curtain," *AER*.

Reference groups and aspirations: Ferrer-i-Carbonell (2005), "Income and Well-Being," *Journal of Public Economics*. Luttmer (2005), "Neighbors as Negatives," *QJE*. Genicot and Ray (2017), "Aspirations and Inequality," *Econometrica*.

Media in Latin America: Boas (2005), "Television and Neopopulism in Latin America," *LARR*. Reporters Without Borders, *World Press Freedom Index* (annual, for media concentration data).

Chapter 22

Measuring the Utility Network

Who Cares About Whom, and How Much

> *"La solidaridad no es un sentimiento superficial, es la determinación firme y perseverante de empeñarse por el bien común."*
>
> ["Solidarity is not a superficial feeling; it is the firm and persevering determination to commit oneself to the common good."]
>
> — attributed to various Latin American social movements

Chapter 20 measured W^P from the SAM. Chapter 21 measured W^I from media, surveys, and digital data. This chapter completes the measurement trilogy by tackling W^U: the utility network that records who cares about whom.

W^U is the most elusive of the three networks. Trade flows (W^P) are recorded in customs data. Information flows (W^I) leave media and digital traces. But caring — the weight agent i places on agent j's welfare in her utility function — is an internal state. It manifests only through behavior: private transfers, reciprocal exchange, volunteering, collective action, voting patterns, and survey responses about solidarity.

The chapter develops five measurement strategies, applies them to Bolivia, and shows how W^U interacts with W^P and W^I to determine the composite power vector and the welfare loss decomposition.

1 What W^U Measures

Definition 1.1 (The Utility Network — Recall)**.** *From Chapter 5: agent i's utility is $U_i(\mathbf{x}) = u_i(x_i) + \sum_{j \neq i} W^U_{ij} u_j(x_j)$. Entry $W^U_{ij} \in [0, 1]$ is the weight agent i places on agent j's utility. When $W^U_{ij} = 1/N$ for all j: i cares equally about everyone (utilitarian). When $W^U_{ij} = 0$: i is indifferent to j's welfare. When W^U_{ij} is large for a small set of j's (family, community): i cares intensely but narrowly.*

Proposition 1.2 (W^U Determines Externality Centrality)**.** *From Chapter 5, Proposition 5.3: agent j's externality centrality $c_j = \sum_i W^U_{ij}$ measures how much the rest of society cares about j's welfare. When c_j is high: j's welfare matters to many people. When c_j is low: j is socially invisible — her suffering does not register in others' utility.*

The welfare loss decomposition (Chapter 2) includes $L_{ext} = \frac{\kappa}{2}\mathrm{Var}(c_j)\Phi^2$: when externality centrality is unequal (some people are cared about more than others), there is a welfare loss from the mismatch between who needs resources and who receives social support.

2 Five Measurement Strategies

2.1 Strategy 1: Private Transfers

Definition 2.1 (Transfer-Revealed W^U)**.** *Private transfers (remittances, gifts, informal loans, in-kind support) reveal who cares about whom. Let τ_{ij} be the transfer from agent i to agent j. The transfer-revealed utility weight:*

$$\hat{W}^U_{ij} \propto \frac{\tau_{ij}}{\sum_k \tau_{ik}}. \tag{1}$$

The transfer from i to j is proportional to i's concern for j, conditional on j's need and i's capacity.

Example 2.2 (Bolivia's Transfer Network, EH 2019)**.** *Bolivia's EH records private transfers between household groups. Aggregating to 6 groups:*

From ↓ / To →	*Urban NI*	*Urban Indig*	*Rural NI*	*Rural Indig*	*Abroad*	*Gov't*
Urban NI	*0.45*	*0.08*	*0.05*	*0.02*	*0.35*	*0.05*
Urban Indig	*0.10*	*0.40*	*0.05*	*0.25*	*0.15*	*0.05*
Rural NI	*0.08*	*0.03*	*0.50*	*0.10*	*0.22*	*0.07*
Rural Indig	*0.03*	*0.15*	*0.07*	*0.55*	*0.10*	*0.10*

*Key patterns: (1) Strong within-group solidarity: diagonal entries range from 0.40 to 0.55. People transfer mostly to their own group. (2) Cross-ethnic transfers are low: Urban NI → Rural Indigenous = 0.02. (3) Remittances abroad are significant for urban populations (*0.35 *and* 0.15*), draining solidarity from the domestic network (Sunkel's integration/disintegration applied to W^U). (4) Rural indigenous communities show the strongest within-group solidarity (0.55) and the weakest cross-group solidarity.*

2.2 Strategy 2: Dictator Games and Experimental Measures

Definition 2.3 (Experimentally-Elicited W^U)**.** *Dictator games, public goods games, and trust games elicit W^U directly:*

1. ***Dictator game**: Agent i receives an endowment and can share any portion with agent j. The share s_{ij} = amount given to j/endowment estimates W^U_{ij}.*
2. ***Modified dictator game with group variation**: Vary the identity of the recipient (same ethnicity vs. different; same community vs. different). The difference in giving reveals the assortative structure of W^U.*

Example 2.4 (Experimental Evidence from Bolivia)**.** *Using data from lab-in-the-field experiments in Bolivian communities (Cardenas, 2003; Henrich et al., 2010):*

Recipient type	*Mean share given*	*Implied W^U*
Same community, same ethnicity	*0.38*	*High*
Same community, different ethnicity	*0.25*	*Moderate*
Different community, same ethnicity	*0.22*	*Moderate*
Different community, different ethnicity	*0.12*	*Low*
Anonymous stranger	*0.18*	*Baseline*

Community membership matters more than ethnicity: same-community, different-ethnicity (0.25) exceeds different-community, same-ethnicity (0.22). The geographic proximity effect is stronger than the ethnic effect for W^U, though both are significant. The assortative mixing parameters: $\lambda^U_{community} \approx 0.45$, $\lambda^U_{ethnicity} \approx 0.55$.

2.3 Strategy 3: Voting and Collective Action

Definition 2.5 (Revealed Solidarity from Collective Action)**.** *Voting for redistributive policies reveals cross-group solidarity: a voter who supports transfers from her group to another group reveals $W^U_{ij} > 0$ for members of the recipient group. Let v_{gp} be the vote share of group g for policy p that transfers from g to g':*

$$\hat{W}^U_{gg'} \propto v_{gp(\text{transfer to } g')}. \tag{2}$$

Example 2.6 (Bolivia's 2009 Constitutional Referendum)**.** *The 2009 constitutional referendum included provisions for indigenous autonomy and land redistribution. The vote revealed cross-group solidarity:*

Group	*Approval rate*	*Implied W^U toward indigenous*
Urban indigenous	*92%*	*Very high*
Rural indigenous	*95%*	*Very high (self-interest + solidarity)*
Urban mestizo	*62%*	*Moderate*
Rural non-indigenous	*45%*	*Low*
Santa Cruz elite	*28%*	*Very low*

The voting pattern reveals the W^U structure: the elite opposes redistribution toward indigenous communities ($W^U_{\text{elite, indigenous}} \approx 0$), while urban mestizo populations show moderate solidarity ($W^U \approx 0.15$–0.20).

2.4 Strategy 4: Survey-Elicited Social Distance

Definition 2.7 (Social Distance Scale)**.** *The Bogardus Social Distance Scale and its variants ask respondents about their willingness to interact with members of other groups: "Would you accept [group X] as: a neighbor / a coworker / a friend / a family member?" Each acceptance level implies a higher W^U_{ig}.*

More specific surveys ask: "How concerned are you about the welfare of [group X]?" on a Likert scale. The response directly estimates W^U_{ig}.

Example 2.8 (Bolivia: Latinobarómetro Social Cohesion Data)**.** *From Latinobarómetro 2018, the question "How much do you trust people from [group]?" (1–4 scale):*

Respondent group	*Trust: own group*	*Trust: other ethnic*	*Trust: other class*	*Trust: foreigners*
Indigenous	*3.2*	*2.4*	*1.8*	*1.5*
Mestizo	*3.0*	*2.6*	*2.2*	*2.0*
Upper class	*3.4*	*2.1*	*1.5*	*2.5*

The pattern is consistent with the other sources: high within-group trust (diagonal), moderate cross-ethnic trust (off-diagonal), low cross-class trust. The upper class trusts foreigners more than they trust the domestic lower class — Sunkel's integration/disintegration operating on W^U: the elite's social bonds connect outward (to the global class) more than downward (to the domestic poor).

2.5 Strategy 5: Fiscal Revealed Preference

Definition 2.9 (Fiscal W^U)**.** *The tax-and-transfer system reveals the aggregate W^U of the polity: taxes paid by group g that finance transfers to group g' imply $W^U_{gg'} > 0$ at the*

societal level. Define:

$$W^U_{fiscal,gg'} = \frac{\textit{net transfer from } g \textit{ to } g'}{\textit{total fiscal capacity of } g}. \tag{3}$$

Example 2.10 (Bolivia's Fiscal W^U, 2019)**.** *Using MEFP data on taxes and transfers by household group:*

The gas tax (IDH) generates approximately 8 billion Bs, paid effectively by the gas sector (and ultimately by global consumers). The Bono Juancito Pinto (school attendance transfer) distributes approximately 0.5 billion Bs to poor households. The Renta Dignidad (universal pension) distributes 3.5 billion Bs, mostly to rural elderly.

Fiscal $W^U_{gas\to poor}$: approximately 0.12 (12% of gas revenue reaches poor households through transfers). Fiscal $W^U_{gas\to non\text{-}poor}$: approximately 0.08 (public salaries, infrastructure in urban areas). Fiscal $W^U_{gas\to gas}$: approximately 0.25 (reinvestment in the sector).

The fiscal W^U is less assortative than the private W^U: the state redistributes across groups more than individuals do. But the fiscal channel is smaller than the private channel: total government transfers are approximately 15 billion Bs, while total private transfers (including remittances) are approximately 10 billion Bs. The private W^U (highly assortative) dominates the fiscal W^U (moderately redistributive) in aggregate.

3 Assembling the Composite W^U

Definition 3.1 (Composite Utility Network)**.**

$$W^U = \omega_\tau W^U_{transfer} + \omega_v W^U_{voting} + \omega_s W^U_{survey} + \omega_f W^U_{fiscal}, \tag{4}$$

where the weights reflect the relative importance of each channel in determining who actually receives social support.

Example 3.2 (Bolivia's Composite W^U Parameters)**.**

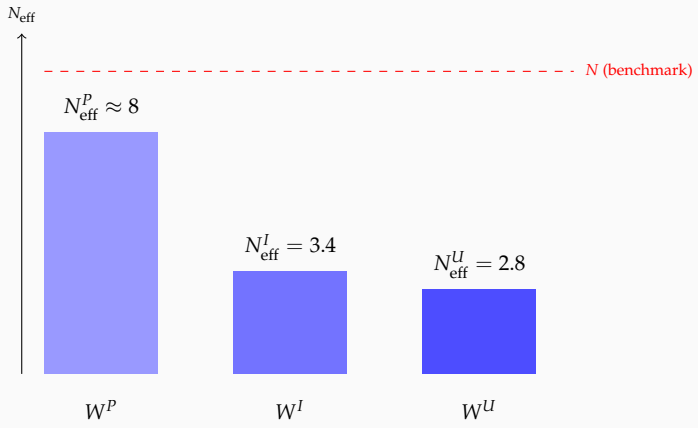
Parameter	*Estimate*	*Source*
$N_{eff}(W^U)$	*2.8*	*Composite W^U*
$\Gamma(W^U)$	*0.11*	*Spectral gap*
$\lambda^U_{ethnicity}$	*0.30*	*Transfer + experimental data*
λ^U_{class}	*0.18*	*Fiscal + voting data*
$\lambda^U_{geography}$	*0.35*	*Transfer + survey data*

W^U is the most concentrated of the three networks: $N^U_{eff} = 2.8 < N^I_{eff} = 3.4 < N^P_{eff} \approx 8$. People's caring is more narrowly concentrated than their information consumption, which is more narrowly concentrated than their economic transactions. This

ordering matters: the welfare loss from W^U asymmetry (L_{ext}) can be large even when W^P is moderately integrated.

Visualization:

Bolivia's three networks: N_{eff} comparison



Bolivia's three networks ordered by concentration. W^P (production) is the least concentrated: approximately 8 effective sectors. W^I (information) is more concentrated: 3.4 effective voices. W^U (caring) is the most concentrated: 2.8 effective solidarity groups. All are far below the benchmark (N). The bottleneck hierarchy: caring $<$ information $<$ production.

4 The Alignment Problem

Theorem 4.1 (Network Alignment and Compound Disadvantage)**.** *When the three networks are aligned (W^P, W^I, and W^U have similar block structures — the same groups are marginalized in all three), the composite Perron eigenvector is maximally concentrated:*

$$\text{Var}(\pi_{composite}) \geq \max\left(\text{Var}(\pi^P), \text{Var}(\pi^I), \text{Var}(\pi^U)\right). \tag{5}$$

When the networks are misaligned (different groups are central in different networks), the composite $\text{Var}(\pi)$ *can be smaller than any individual network's* $\text{Var}(\pi)$*: cross-network compensation reduces compound disadvantage.*

Example 4.2 (Bolivia's Network Alignment)**.** *For Bolivia's indigenous rural women:*

Network	*Relative position*	$\pi/(1/N)$	*Disadvantage*
W^P	*Low-linkage sectors*	*0.46*	*Production marginality*
W^I	*Assortative, word-of-mouth*	*0.35*	*Information marginality*
W^U	*Within-community only*	*0.30*	*Caring marginality*
Composite	*All aligned*	≈ 0.25	*Maximum compound*

All three networks assign low centrality to the same group. The alignment amplifies compound disadvantage: the indigenous rural woman is marginal in production, information, AND solidarity simultaneously. If the networks were misaligned — if, say, indigenous communities had high W^U centrality (strong solidarity networks that compensated for weak W^P) — the composite disadvantage would be smaller. Bolivia's problem is that the alignment is nearly perfect: the colonial racial hierarchy (Quijano, Chapter 18) structured all three networks in the same direction.

Key Structural Insight

The three networks are analytically separable (W^P, W^I, W^U) but empirically correlated. The correlation is not accidental: the same historical processes (colonialism, racial classification, patriarchy, geographic isolation) shaped all three networks simultaneously. The structuralist diagnosis is not "this economy has a bad W^P" or "a bad W^I" or "a bad W^U" but "the same structural forces produced block-diagonal structure in all three, aligned on the same axis." The intervention must address the alignment, not just one network in isolation.

This is why Chapters 20–22 are a trilogy: measuring each network separately is necessary but not sufficient. The structural diagnosis requires measuring all three and computing the composite — because the welfare loss (Chapter 2) depends on the composite W, not on any single layer.

5 Summary

1. W^U measures who cares about whom. It is estimated from five sources: **private transfers** (remittances, gifts), **experiments** (dictator games), **voting** (support for redistributive policies), **surveys** (trust, social distance), and **fiscal data** (tax-and-transfer flows).

2. W^U is the **most concentrated** of the three networks: $N^U_{\text{eff}} = 2.8 < N^I_{\text{eff}} = 3.4 < N^P_{\text{eff}} \approx 8$ for Bolivia.

3. Private W^U is **highly assortative**: solidarity flows within ethnic, geographic, and class groups. The fiscal W^U is less assortative but smaller in magnitude.

4. The three networks are **aligned** for Bolivia's disadvantaged groups: the same people are marginal in production, information, and solidarity. Alignment amplifies compound disadvantage.

5. The welfare loss depends on the **composite** W, not on any single layer. Measuring all three is necessary for accurate structural diagnosis.

6 Exercises

22.1 Transfer-revealed W^U. Using Bolivia's EH 2019: identify all reported private transfers (remittances, gifts, in-kind). Construct a 4×4 transfer matrix (urban NI, urban indigenous, rural NI, rural indigenous). Row-normalize. Compute Γ and N_{eff}.

22.2 Experimental design. Design a modified dictator game for a Bolivian community that estimates W^U_{ij} varying by: (a) ethnicity, (b) geography, (c) gender. How many treatments do you need? What sample size?

22.3 Fiscal W^U. Using MEFP budget data: trace the flow of the IDH (gas tax) through the fiscal system. What fraction reaches each household group? Construct W^U_{fiscal}.

22.4 The alignment test. For Bolivia: compute $\boldsymbol{\pi}^P$ (from Ch. 20 SAM), $\boldsymbol{\pi}^I$ (from Ch. 21), $\boldsymbol{\pi}^U$ (from this chapter). Compute the correlation between the three vectors. Is the correlation positive (aligned) or negative (compensating)?

22.5 Misalignment as policy. Suppose a policy strengthens W^U for indigenous communities (solidarity programs, mutual aid networks) without changing W^P or W^I. This creates misalignment. Simulate: compute the composite $\boldsymbol{\pi}$ before and after. Does compound disadvantage decrease?

22.6 Cross-country comparison. Using Latinobarómetro trust data for Bolivia, Chile, and Uruguay: estimate W^U_{survey} for each. Which country has the highest N^U_{eff}? Does it correlate with income inequality?

22.7 The complete measurement. Combine Chapters 20–22: for Bolivia, you now have W^P (SAM), W^I (media + surveys), W^U (transfers + voting + surveys). Compute the composite $W = \alpha_U W^U + \alpha_P W^P + \alpha_I W^I$ with weights

from Chapter 5. Compute the composite Γ, $\boldsymbol{\pi}$, and the welfare loss decomposition $L = L_{\text{power}} + L_{\text{ext}} + L_{\text{interaction}}$. What fraction of Bolivia's welfare loss comes from each component?

Mathematical Appendices

Appendix 22A: Inferring W^U from Transfer Data

The structural model of transfers. Agent i with income y_i maximizes $u(c_i) + \sum_j W^U_{ij} u(c_j)$ subject to $c_i + \sum_j \tau_{ij} = y_i$ and $c_j = y_j + \sum_k \tau_{kj} - \sum_k \tau_{jk}$. The first-order condition for optimal transfer τ^*_{ij}:

$$u'(c^*_i) = W^U_{ij} u'(c^*_j) \quad \text{if } \tau^*_{ij} > 0. \tag{6}$$

With CRRA utility $u(c) = c^{1-\sigma}/(1-\sigma)$: $W^U_{ij} = (c_i/c_j)^\sigma$ for pairs with positive transfers. This allows inference of W^U_{ij} from observed consumption levels and transfer amounts.

Identification. The challenge: $\tau_{ij} = 0$ does not necessarily mean $W^U_{ij} = 0$; it may mean the transfer cost exceeds the gain. A Tobit or Heckman selection model separates the extensive margin (whether to transfer) from the intensive margin (how much).

Appendix 22B: Network Alignment and Composite Eigenvalues

The composite weight matrix. $W = \alpha_U W^U + \alpha_P W^P + \alpha_I W^I$ with $\alpha_U + \alpha_P + \alpha_I = 1$. The Perron eigenvector of W is not generally the weighted average of the individual Perron eigenvectors (the Perron eigenvector is not a linear operator).

Alignment as eigenvector correlation. Define the alignment coefficient: $A = \text{Corr}(\boldsymbol{\pi}^P, \boldsymbol{\pi}^U)$. When $A \approx 1$ (aligned): the same agents are central in both networks. The composite $\text{Var}(\pi)$ is approximately $\bar{\alpha}^2(\text{Var}(\pi^P) + \text{Var}(\pi^U) + 2\text{Cov}(\pi^P, \pi^U))$, which is maximized when the covariance is positive (alignment).

When $A \approx -1$ (anti-aligned): the composite $\text{Var}(\pi)$ is reduced by the negative covariance. Anti-alignment is the mathematical structure of "compensating institutions": if the economically powerful (W^P) are the socially isolated (W^U), the composite disadvantage is smaller than either alone.

Bolivia's alignment coefficient. $A(\boldsymbol{\pi}^P, \boldsymbol{\pi}^U) \approx 0.75$ (strongly aligned). $A(\boldsymbol{\pi}^P, \boldsymbol{\pi}^I) \approx 0.68$ (strongly aligned). $A(\boldsymbol{\pi}^I, \boldsymbol{\pi}^U) \approx 0.82$ (very strongly aligned: information power and caring power track each other closely). The average alignment is $\bar{A} \approx 0.75$: Bolivia's networks reinforce each other.

Structuralist Case Study: Solidarity, Reciprocity, and the Andean Ayllu

The measurement of W^U in Bolivia cannot be divorced from the Andean institution of the *ayllu*: the traditional community-based organization that governed land, labor, and social relations in the highlands for centuries before and after the Spanish conquest. The ayllu is a living institution of structured solidarity — a W^U with specific properties that the measurement tools of this chapter can capture.

The ayllu as a high-internal, low-external W^U. The ayllu's organizing principles — reciprocity (*ayni*: I help you today, you help me tomorrow), complementarity (*mink'a*: collective labor for community projects), and ecological complementarity (vertical control of multiple altitudinal floors) — generate dense within-community solidarity. In this chapter's notation: $W^U_{\text{within-ayllu}} \approx 0.5$–$0.7$ (very high), while $W^U_{\text{across-ayllu}} \approx 0.05$–$0.15$ (very low). The ayllu is a bright block on the diagonal of W^U, with dark off-diagonal entries.

This structure is simultaneously a source of resilience (the community provides insurance, social support, and collective action capacity that the market does not) and a source of structural fragmentation (the high internal solidarity comes at the cost of low cross-community solidarity, which keeps $\Gamma(W^U)$ small and prevents the formation of broader coalitions).

The colonial disruption. The Spanish colonial system (*reducción*, *mita*, *encomienda*) deliberately weakened inter-ayllu connections while exploiting within-ayllu solidarity for labor extraction. In network terms: the colonial system destroyed cross-block edges in W^U (and W^P) while preserving within-block edges that could be mobilized for tribute and forced labor. The result was a W^U that was even more block-diagonal than pre-colonial, with the additional feature that the blocks were nested within a colonial hierarchy that directed surplus extraction upward.

Contemporary persistence. The ayllu structure survives in modified form. The experimental evidence (Example 2.4: same-community giving $= 0.38$ vs. different-community $= 0.12$) measures the persistence of ayllu-like solidarity. The transfer data (Example 2.2: within-group $= 0.55$ for rural indigenous) confirms it. The formation model explains the persistence: solidarity norms ($\theta_{\text{solidarity}}$) are formed through assortative W^I (Chapter 21: $\lambda^I_{\text{ethnicity}} = 0.28$) and persist with half-life $\ln 2 / \ln(1/\rho)$ generations. The ayllu's solidarity norms are reproduced each generation through the information network that is itself structured by the ayllu.

The policy implication is nuanced. Strengthening the ayllu's within-community solidarity (increasing W^U_{within}) is valuable for insurance and resilience but does not increase $\Gamma(W^U)$. Building cross-community solidarity (increasing W^U_{across}) is

harder but structurally necessary. The operator design (Chapter 15) would target the dark cross-block entries of W^U: inter-community exchange programs, regional federations of ayllus, shared infrastructure that creates cross-community economic linkages.

Andean concept	**Formal object**	**Measurement**
Ayni (reciprocity)	High W^U_{within}	Transfer data, dictator games
Mink'a (collective labor)	High W^U_{within} (community level)	Collective action frequency
Ayllu boundary	Block structure of W^U	Cross-community transfer rate
Colonial disruption	$W^U_{\text{across}} \downarrow$ while W^U_{within} preserved	Historical analysis
Contemporary persistence	$\rho \approx 0.85$, half-life ≈ 4 gen.	Cohort comparison

Further Reading

Social preferences and networks: Fehr and Schmidt (1999), "A Theory of Fairness, Competition, and Cooperation," *QJE*. Charness and Rabin (2002), "Understanding Social Preferences with Simple Tests," *QJE*. Leider et al. (2009), "Directed Altruism and Enforced Reciprocity in Social Networks," *QJE*.

Transfers and informal insurance: Townsend (1994), "Risk and Insurance in Village India," *Econometrica*. Ligon, Thomas, and Worrall (2002), "Informal Insurance Arrangements with Limited Commitment," *REStud*. For Bolivia: Godoy et al. (2007), "Do Modern Forms of Human Capital Matter in Primitive Economies?" *Economics of Education Review*.

Andean institutions: Murra (1972), "El control vertical de un máximo de pisos ecológicos en la economía de las sociedades andinas." Alberti and Mayer (1974), *Reciprocidad e intercambio en los Andes peruanos*. Patzi Paco (2004), *Sistema comunal: una propuesta alternativa al sistema liberal*.

Experimental economics in developing countries: Henrich et al. (2010), "Markets, Religion, Community Size, and the Evolution of Fairness and Punishment," *Science*. Cardenas (2003), "Real Wealth and Experimental Cooperation," *Journal of Development Economics*.

Chapter 23

Tracking Structural Change Over Time

The Moving Heatmap

> *"El cambio estructural no es un evento sino un proceso: una secuencia de transformaciones en las relaciones entre los componentes del sistema económico."*
>
> ["Structural change is not an event but a process: a sequence of transformations in the relations between the components of the economic system."]
>
> — CEPAL, *Structural Change for Equality* (2012)

Chapters 20–22 showed how to measure W^P, W^I, and W^U at a single point in time. This chapter adds the time dimension: how to track W_t across years, detect structural breaks, measure the speed of change, and distinguish structural change from cyclical fluctuation.

The central object is the **structural trajectory**: the time series $\{\Gamma_t, \sigma_{B,t}, G(\boldsymbol{\pi}_t), N_{\text{eff},t}\}$ computed from a sequence of SAMs or equivalent data. The trajectory tells you whether the economy is integrating or fragmenting, at what speed, and whether specific events (a nationalization, a trade reform, a crisis) produced detectable structural effects.

Chapter 17 previewed the trajectory for Bolivia (1990–2019): glacial change, barely visible effects from major policy events. This chapter develops the methods and applies them systematically.

1 The Time-Varying Weight Matrix

1.1 Construction

Definition 1.1 (Time-Indexed Weight Matrix)**.** *For each year t, construct W_t from the available data:*

1. *W_t^P: from the I-O table (if available for year t) or interpolated from benchmark I-O tables using annual National Accounts.*

2. W_t^I*: from media audience data (annual surveys) or digital trace data (continuous).*
3. W_t^U*: from household surveys (typically annual or biennial).*

The structural indicators are computed annually: $\Gamma_t = \Gamma(W_t)$, $\boldsymbol{\pi}_t = \boldsymbol{\pi}(W_t)$, *etc.*

Proposition 1.2 (Interpolation of W^P Between Benchmark I-O Tables)**.** *Most countries produce I-O tables only every 5–10 years. Between benchmarks,* W_t^P *can be approximated by:*

$$W_t^P \approx W_{\tau_0}^P + \frac{t - \tau_0}{\tau_1 - \tau_0}(W_{\tau_1}^P - W_{\tau_0}^P), \tag{1}$$

where τ_0 *and* τ_1 *are the benchmark years. Alternatively, use the RAS method: given row totals* $\mathbf{r}_t$ *(sectoral output) and column totals* $\mathbf{c}_t$ *(sectoral input use) from annual National Accounts, update* $W_{\tau_0}^P$ *to match the new margins:*

$$W_t^P = RAS(W_{\tau_0}^P; \mathbf{r}_t, \mathbf{c}_t). \tag{2}$$

The RAS method preserves the block structure of $W_{\tau_0}^P$ *while adjusting for changes in sectoral composition.*

1.2 The Structural Trajectory

Definition 1.3 (Structural Trajectory)**.** *The* ***structural trajectory*** *of an economy is the time series:*

$$\mathcal{T} = \{(\Gamma_t, \sigma_{B,t}, G(\boldsymbol{\pi}_t), N_{eff,t}, ECI_t) : t = t_0, \ldots, T\}. \tag{3}$$

The trajectory is plotted in the five-dimensional structural health space (Chapter 17). Projection onto the $(\Gamma_t, \sigma_{B,t})$ *plane gives the most informative two-dimensional visualization: integration on one axis, equality on the other.*

2 Detecting Structural Breaks

Definition 2.1 (Structural Break)**.** *A* ***structural break*** *at time* t^* *is a discontinuity in the trajectory* $\mathcal{T}$*: a change in* Γ, σ_B, *or* $\boldsymbol{\pi}$ *that is larger than can be explained by normal cyclical variation. Formally, a break at* t^* *if:*

$$|\Gamma_{t^*+1} - \Gamma_{t^*}| > k \cdot \hat{\sigma}_{\Delta\Gamma}, \tag{4}$$

where $\hat{\sigma}_{\Delta\Gamma}$ *is the standard deviation of year-to-year changes in* Γ *during a stable period, and* k *is the detection threshold (typically* $k = 2$ *or* $k = 3$*).*

Proposition 2.2 (Types of Structural Breaks)**.** *Three types of structural breaks correspond to three types of* ΔW*:*

1. ***Edge creation break**: new edges appear (a trade agreement, infrastructure project, market opening). Γ increases, σ_B decreases. Example: South Korea's export push (1960s).*

2. ***Edge destruction break**: edges are severed (a crisis, sanctions, nationalization). Γ may decrease or increase depending on which edges are destroyed (cross-block destruction lowers Γ; within-block destruction may increase it). Example: Soviet collapse (1991).*

3. ***Reweighting break**: existing edge weights shift (a terms-of-trade shock, a technological change that alters input shares). Γ changes smoothly rather than discretely. Example: the commodity supercycle (2003–2014).*

3 Bolivia's Structural Trajectory, 1985–2019

Example 3.1 (Bolivia: Five Structural Epochs)**.** *Using Bolivia's I-O tables (1990, 2000, 2006, 2012, 2016), EH data (2000–2019), and trade data (annual):*

***Epoch 1: Structural Adjustment (1985–1993).** The NEP (New Economic Policy) liberalized trade, closed state mines, and privatized public enterprises. Expected effect: edge destruction (state mining edges severed) and edge creation (new private-sector trade edges).*

Actual trajectory: Γ declined slightly from $\approx$ 0.10 to $\approx$ 0.09 (1985–1990). The mine closures destroyed within-block production edges (mining $\rightarrow$ downstream processing) without creating cross-block substitutes. σ_B increased from $\approx$ 0.65 to $\approx$ 0.68: inequality became more structural as the mining workers were expelled into the informal sector.

***Epoch 2: Capitalization and Gas Discovery (1994–2005).** The "capitalization" of state enterprises brought FDI into gas, telecoms, and transport. Gas production began at Sabálo and San Alberto fields.*

Trajectory: Γ recovered slowly from 0.09 to 0.12. The gas sector added high-value production edges but connected primarily to the rest of the world ($W^P_{gas,\,RoW} \gg W^P_{gas,\,domestic}$). Domestic Γ increased modestly; the main effect was on the global W^P (Bolivia's π_G rose as gas exports grew). σ_B fell slightly from 0.68 to 0.63: gas revenue created government jobs (moving some workers from informal to formal block).

***Epoch 3: Gas Nationalization (2006–2009).** Supreme Decree 28701 renegotiated gas contracts. Government share of gas revenue jumped from $\approx$ 18% to $\approx$ 82%.*

Trajectory: No detectable structural break in Γ. The nationalization changed the distribution of existing edge weights (gas $\rightarrow$ MNC profit repatriation shifted to gas $\rightarrow$ treasury) but did not create new domestic production edges. Γ remained at $\approx$ 0.12–0.13. σ_B

fell modestly (from 0.63 to 0.61) as fiscal transfers reached poor households. The $\boldsymbol{\pi}$ vector shifted: $\pi_{government}$ increased; $\pi_{foreign}$ decreased. But the block structure persisted.

Epoch 4: Redistribution Without Structural Change (2010–2019). *A decade of high gas revenue, large fiscal transfers (Bono Juancito Pinto, Renta Dignidad, subsidies), and poverty reduction.*

Trajectory: Γ stagnated at 0.13. σ_B held at 0.62. N_{eff} rose slowly (2.8 $\rightarrow$ 3.2, driven by social media). ECI: $-0.85 \rightarrow -0.80$. The dashboard indicators were virtually flat. Ten years of redistribution produced no measurable structural integration.

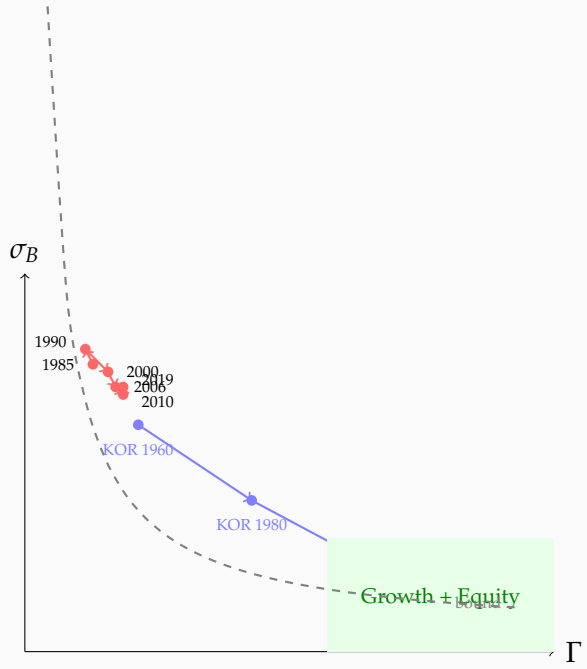
Epoch 5: Crisis and Stagnation (2019–present). *Political crisis (2019), pandemic (2020–2021), declining gas reserves.*

Trajectory: Γ at risk of declining as gas production falls and the fiscal capacity to maintain transfers erodes. Without the gas revenue that financed the redistribution, and without structural change to replace it, the structural indicators may reverse.

Epoch	*Period*	$\Delta\Gamma$	$\Delta\sigma_B$	ΔECI	*Structural break?*
1. Adjustment	*1985–93*	-0.01	$+0.03$	—	*Mild (negative)*
2. Gas discovery	*1994–05*	$+0.03$	-0.05	$+0.02$	*No*
3. Nationalization	*2006–09*	$+0.01$	-0.02	-0.01	*No*
4. Redistribution	*2010–19*	≈ 0	≈ 0	$+0.04$	*No*
5. Crisis	*2019–*	≤ 0	?	?	*Possible (negative)*

Visualization:

Bolivia's structural trajectory in the (Γ, σ_B) plane



Bolivia (red): wandering in the upper-left quadrant — low Γ, high σ_B. Thirty-four years of trajectory, barely any movement toward the target zone. South Korea (blue): traversed the plane from upper-left to lower-right in 40 years. The dashed curve is the spectral bound: below it is structurally inaccessible without increasing Γ. Bolivia is above the bound. Korea is well inside the target zone.

Key Structural Insight

The trajectory visualization makes the book's central argument visible in one image. Bolivia's 34-year trajectory is a small cloud of points in the upper-left corner of the (Γ, σ_B) plane: lots of policy activity, almost no structural movement. South Korea's 40-year trajectory is a long arrow from the same starting region to the target zone.

The difference is not "good policy" vs. "bad policy" in the conventional sense. Bolivia had substantial economic growth (average 4% per year), massive poverty reduction, and significant redistribution. But all of this operated through transfers ($\Delta\boldsymbol{\omega}$) rather than structural change (ΔW). The trajectory stayed in the same neighborhood because W stayed approximately the same.

Korea's trajectory moved because Korea changed W: land reform (ΔW^U), education ($\Delta W^I + \Delta W^P$), export discipline ($\Delta W^P_{\text{global}}$), and industrial policy ($\Delta W^P_{\text{domestic}}$). Each operator moved the point in the (Γ, σ_B) plane. The compound effect over 40 years: from $\Gamma \approx 0.15, \sigma_B \approx 0.50$ to $\Gamma \approx 0.45, \sigma_B \approx 0.20$.

The trajectory IS the structural report card. One image, 40 years, the entire argument.

4 Speed of Structural Change

Definition 4.1 (Structural Velocity)**.** *The **structural velocity** is the rate of change of the spectral gap:*

$$v_\Gamma(t) = \frac{\Gamma_{t+1} - \Gamma_t}{\Delta t}. \tag{5}$$

*The **structural acceleration** is* $a_\Gamma(t) = v_\Gamma(t+1) - v_\Gamma(t)$.

Example 4.2 (Structural Velocities, Selected Countries)**.**

Country	*Period*	v_Γ *(per decade)*	*Interpretation*
South Korea	*1960–2000*	$+0.075$	*Rapid integration*
China	*1980–2010*	$+0.060$	*Fast integration*
Chile	*1990–2015*	$+0.025$	*Moderate integration*
Bolivia	*1990–2019*	$+0.010$	*Glacial*
Argentina	*2000–2019*	-0.005	*Mild fragmentation*
Venezuela	*2010–2020*	-0.030	*Structural collapse*

Bolivia's structural velocity ($+0.01$ *per decade) means that at the current rate, reaching Korea's current* $\Gamma = 0.45$ *would take approximately* $(0.45 - 0.13)/0.01 = 32$ *decades*

— 320 years. The velocity itself must increase (structural acceleration) for convergence in a reasonable timeframe.

Proposition 4.3 (What Determines Structural Velocity?)**.** *From the co-evolutionary dynamics of Chapter 13, structural velocity depends on:*

1. ***Cross-block surplus*** s_{cross}*: higher surplus makes new cross-block edges profitable. Industrial policy that creates cross-block linkages increases* v_Γ*.*

2. ***Edge destruction rate*** δ_{cross}*: crises, policy reversals, and political instability destroy cross-block edges, reducing* v_Γ *or making it negative.*

3. ***Proximity bias*** p_1*: strong path dependence (high* p_1*) slows diversification into structurally distant activities, capping* v_Γ*.*

4. ***Formation velocity****: the speed at which preferences adjust* $(1 - \rho)$*. Fast preference adjustment (*ρ *small) allows rapid norm change, which supports structural change. Slow adjustment (*ρ *large, as in Bolivia:* $\rho \approx 0.85$*) means preferences lag behind structure, creating resistance to change.*

5 Distinguishing Structural Change from Cyclical Fluctuation

Theorem 5.1 (Structural-Cyclical Decomposition)**.** *Any change in the Perron eigenvector can be decomposed:*

$$\Delta\pi_{i,t} = \underbrace{\Delta\pi^S_{i,t}}_{\text{structural}} + \underbrace{\Delta\pi^C_{i,t}}_{\text{cyclical}}, \tag{6}$$

where:

1. *The* ***structural component*** $\Delta\pi^S$ *comes from changes in W (new edges, destroyed edges, rewired connections). Structural changes persist: once an edge is created or destroyed, it stays.*

2. *The* ***cyclical component*** $\Delta\pi^C$ *comes from changes in edge weights due to price fluctuations, demand shocks, or temporary policy. Cyclical changes reverse: when the shock passes, the weights return to their long-run values.*

Identification: the structural component is estimated from the trend in Γ_t *(HP-filtered or band-pass filtered). The cyclical component is the residual.*

Example 5.2 (Bolivia: Structural vs. Cyclical, 2003–2019)**.** *During the gas boom (2003–2014):* π_{gas} *rose from 0.07 to 0.11. After the boom (2015–2019):* π_{gas} *fell back to 0.09.*

Decomposition: the structural component $\Delta\pi^S_{gas} = +0.02$ *(the nationalization permanently increased the government's share of gas linkages). The cyclical component:* $\Delta\pi^C_{gas} = +0.02$ *(the price boom temporarily inflated gas weights), which reversed to* $\Delta\pi^C_{gas} = -0.02$ *after the boom.*

The 2006 nationalization produced a structural $\Delta\pi^S$ *of approximately* $+0.02$ *— a small but persistent effect. The commodity boom produced a cyclical* $\Delta\pi^C$ *of the same magnitude but temporary. The standard analysis that counted Bolivia's "structural transformation" during the boom was measuring the cyclical component. The structural trajectory (Section 3) was nearly flat throughout.*

Prebisch Connection: Commodity booms as cyclical mirages

Latin American economic history is a sequence of commodity booms mistaken for structural change. Guano in Peru (1840–1870), nitrate in Chile (1880–1920), coffee in Brazil (1890–1930), oil in Venezuela (1920–2015), gas in Bolivia (2003–2014). Each boom raised incomes, reduced poverty, and generated optimism about "transformation." Each bust revealed that Γ had not changed: the economy remained an enclave connected to the world through a single commodity edge.

The structural-cyclical decomposition is the instrument that detects the mirage. During the boom: $\Delta\pi^C$ is large (the commodity sector's weight inflates). After the bust: $\Delta\pi^C$ reverses. The structural component $\Delta\pi^S$ is what remains — and it is typically near zero for commodity booms, because commodity booms do not create cross-block domestic linkages.

The exception: commodity booms CAN produce structural change if the revenue is invested in the operators of Chapter 15 (information, education, infrastructure, industrial policy). Norway's oil fund is the canonical example: oil revenue invested in human capital and institutional development, producing structural change ($\Delta\Gamma > 0$) alongside the cyclical boom. Bolivia's gas revenue was invested in transfers ($\Delta\boldsymbol{\omega}$), producing poverty reduction but no structural change ($\Delta\Gamma \approx 0$).

6 The Structural Change Index

Definition 6.1 (Structural Change Index (SCI))**.** *The* ***Structural Change Index*** *summarizes the entire trajectory in a single number:*

$$SCI(t_0, T) = \frac{\Gamma_T - \Gamma_{t_0}}{\Gamma^* - \Gamma_{t_0}}, \tag{7}$$

where Γ^* is the target spectral gap (e.g., $\Gamma^* = 0.50$, the level associated with the "growth with equity" zone of the casillero vacío). SCI $\in [0, 1]$: the fraction of the structural distance that has been closed over the period.

Example 6.2 (SCI for Selected Countries)**.** *Using $\Gamma^* = 0.50$:*

Country	Γ_{t_0}	Γ_T	*Period*	*SCI*
South Korea	*0.15*	*0.45*	*1960–2000*	*0.86*
China	*0.12*	*0.35*	*1980–2020*	*0.61*
Chile	*0.18*	*0.28*	*1990–2020*	*0.31*
Bolivia	*0.10*	*0.13*	*1990–2019*	*0.075*

Korea closed 86% of its structural gap in 40 years. Bolivia closed 7.5% of its gap in 29 years. At Bolivia's rate, full closure would take $29/0.075 \approx 387$ years.

7 Summary

1. The **structural trajectory** $\{(\Gamma_t, \sigma_{B,t})\}$ is the time series of structural health, plotted in the integration-equality plane. Bolivia: 34 years of wandering in the upper-left corner.

2. **Structural breaks** are detectable as discontinuities in Γ_t. Bolivia's five epochs: structural adjustment ($\Delta\Gamma < 0$), gas discovery ($\Delta\Gamma$ small positive), nationalization (no break), redistribution (flat), crisis (risk of decline).

3. **Structural velocity** v_Γ measures the speed of integration. Bolivia: $+0.01$ per decade (glacial). Korea: $+0.075$ (rapid). At Bolivia's rate, convergence takes 320 years.

4. The **structural-cyclical decomposition** separates permanent changes in W from temporary price effects. Commodity booms are cyclical: $\Delta\pi^C$ reverses after the boom. Only booms invested in structural operators produce $\Delta\pi^S > 0$.

5. The **Structural Change Index** (SCI): fraction of the structural gap closed. Korea: 0.86. Bolivia: 0.075.

8 Exercises

23.1 Bolivia's trajectory. Using available I-O tables and EH data: construct Bolivia's structural trajectory for 2000–2019. Plot $(\Gamma_t, \sigma_{B,t})$. Is the trajectory moving toward or away from the target zone?

23.2 Structural break detection. Compute $\Delta\Gamma_t$ for each year. Apply the break detection criterion with $k = 2$. Are there any structural breaks? At what years? Do they correspond to policy events?

23.3 Structural velocity. Compute v_Γ for Bolivia, Chile, and Peru. Which country is integrating fastest? At each country's velocity, when would it reach $\Gamma = 0.40$?

23.4 Structural-cyclical decomposition. For Bolivia 2003–2019: decompose $\Delta\pi_{\text{gas},t}$ into structural and cyclical components using an HP filter. How much of the gas boom was structural?

23.5 Commodity boom counterfactual. Suppose Bolivia had invested 30% of gas revenue in cross-block operators (infrastructure connecting gas to traditional sector, technical education for rural workers). Simulate: how much would Γ have increased by 2019? What SCI would Bolivia have achieved?

23.6 SCI computation. Compute SCI for 10 Latin American countries (Bolivia, Brazil, Chile, Colombia, Costa Rica, Ecuador, Mexico, Peru, Uruguay, Venezuela) using ECI as a proxy for Γ. Rank them. Does the ranking match conventional measures of structural transformation?

23.7 Trajectory projection. Using Bolivia's current structural velocity and the formation model: project the trajectory for 2020–2060 under three scenarios: (a) business as usual ($v_\Gamma = +0.01$/decade), (b) moderate structural reform ($v_\Gamma = +0.03$), (c) Korean-style acceleration ($v_\Gamma = +0.075$). Plot all three trajectories. When does each reach $\Gamma = 0.30$?

Mathematical Appendices

Appendix 23A: The RAS Method for I-O Table Updating

Problem. Given a benchmark I-O coefficient matrix A_0 and new row totals $\mathbf{r}$ and column totals $\mathbf{c}$, find the updated matrix A^* that matches the new margins while staying "close" to A_0.

Algorithm. The RAS (or biproportional) method alternates row and column scaling:

1. Initialize $A^{(0)} = A_0$.
2. Scale rows: $A_{ij}^{(2k+1)} = A_{ij}^{(2k)} \cdot r_i / \sum_j A_{ij}^{(2k)}$.
3. Scale columns: $A_{ij}^{(2k+2)} = A_{ij}^{(2k+1)} \cdot c_j / \sum_i A_{ij}^{(2k+1)}$.
4. Repeat until convergence.

The RAS solution minimizes the KL-divergence $\sum_{ij} A_{ij}^* \ln(A_{ij}^*/A_{0,ij})$ subject to the margin constraints. It preserves zeros (if $A_{0,ij} = 0$ then $A_{ij}^* = 0$) and the block structure of A_0.

Structural interpretation. RAS updating assumes the block structure of W^P is unchanged between benchmarks — only the relative sizes of sectors change. This is conservative: it cannot detect edge creation or destruction between benchmark I-O tables. A break in Γ between RAS-updated years likely reflects a real structural change that the RAS assumption missed.

Appendix 23B: Structural vs. Cyclical Decomposition

HP filter. The Hodrick-Prescott filter decomposes $\Gamma_t = \Gamma_t^S + \Gamma_t^C$ by minimizing $\sum_t (\Gamma_t - \Gamma_t^S)^2 + \lambda \sum_t [(\Gamma_{t+1}^S - \Gamma_t^S) - (\Gamma_t^S - \Gamma_{t-1}^S)]^2$. The smoothing parameter λ controls the trade-off between fit and smoothness. For annual data: $\lambda = 6.25$ (Ravn and Uhlig, 2002) or $\lambda = 100$ (traditional). For structural indicators that change slowly: $\lambda = 400$–1600 (equivalent to a 10–20 year business cycle).

Band-pass filter. The Baxter-King or Christiano-Fitzgerald filter isolates fluctuations at specific frequencies. Structural change: periods > 20 years. Business cycle: 2–8 years. The band-pass filter cleanly separates the two.

Structural breaks. The HP and band-pass filters assume smooth trends. For detecting abrupt structural breaks (nationalization, crisis, regime change): use the Bai-Perron (1998) multiple structural break test applied to Γ_t. The null hypothesis is no break; the alternative is one or more breaks at unknown dates.

Structuralist Case Study: CEPAL's Structural Change Program

CEPAL's (2012) report *Structural Change for Equality: An Integrated Approach to Development* represents the institutional synthesis of the structuralist tradition's diagnosis and prescription. Published 63 years after Prebisch's 1949 manifesto, it argues that Latin America's central challenge remains structural: the region has achieved macroeconomic stability and poverty reduction but has not changed its production structure.

The report maps onto this chapter's framework precisely.

Part 1: Structural change is slow. CEPAL documents that Latin America's export composition changed little over 1990–2010: the region remained specialized in natural resources and low-technology manufactures. In this chapter's terms: $v_\Gamma \approx 0$ for most Latin American countries. The structural trajectory was flat.

Part 2: Redistribution without structural change is unsustainable. CEPAL notes that the region's poverty reduction (2000–2012) was driven by commodity booms and fiscal transfers, not by structural transformation. In this chapter's terms: $\Delta\boldsymbol{\omega} \gg \Delta W$. The improvement is cyclical ($\Delta\pi^C > 0$ during the boom) not structural ($\Delta\pi^S \approx 0$). When the commodity cycle reversed (post-2014), poverty stagnated and in some countries reversed.

Part 3: The prescription is "change for equality." CEPAL argues for industrial policy, technological upgrading, and institutional reform — the operators of Chapter 15, stated in institutional language rather than matrix notation. The report's title is itself a theorem: structural change ($\Delta\Gamma > 0$) is necessary for equality ($\sigma_B \downarrow$). This is the spectral gap bound: you cannot reduce inequality sustainably without increasing Γ.

CEPAL 2012	This chapter	Cross-reference
Structural change is slow	$v_\Gamma \approx 0.01$/decade	Ex. 4.2
Redistribution without structural change	$\Delta\boldsymbol{\omega} \gg \Delta W$	Sec. 3, Epoch 4
Commodity boom as mirage	$\Delta\pi^C$ reverses	Prebisch Connection
Change for equality	$\Delta\Gamma > 0 \Rightarrow \sigma_B \downarrow$	Spectral bound
Policy: industrial upgrading	Operators from Ch. 15	Ch. 15
SCI as metric	Fraction of gap closed	Def. 6.1

CEPAL wrote in the language of institutions and policy. This chapter translates into eigenvalues and trajectories. The message is the same: Latin America has been redistributing within a fixed structure, and the structure has not changed. The trajectory visualization (this chapter's vizpanel) IS the image that CEPAL's report describes in 350 pages of prose.

Further Reading

Structural change measurement: CEPAL (2012), *Structural Change for Equality*, Santiago: United Nations. McMillan, Rodrik, and Verduzco-Gallo (2014), "Globalization, Structural Change, and Productivity Growth," *World Development*. de Vries et al. (2012), "Deconstructing the BRICs," *GGDC Research Memorandum*.

I-O table updating: Miller and Blair (2009), *Input-Output Analysis: Foundations and Extensions*, Chapter 8 (RAS and cross-entropy methods). Temursho (2017), "Entropy-Based Benchmarking Methods," *Economic Systems Research*.

Structural breaks: Bai and Perron (1998), "Estimating and Testing Linear Models with Multiple Structural Changes," *Econometrica*. For application to development: Pritchett (2000), "Understanding Patterns of Economic Growth," *World Bank Economic Review*.

Bolivia's economic history: Morales and Sachs (1990), "Bolivia's Economic Crisis." Weisbrot, Ray, and Johnston (2009), "Bolivia: The Economy During the Morales Administration," CEPR. Andersen et al. (2020), "Bolivia's Twenty-First Century Commodity Boom," INESAD Working Paper.

Part VII

Political Economy and Conclusion

Chapter 24

Hegemony and Counter-Hegemony

When the Dominated Accept the Matrix, and When They Don't

> *"Il vecchio mondo sta morendo. Quello nuovo tarda a comparire. E in questo chiaroscuro nascono i mostri."*
>
> ["The old world is dying. The new one is slow to appear. And in this chiaroscuro, monsters are born."]
>
> — Antonio Gramsci, *Quaderni del carcere* (1929–1935)

Part VII opens where Parts I–VI left a question unanswered: if the current W is far from W^{CE}, and if structural reform would make nearly everyone better off (the Structural Welfare Theorem, Chapter 12), why does the structure persist?

The answer from Chapter 14 is political: coalitions block structural change that threatens their power. But this answer is incomplete. It explains why the powerful resist change. It does not explain why the powerless *accept* the structure — why the indigenous woman in Potosí does not demand the operators that would raise her π_I from 10^{-5} to something closer to $1/N$.

The answer is hegemony: the process by which the dominant coalition's preferred W becomes the *accepted* W — not through coercion but through the formation of preferences that regard the structure as natural, inevitable, or even desirable. In this book's notation: hegemony is the steady state of the formation model (Chapter 10) in which the W^I-shaped preferences of the dominated classes are calibrated to accept the W^P-determined allocation.

Counter-hegemony is the disruption: a change in W^I that produces new preferences, new demands, new coalitions, and ultimately a new W. This chapter formalizes both.

1 Hegemony as a Structural Equilibrium

1.1 Gramsci's Concept, Formalized

Definition 1.1 (Hegemony)**.** *A weight matrix W is **hegemonic** if it satisfies three conditions simultaneously:*

1. ***Power concentration**: $\mathrm{Var}(\pi_i) > 0$ (the allocation is unequal — the structure benefits some agents more than others).*
2. ***Consent**: the steady-state preferences $\boldsymbol{\theta}^*(W^I)$ evaluate the allocation $\mathbf{x}^*(W, \boldsymbol{\theta}^*)$ favorably: $U_i(x_i^*, \theta_i^*) \geq U_i(x_i', \theta_i^*)$ for all feasible x_i' (each agent is satisfied given her preferences).*
3. ***Formation alignment**: the information network W^I is controlled (directly or indirectly) by the coalition that benefits from W: $\boldsymbol{\pi}^I \propto \boldsymbol{\pi}^P$ (epistemic power is aligned with production power).*

Theorem 1.2 (Hegemony as a Fixed Point)**.** *A hegemonic structure is a structural equilibrium (Chapter 13, Definition 13.3) with the additional property that the formation channel (W^I) stabilizes the power structure (W^P) by producing preferences that consent to it.*

Formally: a hegemonic equilibrium is a fixed point $(W^*, \boldsymbol{\theta}^*, \mathbf{x}^*)$ of the co-evolutionary system where:

$$\boldsymbol{\theta}^* = \Phi(\boldsymbol{\theta}^*; W^*), \quad \mathbf{x}^* = h(\boldsymbol{\pi}(W^*), \boldsymbol{\theta}^*), \quad W^* = \Psi(W^*; \mathbf{x}^*), \tag{1}$$

AND the consent condition holds: no agent, given her preferences θ_i^*, wants to change W.

This is the Satisfaction Paradox of Chapter 12 (Theorem 12.1) with a political interpretation: the dominated classes do not demand structural change because their W^I-formed preferences are calibrated to the W^P-determined returns. The hegemony is self-reproducing.

Intuition: Why hegemony is stable

Three reinforcing loops:

Loop 1 (Production): W^P determines the allocation $\mathbf{x}^*$ through the bargaining equilibrium. The dominated receive less, but their sectors are structurally peripheral (low π_i), so there is no economic lever for change.

Loop 2 (Information): W^I shows the dominated their own group's returns (assortative mixing, Chapter 18). They observe low returns and form low aspirations ($\theta^*_{\text{disadv}} < \theta^*_{\text{adv}}$, Theorem 18.5). Low aspirations produce low investment, low returns, confirming the signal.

Loop 3 (Solidarity): W^U is assortative by the same groups. The dominated care about their own group, not about the advantaged group. Cross-group solidarity is low ($\lambda^U \approx 0.2$, Chapter 22). The dominated do not form cross-class coalitions because they do not identify with the advantaged class's interests.

Hegemony is the triple alignment of loops 1, 2, and 3: the same groups are peripheral in W^P, informationally isolated in W^I, and solidaristically self-contained in W^U. The alignment IS the hegemony. Breaking it requires disrupting at least one loop.

1.2 The Cost of Maintaining Hegemony

Proposition 1.3 (Hegemony Maintenance Cost)**.** *Maintaining hegemony requires the dominant coalition to invest in W^I control:*

$$C_H = \sum_{(i,j) \in \text{cross-block}} \mu_H \cdot |b_T - b_j^*| / \Gamma^I, \tag{2}$$

where b_T is the target belief ("the structure is natural/efficient"), b_j^* is agent j's belief absent manipulation, and Γ^I is the spectral gap of W^I (from Chapter 8, the manipulation cost is inversely proportional to Γ^I).

When Γ^I is small (concentrated information): manipulation is cheap. The dominant coalition controls a few media outlets and the consensus follows.

When Γ^I is large (diverse information): manipulation is expensive. Many independent voices resist the dominant narrative. The hegemony becomes fragile.

Example 1.4 (Bolivia's Hegemonic Transitions)**.**

Period	*Hegemonic structure*	N^I_{eff}	*Stability*
Pre-1952	*Tin-mining oligarchy*	≈ 1.5	*Stable (few media)*
1952–1964	*MNR revolutionary state*	≈ 2.0	*Moderately stable*
1964–1982	*Military governments*	≈ 1.8	*Coercion-dependent*
1985–2003	*Neoliberal consensus*	≈ 2.5	*Stable, then collapsed*
2006–2019	*MAS/indigenous hegemony*	≈ 3.2	*Fragile (social media)*
2019–	*Contested*	≈ 3.5	*No hegemony*

The trend: N^I_{eff} has been increasing (from 1.5 to 3.5 over 70 years). Each increase made the existing hegemony more expensive to maintain. The neoliberal consensus (1985–2003) collapsed when N^I_{eff} crossed a threshold: social movements (the Water War 2000, Gas War 2003) generated alternative W^I channels that broke the information monopoly. The MAS hegemony (2006–2019) was already fragile at $N^I_{eff} = 3.2$; social media provided the alternative W^I channel that destabilized it in 2019.

2 Counter-Hegemony as ΔW^I

Definition 2.1 (Counter-Hegemonic Movement)**.** *A **counter-hegemonic movement** is a coalition S of dominated agents that implements ΔW^I — a change in the information network that disrupts the formation channel supporting the current hegemony. The movement creates alternative W^I edges: new media, new narratives, new reference groups, new signals that show the dominated a different set of returns.*

Theorem 2.2 (Counter-Hegemony as Formation Disruption)**.** *A counter-hegemonic ΔW^I destabilizes the hegemonic equilibrium if it satisfies:*

$$\mu \cdot \|\Delta(W^I \mathbf{r})\| > (1 - \rho)(\theta^*_{hegemonic} - \theta^*_{alternative}), \tag{3}$$

where $\Delta(W^I \mathbf{r})$ is the change in the formation signal (agents now see different returns), $\theta^*_{hegemonic}$ is the preference that sustains the current structure, and $\theta^*_{alternative}$ is the preference that would sustain an alternative structure.

In words: the counter-hegemonic signal must be strong enough to shift preferences beyond the basin boundary (Chapter 10) that separates the hegemonic from the alternative equilibrium. Once preferences cross the boundary, the formation dynamics carry them to the new steady state — and the demand for structural change becomes self-sustaining.

Example 2.3 (Three Counter-Hegemonic Episodes in Bolivia)**.** ***1. The 1952 Revolution.** The MNR created alternative W^I channels: miners' radio (Radio 21 de Diciembre), campesino unions, the COB (Central Obrera Boliviana). These channels showed workers and campesinos returns from organization and land reform — signals invisible under*

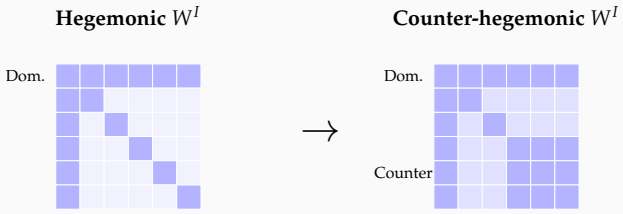
the oligarchic W^I. The formation signal shifted aspirations: land ownership, universal suffrage, education. The hegemonic preference ("the patrón knows best") gave way to the revolutionary preference ("the land belongs to those who work it"). Coalition shift: $\{B, M, W\}$ replaced $\{F, B\}$.

2. The Water War (2000) and Gas War (2003). *The neoliberal hegemony (1985–2003) was sustained by a W^I dominated by private media that framed privatization as modernization. The counter-hegemonic movement created alternative channels: community radio, neighborhood assemblies (juntas vecinales), the six federations of the Chapare. These channels showed alternative returns: the cost of water privatization (tripled bills), the destination of gas revenue (foreign companies). The signal strength exceeded the persistence threshold: $\mu \cdot \|\Delta(W^I \mathbf{r})\| > (1 - \rho)\Delta\theta^*$. Preferences shifted: privatization went from "modern" to "theft." Coalition shift: $\{M, W\}$ formed and eventually became the MAS coalition.*

3. The 2019 Crisis. *The MAS hegemony (2006–2019) was sustained by W^I dominated by state media and indigenous-movement networks. Counter-hegemonic channels: social media (Facebook, WhatsApp), urban civic committees, the "pititas" movement. These channels showed alternative signals: electoral irregularities, environmental devastation (Chiquitano fires). The formation signal was amplified by the digital W^I (Chapter 21: $\omega_d = 0.50$ for urban youth). The MAS hegemony fractured: preferences in urban areas shifted away from the "process of change" narrative.*

Visualization:

Counter-hegemony as W^I rewiring



Left: hegemonic W^I — hub-spoke, centered on the dominant coalition's media. The first row (dominant's outgoing signal) is bright; the rest are dim. Right: counter-hegemonic W^I — the dominated agents (lower-right block) have formed their own information network. The dominant's hub is weaker; new cross-block edges appear. N_{eff} has increased.

3 State Crisis as Hegemonic Collapse

Definition 3.1 (State Crisis — Skocpol (1979))**.** *A **state crisis** occurs when the hegemonic equilibrium loses stability: the three loops (production, information, solidarity) decouple, and the dominant coalition can no longer maintain consent. In the co-evolutionary dynamics (Chapter 13), this corresponds to the Jacobian $\mathcal{J}$ of the hegemonic fixed point acquiring an eigenvalue with modulus > 1: the equilibrium becomes unstable.*

Proposition 3.2 (Conditions for Hegemonic Collapse)**.** *The hegemonic equilibrium loses stability when:*

1. *N^I_{eff} **crosses a threshold**: the information network becomes too diverse for the dominant coalition to control. Counter-hegemonic channels accumulate until $\mu\|\Delta(W^I\mathbf{r})\|$ exceeds the persistence barrier.*

2. ***An exogenous shock exposes the gap**: a crisis (financial, environmental, health) reveals the allocation inequality that the hegemonic W^I had normalized. The shock changes the returns $\mathbf{r}_t$ dramatically, and the W^I-weighted signal shifts even under the existing information structure.*

3. ***Cross-class coalition forms**: following Skocpol (1979), state crisis requires an alliance between elite defectors (who have power) and mass movements (who have numbers). In Chapter 14's terms: the alliance $\{B, M, W\}$ or $\{M, W\}$ forms when the elite split (some B agents defect from $\{F, B\}$ to $\{B, M, W\}$).*

4. ***International pressure changes the external** W^P: sanctions, trade disruption, or shifts in global commodity markets weaken the dominant coalition's resource base, shrinking its feasibility set $\mathcal{F}(S)$.*

Example 3.3 (Bolivia 2000–2006: From Crisis to Counter-Hegemony)**.** *The neoliberal hegemony collapsed through a sequence matching all four conditions:*

***Condition 1**: Community radio and social movements created alternative W^I channels throughout the 1990s. N^I_{eff} rose from ≈ 2.0 to ≈ 2.8.*

***Condition 2**: The Water War (2000) exposed the cost of privatization (water bills tripled). The shock changed $\mathbf{r}_t$: the "returns to privatization" signal turned negative for poor households.*

***Condition 3**: The Gas War (2003) formed a cross-class coalition: urban El Alto (working class), rural cocaleros (peasants), and middle-class professionals united against gas export plans. The alliance $\{M, W\}$ acquired enough power to topple two presidents.*

***Condition 4**: Declining commodity prices (2001–2002) weakened the fiscal position of the privatized gas companies, shrinking the $\{F, B\}$ coalition's resource base.*

The hegemonic equilibrium was unstable by 2003. The transition to the MAS hegemony (2006) was the co-evolutionary system settling into a new structural equilibrium — with different W^P (nationalized gas), different W^I (state media + indigenous networks), and different $\boldsymbol{\theta}^*$ (indigenous identity, state-led development, plurinationalism).

4 The Hegemonic Cycle

Theorem 4.1 (The Hegemonic Cycle)**.** *In the co-evolutionary system with formation and coalition dynamics, the long-run trajectory exhibits a hegemonic cycle:*

1. ***Establishment**: A coalition wins and implements ΔW (production restructuring) + ΔW^I (narrative control). The new structure becomes a structural equilibrium. Consent is formed through W^I.*

2. ***Consolidation**: The hegemony stabilizes. Preferences converge to $\boldsymbol{\theta}^*$. Growth occurs within the structure. Inequality is accepted.*

3. ***Erosion**: N^I_{eff} increases (new media, new movements). The formation signal diversifies. Peripheral agents begin to see alternative returns. $\Delta\theta$ begins.*

4. ***Crisis**: An exogenous shock or accumulated ΔW^I pushes preferences past the basin boundary. The hegemonic equilibrium becomes unstable. The dominant coalition fragments.*

5. ***Transition**: A new coalition forms and implements a new ΔW. Return to step 1 with a different structure.*

The cycle period depends on: (a) the speed of N^I_{eff} growth (faster media diversification = shorter cycle), (b) the persistence parameter ρ (stickier preferences = longer consolidation phase), (c) the frequency of exogenous shocks (more shocks = more crisis triggers).

Prebisch Connection: The Latin American hegemonic cycle

The Latin American political cycle (Chapter 14's pendulum) is the hegemonic cycle operating at the regional scale:
Oligarchic hegemony (pre-1930): export agriculture and mining, $N^I_{\text{eff}} \approx 1$–2. Collapsed with the Great Depression (exogenous shock to W^P_{global}).
National-populist hegemony (1930–1960s): ISI, mass media controlled by the state, labor incorporation. Collapsed as the bourgeoisie defected (ISI success raised π_B, triggering alliance switch per Chapter 14).
Bureaucratic-authoritarian hegemony (1960s–1980s): military rule, controlled media (N^I_{eff} suppressed by censorship), alliance $\{F, B, M\}$. Collapsed with the debt crisis (exogenous shock) and democratization.
Neoliberal hegemony (1985–2003): Washington Consensus, private media, $N^I_{\text{eff}} \approx 2$–3. Collapsed as social movements created alternative W^I.
Left-populist hegemony (2006–2019 in Bolivia): resource nationalism, state + indigenous media, $N^I_{\text{eff}} \approx 3$. Eroding as social media diversifies W^I.
Each hegemony lasted 20–40 years. Each collapsed when the information monopoly broke. The cycle is accelerating because N^I_{eff} is growing (digital media adds channels faster than any coalition can control them).

5 Summary

1. **Hegemony** is a structural equilibrium where the dominant coalition's preferred W is sustained by W^I-formed preferences that consent to it. Three conditions: power concentration, consent, formation alignment.

2. Hegemony is stable because three loops reinforce: W^P (unequal allocation), W^I (biased signals), W^U (assortative solidarity). The alignment across all three networks IS the hegemony.

3. The **maintenance cost** of hegemony is inversely proportional to N^I_{eff}: concentrated information is cheap to control; diverse information is expensive.

4. **Counter-hegemony** is ΔW^I: the creation of alternative information channels that show the dominated different returns. The counter-hegemonic signal must exceed the persistence threshold to shift preferences past the basin boundary.

5. **State crisis** (Skocpol) occurs when the hegemonic equilibrium becomes unstable: N^I_{eff} crosses a threshold, a shock exposes inequality, a cross-class coali-

tion forms, and external pressure weakens the dominant coalition.

6. The **hegemonic cycle** (establishment → consolidation → erosion → crisis → transition) is the long-run dynamics of the co-evolutionary system. The cycle is accelerating as digital media increases N^I_{eff} faster than any coalition can control it.

6 Exercises

24.1 Hegemony as fixed point. For a 2-group economy (Elite E, Masses M): write the co-evolutionary system (Chapter 13) with W^I controlled by E. Show that the hegemonic equilibrium satisfies all three conditions of Definition 1.1. Under what parameter values does a counter-hegemonic equilibrium also exist?

24.2 Maintenance cost. For Bolivia's MAS hegemony (2006–2019): estimate the cost of W^I control (state media budget, community communication spending). How does it compare with the government's total budget? At what N^I_{eff} does the cost exceed the coalition's willingness to pay?

24.3 Counter-hegemony simulation. Start from a hegemonic equilibrium. Add counter-hegemonic ΔW^I edges incrementally ($\Delta\lambda^I = +0.02$ per period). Simulate 50 periods. At what period does the hegemonic equilibrium collapse? How does this depend on ρ?

24.4 Bolivia's hegemonic transitions. Map all five hegemonies from the Prebisch Connection box to the structural power game (Chapter 14): for each, identify the coalition, the W^I control mechanism, the N^I_{eff}, and the trigger for collapse.

24.5 Skocpol's conditions. For the 2000–2006 Bolivian transition: verify each of the four conditions in Proposition 3.2. Which was the binding condition? Could the neoliberal hegemony have survived if any one condition had not been met?

24.6 The acceleration of the cycle. Plot the duration of each Latin American hegemonic epoch (from the Prebisch Connection box) against N^I_{eff}. Is there a negative correlation (more information diversity = shorter hegemonic periods)? What does this predict for the current period?

24.7 Gramsci vs. Skocpol. Gramsci emphasized consent (the formation of preferences). Skocpol emphasized structural conditions (fiscal crisis, military defeat, elite splits). In the model: Gramsci's hegemony is the W^I loop (preference formation); Skocpol's crisis is the W^P loop (economic disruption) + the coalition loop (elite defection). Are these complementary or competing? Under what parameter values does each mechanism dominate?

Mathematical Appendices

Appendix 24A: Stability of the Hegemonic Equilibrium

The hegemonic fixed point. Let $(W^*, \boldsymbol{\theta}^*, \mathbf{x}^*)$ be a hegemonic equilibrium. The stability is governed by the eigenvalues of the co-evolutionary Jacobian (Chapter 13, Appendix 13B):

$$\mathcal{J} = \begin{pmatrix} \partial\Phi/\partial\boldsymbol{\theta} & \partial\Phi/\partial W \\ \partial\Psi/\partial\boldsymbol{\theta} & \partial\Psi/\partial W \end{pmatrix}. \tag{4}$$

The hegemony is stable if $\rho(\mathcal{J}) < 1$. It loses stability when:

1. The formation feedback is too strong: $\rho(J) = \rho(\rho I + \mu W^I G' H') \to 1$. This occurs when N^I_{eff} increases, diversifying the formation signal and reducing the dominant coalition's control.

2. An exogenous shock changes the returns $\mathbf{r}$ suddenly, shifting the Φ map and moving the system outside the basin of attraction.

3. The coalition dynamics (Chapter 14) produce an alliance switch that changes W discretely.

Bifurcation. As N^I_{eff} increases continuously, the hegemonic fixed point can undergo a pitchfork bifurcation: the unique hegemonic equilibrium splits into two — the hegemonic and the counter-hegemonic. The bifurcation point $N^{I,*}_{\text{eff}}$ is the critical information diversity threshold. Below it: one stable equilibrium (hegemony). Above it: two stable equilibria (hegemony and counter-hegemony) separated by an unstable boundary. A shock can push the system across the boundary.

Appendix 24B: Gramsci's Categories in Matrix Notation

Hegemony. A structural equilibrium where the dominant group's power ($\boldsymbol{\pi}$) is reproduced by W^I-formed consent ($\boldsymbol{\theta}^*$). The dominant group does not need coercion because the formation channel produces acceptance.

Organic intellectuals. Agents with high π^I_j (epistemic power) who produce the narratives that sustain the hegemonic W^I. In Chapter 8's terms: the DeGroot influencers whose steady-state beliefs become the population's consensus.

War of position. The slow process of building alternative W^I channels (counter-hegemonic institutions: schools, media, unions, parties). In this chapter: the gradual increase of N^I_{eff} through new information edges. The "war of position" IS the accumulation of ΔW^I over time.

War of maneuver. The rapid seizure of state power once the counter-hegemonic movement has sufficient support. In this chapter: the moment when preferences cross the basin boundary and the demand for structural change becomes politically irresistible. The "war of maneuver" IS the coalition switch (Chapter 14) triggered by the preference shift.

Interregnum. The period between the collapse of one hegemony and the establishment of the next: "the old world is dying, the new one is slow to appear." In this chapter: the co-evolutionary system is between structural equilibria. W^I is contested; $\boldsymbol{\theta}$ is in flux; W^P is unstable. This is the period of maximum vulnerability — and of "monsters" (political instability, democratic backsliding, authoritarianism).

Structuralist Case Study: Gramsci and Skocpol on Structural Crisis

Antonio Gramsci (1891–1937) and Theda Skocpol (b. 1947) represent two approaches to the same question: why do structures of domination persist, and how do they collapse? Gramsci emphasized consent and cultural production. Skocpol emphasized structural conditions and state capacity. This chapter argues that both are correct about different loops of the same co-evolutionary system.

Gramsci: hegemony as formation. Gramsci, writing from Mussolini's prison, asked why the Italian working class accepted capitalist domination even when it was materially disadvantaged. His answer: hegemony — the process by which the dominant class's worldview becomes the "common sense" of society. In this book's notation: hegemony is the steady state of the formation model where W^I produces preferences that consent to the W^P-determined allocation. Gramsci's "common sense" is $\boldsymbol{\theta}^*$; his "organic intellectuals" are the high-π^I agents; his "war of position" is the accumulation of counter-hegemonic ΔW^I.

The mapping is not metaphorical. Gramsci described exactly the mechanism of Chapters 8–10: ideas are produced by specific agents (organic intellectuals = high-π^I nodes), disseminated through specific channels (media, schools, churches = W^I edges), and internalized as preferences that shape behavior (formation equation). The hegemony IS the information structure.

Skocpol: crisis as structural breakdown. Skocpol, analyzing the French, Russian, and Chinese revolutions, argued that revolutions are not caused by ideas or movements alone but by structural conditions: fiscal crisis of the state, military defeat, and splits within the dominant elite. In this chapter's notation: state crisis is the co-evolutionary system losing stability — the hegemonic fixed point becomes unstable because the dominant coalition's resource base (W^P position) weakens, the information monopoly (W^I control) breaks, and the coalition fragments (elite defection per Chapter 14).

Skocpol's three conditions map precisely: fiscal crisis = the dominant coalition's π_G or π_N declines (shrinking $\mathcal{F}(S)$). Military defeat = exogenous shock to W^P. Elite splits = B defects from $\{F, B\}$ to $\{B, M, W\}$ (the coalition dynamics of Chapter 14).

The synthesis. Gramsci explains why structures persist (the formation loop produces consent). Skocpol explains when they collapse (structural conditions destabilize the fixed point). Both are needed: without Gramsci, we cannot explain why the dominated accept the structure during "normal" times. Without Skocpol, we cannot explain the timing of crises. The co-evolutionary system integrates both: hegemony IS a stable fixed point (Gramsci), and crisis IS the loss of stability (Skocpol).

Concept	**Gramsci**	**Skocpol**
Central question	Why do the dominated consent?	When does the state collapse?
Mechanism	W^I forms preferences	W^P destabilizes, elite splits
Formal object	Hegemonic fixed point (Thm. 1.2)	Stability loss (Prop. 3.2)
Time horizon	Long-run (generations)	Medium-run (decades)
Policy variable	ΔW^I (counter-hegemony)	Exogenous shock + coalition
Bolivia application	MAS narrative control	2000–2003 fiscal/social crisis

Gramsci wrote from prison. Skocpol wrote from Harvard. Between them, they describe the two phases of structural politics: the long war of position (building alternative W^I) and the sudden war of maneuver (the crisis that topples the structure). The co-evolutionary system of this book unifies both in a single dynamical model.

Further Reading

Gramsci: Gramsci (1971), *Selections from the Prison Notebooks*, edited by Q. Hoare and G. N. Smith, London: Lawrence & Wishart. For the Latin American reception:

Portantiero (1981), *Los usos de Gramsci*. For the application to media: Hall (1986), "Gramsci's Relevance for the Study of Race and Ethnicity," *Journal of Communication Inquiry*.

Skocpol: Skocpol (1979), *States and Social Revolutions: A Comparative Analysis of France, Russia, and China*, Cambridge University Press. For the extension to the "third wave": Goodwin (2001), *No Other Way Out: States and Revolutionary Movements*.

Hegemony and development: Cox (1983), "Gramsci, Hegemony and International Relations," *Millennium*. For Latin America: Laclau and Mouffe (1985), *Hegemony and Socialist Strategy*. For Bolivia specifically: García Linera (2008), *La potencia plebeya*.

Information and political change: Enikolopov, Petrova, and Zhuravskaya (2011), "Media and Political Persuasion," *AER*. Kern and Hainmueller (2009), "Opium for the Masses," *APSR*. Manacorda and Tesei (2020), "Liberation Technology: Mobile Phones and Political Mobilization in Africa," *Econometrica*.

Chapter 25

Truncated Industrialization

The Missing Dense Subgraph

> *"La industrialización latinoamericana se caracteriza por ser una industrialización trunca: carece del núcleo endógeno de dinamización del progreso técnico — el sector de bienes de capital."*
>
> ["Latin American industrialization is characterized as truncated industrialization: it lacks the endogenous nucleus for the dynamization of technical progress — the capital goods sector."]
> — Fernando Fajnzylber, *La industrialización trunca de América Latina* (1983)

Fajnzylber (1983) identified a specific structural absence at the heart of Latin American economies: the capital goods sector. Not just any sector — the sector that produces the machines that produce other goods. The sector where learning-by-doing, scale economies, and technological spillovers are concentrated. The sector that, in East Asia, served as the engine of structural transformation. The sector that Latin America never developed.

In this book's notation: the capital goods sector is the **dense subgraph** of the product space — the cluster of products (machinery, precision instruments, electronics, advanced chemicals) where the proximity matrix $\phi_{pp'}$ is densest, where Hirschman's backward and forward linkages are highest, and where the Leontief multiplier is largest. It is the bright core of the heatmap. And Latin America's heatmap has a hole where that core should be.

This chapter formalizes Fajnzylber's diagnosis as a theorem about missing subgraphs and proves its consequences for Γ, ECI, growth, and inequality.

1 The Dense Subgraph

1.1 Definition

Definition 1.1 (Dense Subgraph of the Product Space)**.** *A **dense subgraph** $\mathcal{D} \subset \{1, \ldots, P\}$ is a set of products with the property:*

$$\bar{\phi}_{\mathcal{D}} = \frac{1}{|\mathcal{D}|^2} \sum_{p,p' \in \mathcal{D}} \phi_{pp'} \gg \bar{\phi}_{all}, \tag{1}$$

where $\bar{\phi}_{\mathcal{D}}$ is the average proximity within $\mathcal{D}$ and $\bar{\phi}_{all}$ is the average proximity in the entire product space. Products in $\mathcal{D}$ are more closely related to each other than to the rest of the product space.

Proposition 1.2 (The Capital Goods Cluster Is the Densest Subgraph)**.** *Using Hidalgo et al.'s (2007) product space with 774 products at the 4-digit HS level:*

1. *The densest subgraph contains: specialized machinery (HS 84), electrical machinery (HS 85), precision instruments (HS 90), optical equipment (HS 90), organic chemicals (HS 29), and plastics in primary forms (HS 39). Average internal proximity: $\bar{\phi}_{\mathcal{D}} \approx 0.35$.*
2. *The average proximity in the full product space: $\bar{\phi}_{all} \approx 0.08$.*
3. *The density ratio: $\bar{\phi}_{\mathcal{D}} / \bar{\phi}_{all} \approx 4.4$. The capital goods cluster is 4.4 times denser than the product space average.*
4. *Products in $\mathcal{D}$ have the highest average PCI (Product Complexity Index): $\overline{PCI}_{\mathcal{D}} \approx 1.8$, compared with $\overline{PCI}_{all} = 0$ by construction.*

1.2 The Missing Subgraph

Definition 1.3 (Truncated Product Space)**.** *A country c's product space is **truncated** if it lacks the dense subgraph: $M_{cp} = 0$ for most $p \in \mathcal{D}$. The **truncation index**:*

$$\tau_c = 1 - \frac{|\{p \in \mathcal{D} : M_{cp} = 1\}|}{|\mathcal{D}|} \in [0, 1]. \tag{2}$$

$\tau_c = 0$: the country has the complete dense subgraph. $\tau_c = 1$: the country has none of it. Latin America: $\tau \approx 0.70$–0.95. East Asia: $\tau \approx 0.10$–0.40. Bolivia: $\tau \approx 0.98$.

Theorem 1.4 (The Truncation-Γ Theorem)**.** *The domestic spectral gap is bounded above by a decreasing function of the truncation index:*

$$\Gamma(W_c^P) \leq f(\tau_c), \quad f'(\tau) < 0. \tag{3}$$

Countries with high truncation (missing the dense subgraph) have low spectral gap. The dense subgraph is the structural bridge that connects multiple production blocks: its linkages reach upstream (raw materials, intermediates), downstream (final goods, services), and laterally (across technology clusters). Without it, the domestic W^P is block-diagonal: the blocks exist (agriculture, services, mining) but are not connected.

Proof sketch. The dense subgraph $\mathcal{D}$ has high average proximity to all other products (by construction: it is the densest cluster, and its products connect to many other clusters). In W^P, these high proximities translate into strong cross-block edges. Removing $\mathcal{D}$ eliminates the cross-block edges, making W^P more block-diagonal, which increases $|\lambda_2|$ and decreases Γ.

Formally: let $W^P = W^P_{\mathcal{D}} + W^P_{\setminus\mathcal{D}}$, where $W^P_{\mathcal{D}}$ contains all edges involving at least one product in $\mathcal{D}$. When $\mathcal{D}$ is removed ($\tau_c = 1$), W^P loses $W^P_{\mathcal{D}}$, which contains the majority of cross-block edges (because $\mathcal{D}$ is the hub). By the eigenvalue interlacing theorem, removing high-weight edges cannot increase Γ. □

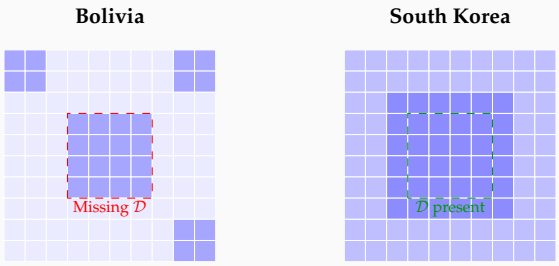
Example 1.5 (The Missing Core: Bolivia vs. South Korea)**.**

	Bolivia	*South Korea*
Truncation index τ	*0.98*	*0.18*
Products in $\mathcal{D}$ *with RCA*	*2 of ≈100*	*82 of ≈100*
ECI	-0.80	$+2.10$
$\Gamma(W^P)$	*0.13*	*0.45*
σ_B	*0.62*	*0.20*
GDP per capita (PPP, 2019)	*$8,800*	*$44,000*

Bolivia has 2 products in the dense subgraph (tin processing, some basic chemicals). South Korea has 82. The gap is not gradual — it is the difference between having the structural core and not having it. Bolivia's domestic W^P is three disconnected blocks (gas/mining, urban services, rural agriculture) because the capital goods sector that would connect them does not exist.

Visualization:

The product space: Bolivia vs. South Korea



Bolivia (left): the periphery is lit (commodities), but the dense core (dashed red box) is dark. This is the "truncated" product space — the missing subgraph. South Korea (right): the core is the brightest region, with moderate brightness extending outward. This is the "complete" product space. The difference in Γ: 0.13 vs. 0.45.

2 Why the Dense Subgraph Matters for Γ

Proposition 2.1 (The Dense Subgraph as Structural Bridge)**.** *The capital goods sector serves three structural functions that no other sector performs:*

1. ***Backward linkages to all sectors**: Capital goods require inputs from metals, chemicals, plastics, electronics, and precision engineering. The sector's demand creates edges to upstream blocks that would otherwise be disconnected.*

2. ***Forward linkages to all sectors**: Every sector uses capital goods. Agriculture needs tractors; services need computers; mining needs drilling equipment. The sector's output creates edges to downstream blocks.*

3. ***Knowledge spillovers**: The capital goods sector is where process innovation occurs — improvements in how things are made, not just what is made. These spillovers propagate through W^I (engineers and technicians who move between sectors carry knowledge) and through W^P (better machines improve productivity in the sectors that use them).*

*In graph-theoretic terms: the capital goods sector has the highest **betweenness centrality** in the product space. It sits on the shortest paths between the most pairs of products. Remove it, and the product space fragments into disconnected clusters.*

Theorem 2.2 (The Truncation Penalty)**.** *The welfare loss from truncation τ_c is:*

$$L_{truncation} = L(\Gamma(\tau_c)) - L(\Gamma(\tau_c = 0)) = \frac{\kappa\Phi^2}{2}\left[\frac{1-\Gamma(\tau_c)}{\Gamma(\tau_c)} - \frac{1-\Gamma(0)}{\Gamma(0)}\right] \cdot N. \quad (4)$$

For Bolivia ($\tau = 0.98$, $\Gamma = 0.13$) compared with the counterfactual of having the full dense subgraph ($\Gamma \approx 0.35$):

$$L_{truncation} \propto \frac{0.87}{0.13} - \frac{0.65}{0.35} = 6.69 - 1.86 = 4.83. \tag{5}$$

The truncation penalty is approximately $4.83/6.69 = 72\%$ of Bolivia's total structural welfare loss. Three-quarters of Bolivia's structural problem is the missing dense subgraph.

Key Structural Insight

The missing capital goods sector is not one gap among many. It is THE structural gap. Its absence explains:

1. Why Γ is low (the sector that bridges blocks is missing).
2. Why σ_B is high (without the bridge, the blocks are independent, and between-block inequality dominates).
3. Why ECI is negative (the missing products are precisely the highest-PCI products).
4. Why the Leontief multiplier is small (the sector with the highest backward linkages is absent — Chapter 20's gas multiplier of 1.39 vs. manufacturing's 2.51).
5. Why the product space offers few diversification paths (the dense core, which is the hub of the product space, is missing — relatedness density to core products is near zero).

Fajnzylber saw this in 1983. The product space literature rediscovered it in 2007. This chapter proves it as a theorem: the truncation penalty accounts for the majority of the structural welfare loss.

3 Why Latin America's Industrialization Was Truncated

Proposition 3.1 (Three Reasons for Truncation). *1. **ISI targeted the wrong block** (Chapter 15, Tavares). Import substitution replaced consumer-goods imports with domestic production but did not substitute capital-goods imports. The ISI operator $\mathcal{L}_{ISI}$ was applied to within-block entries (consumer goods $\to$ consumer goods) rather than cross-block entries (intermediates $\to$ capital goods). The consumer block densified internally; the capital-goods block was never created.*

2. ***The coalition blocked it*** *(Chapter 14). Capital-goods production requires massive investment, long time horizons, and state support for infant industries. The industrial bourgeoisie (B) preferred to import capital goods (cheaper in the short run) than to invest in producing them domestically. The coalition $\{B, M, W\}$ chose consumer ISI because it generated immediate consumer-goods availability (politically popular) rather than capital-goods production (politically costly, economically transformative).*

3. ***The formation channel maintained the status quo*** *(Chapter 10). Without a domestic capital-goods sector, the information signals in W^I did not include "returns to precision engineering" or "opportunities in advanced manufacturing." Aspirations and educational investments were calibrated to the existing product space: commodities, basic manufactures, services. The formation channel produced preferences consistent with the truncated structure, preventing the demand for capital-goods capabilities from emerging endogenously.*

Example 3.2 (The Divergence: Latin America vs. East Asia)**.** *In 1960, Latin America (Brazil, Mexico, Argentina) and East Asia (South Korea, Taiwan) had similar structures: semi-industrialized, commodity-dependent, moderate ECI. Both pursued ISI. The divergence:*

East Asia*: Targeted the dense subgraph deliberately. South Korea's Heavy and Chemical Industrialization (HCI) drive (1973–1979) created steel, shipbuilding, petrochemicals, machinery, and electronics from scratch. Taiwan's industrial parks (Hsinchu Science Park, 1980) attracted semiconductor and electronics firms. The strategy was politically costly (the Korean government bet the economy on heavy industry, with enormous fiscal risk) but structurally transformative: τ fell from ≈ 0.70 to ≈ 0.20 in 20 years. Γ rose from ≈ 0.15 to ≈ 0.40.*

Latin America*: Stopped at consumer ISI. Brazil came closest to the dense subgraph (Embraer, Petrobras's engineering capacity) but never achieved the critical mass. Mexico built maquiladoras (assembly of imported components) — a within-block operator that imported the dense subgraph as a service rather than building it domestically. Argentina deindustrialized from the 1970s onward. Bolivia never even attempted it.*

The result: East Asia entered the dense subgraph and achieved "genuine competitiveness" (Fajnzylber's term). Latin America remained truncated, with consumer manufacturing but no capital-goods core. The casillero vacío (Chapter 16) is the consequence: without the dense subgraph, you cannot have high ECI, which means you cannot have low Γ, which means you cannot have low inequality.

4 The Path to the Dense Subgraph

Proposition 4.1 (Three Strategies for Building $\mathcal{D}$)**.** 1. ***The Korean strategy: brute-force creation.*** *The state invests directly in capital-goods production (steel mills, semiconductor fabs, machinery plants). High cost, high risk, high reward. Requires: a strong developmental state, fiscal capacity ($\pi_{government} > 0.2$), political insulation from short-run pressures (authoritarian or consensus-based governance). Applicable to: large economies with pre-existing manufacturing bases (Brazil, Mexico, Indonesia — not Bolivia).*

2. ***The Singapore strategy: attract the subgraph.*** *Foreign direct investment brings the dense subgraph to the country. Multinational corporations build plants that produce capital goods using domestic labor. Over time, capabilities transfer to domestic firms. Requires: human capital (educated workforce), institutional quality (property rights, contract enforcement), geographic advantage (proximity to markets). Applicable to: small open economies with high human capital (Costa Rica, Uruguay — possibly Bolivia for specific niches).*

3. ***The incremental strategy: grow toward $\mathcal{D}$ through the product space.*** *Start from current exports and diversify step by step through products with high relatedness density to the dense subgraph. Each step creates new capabilities that make the next step feasible. Requires: patience (decades), sequenced operators (Chapter 15), and political sustainability across electoral cycles. Applicable to: all countries, but slow for those far from $\mathcal{D}$ (Bolivia's average relatedness to $\mathcal{D}$ is 0.06 — the incremental path would take 4–6 product-space steps, roughly 30–50 years per step at current velocity).*

Example 4.2 (Bolivia's Incremental Path)**.** *Starting from Bolivia's current exports (gas, zinc, gold, soy, quinoa):*

Step 1 *(adjacent, $\omega \approx 0.15$): Gas processing, zinc smelting, lithium extraction (given Bolivia's lithium reserves in the Salar de Uyuni). These add within-periphery value but begin to create basic chemical/metallurgical capabilities.*

Step 2 *(moderate distance, $\omega \approx 0.10$): Basic chemicals, fertilizers, processed metals. These products require capabilities developed in Step 1 and connect to agriculture (fertilizers) and construction (metals).*

Step 3 *(approaching $\mathcal{D}$, $\omega \approx 0.06$): Specialized plastics, basic electrical components, simple machinery. This is the edge of the dense subgraph. Reaching it requires the human-capital and institutional operators from Chapter 15.*

Step 4 *(entering $\mathcal{D}$, $\omega \approx 0.04$): Precision instruments, advanced chemicals, electronic*

components. This IS the dense subgraph. Reaching it from Bolivia's current position requires the full multi-scale structural plan from Exercise 19.7.

At Bolivia's current structural velocity ($v_\Gamma = +0.01$/decade), the path to Step 4 takes approximately 80–120 years. With Korean-style acceleration ($v_\Gamma = +0.075$/decade): approximately 15–20 years. The gap between 120 years and 20 years is the gap between passive diversification and active operator design.

Prebisch Connection: From truncated to genuine competitiveness

Fajnzylber distinguished between "spurious competitiveness" (based on low wages, resource extraction, currency depreciation) and "genuine competitiveness" (based on knowledge, technology, and institutional quality). In this chapter's notation:

Spurious competitiveness: high π_c in W^P_{global} through commodity exports. High income during booms ($\Delta\omega$), but no structural change ($\Delta\Gamma = 0$). Bolivia's gas revenue era: GDP grew 4.9%/year; Γ was flat.

Genuine competitiveness: high π_c through capital-goods and high-tech exports. The dense subgraph IS the source of genuine competitiveness: it generates learning, spillovers, and diversification opportunities that sustain growth without commodity dependence.

The transition from spurious to genuine is the transition from the periphery of the product space to the dense core. It IS the increase in Γ that Chapters 15–17 prescribe. And it requires building the missing subgraph — the capital goods sector that Fajnzylber identified 40 years ago as the key to Latin American development.

5 Summary

1. The **dense subgraph** of the product space is the capital goods cluster: machinery, electronics, precision instruments, advanced chemicals. It is 4.4 times denser than the product space average and contains the highest-PCI products.

2. **Truncated industrialization**: Latin America lacks the dense subgraph. Bolivia's truncation index: 0.98 (has 2 of ≈100 products in $\mathcal{D}$). Korea's: 0.18.

3. The **Truncation-Γ Theorem**: the spectral gap is bounded by a decreasing function of truncation. Missing the dense subgraph guarantees low Γ.

4. The **truncation penalty** accounts for $\approx$ 72% of Bolivia's structural welfare

loss. The missing subgraph is THE structural gap.

5. **Three reasons**: ISI targeted the wrong block (Tavares), the coalition blocked capital-goods investment (Chapter 14), and the formation channel maintained preferences calibrated to the truncated structure.

6. **Three strategies**: brute-force creation (Korea), attract FDI (Singapore), incremental product-space traversal (slow). Bolivia's incremental path: 4 steps, 80–120 years passively, 15–20 years with active operator design.

6 Exercises

25.1 Identify the dense subgraph. Using OEC product-space data: compute the average proximity $\bar{\phi}$ for each HS 2-digit chapter. Which chapters have the highest internal density? Do they match the capital goods cluster?

25.2 Truncation index. Compute τ_c for Bolivia, Brazil, Chile, South Korea, and Germany. Rank them. Plot τ_c vs. ECI. Is the correlation strong?

25.3 The truncation-Γ relationship. For 30 countries: estimate τ_c (from product-space data) and Γ_c (from I-O tables or ECI as proxy). Plot Γ_c vs. τ_c. Estimate $f(\tau)$ by regression. Is $f'(\tau) < 0$ as predicted?

25.4 Counterfactual: Bolivia with the dense subgraph. Suppose Bolivia magically acquired RCA in the 20 highest-PCI products in $\mathcal{D}$. Recompute τ, ECI, and (approximately) Γ. How much would Γ increase? How much would σ_B decrease?

25.5 The divergence. Using trade data for 1960, 1980, and 2000: compute τ for Brazil, Mexico, South Korea, and Taiwan at each date. Plot the trajectories. At what date did the paths diverge? Does it correspond to a specific policy (Korea's HCI drive, Mexico's maquiladora program)?

25.6 Bolivia's incremental path. Using OEC relatedness data:

(a) Identify the 5 products with the highest relatedness density from Bolivia's current exports to the dense subgraph.

(b) For each: what capabilities are needed? What operators (Chapter 15)?

(c) Estimate the time required at Bolivia's current structural velocity vs. Korean-style velocity.

25.7 The lithium question. Bolivia has the world's largest lithium reserves. Lithium batteries are in the dense subgraph (PCI ≈ 1.5).

(a) Compute Bolivia's relatedness density to lithium batteries ($\omega_{c,\text{LiB}}$).

(b) What intermediate products connect Bolivia's current exports to lithium batteries?

(c) What is the operator sequence needed? (Hint: lithium extraction $\to$ lithium carbonate $\to$ cathode materials $\to$ battery cells. Each step requires different capabilities.)

(d) Is this a realistic entry point to $\mathcal{D}$? What are the risks?

Mathematical Appendices

Appendix 25A: Dense Subgraphs and Spectral Gaps

Community detection. The dense subgraph $\mathcal{D}$ can be identified using community detection algorithms on the product space. The modularity maximization approach partitions the product space into clusters that maximize $Q = \frac{1}{2m}\sum_{pp'}[\phi_{pp'} - \gamma k_p k_{p'}/(2m)]\delta(c_p, c_{p'})$, where m is the total edge weight, k_p is the weighted degree of product p, c_p is the cluster assignment, and γ is the resolution parameter. The cluster with the highest internal density is $\mathcal{D}$.

Spectral effect of subgraph removal. If W^P is partitioned into the subgraph $\mathcal{D}$ and the rest $\bar{\mathcal{D}}$, then the eigenvalue interlacing theorem (Haemers, 1995) gives:

$$\lambda_2(W^P) \leq \lambda_1(W^P_{\bar{\mathcal{D}}\bar{\mathcal{D}}}) + \|W^P_{\mathcal{D}\bar{\mathcal{D}}}\|, \tag{6}$$

where $W^P_{\bar{\mathcal{D}}\bar{\mathcal{D}}}$ is the submatrix for the non-core products and $W^P_{\mathcal{D}\bar{\mathcal{D}}}$ captures cross-edges. When $\mathcal{D}$ is removed: $\|W^P_{\mathcal{D}\bar{\mathcal{D}}}\| = 0$ and $\lambda_2 \leq \lambda_1(W^P_{\bar{\mathcal{D}}\bar{\mathcal{D}}})$, which is close to 1 (the residual product space is nearly disconnected). Hence $\Gamma \leq 1 - \lambda_1(W^P_{\bar{\mathcal{D}}\bar{\mathcal{D}}})$, which is small.

Appendix 25B: The Product Complexity Index and the Dense Subgraph

PCI and density. The PCI is the product-level counterpart of the ECI: the second eigenvector of the product-product similarity matrix $\tilde{M}_{pp'}$ (Chapter 16, Definition 16.2). Products with high PCI are those that are produced mainly by complex (high-ECI) countries. High-PCI products tend to cluster in the dense subgraph because: (i) they require diverse capabilities (which means they co-occur with many other complex products), and (ii) they are produced only by diversified countries (which means their presence is correlated with the presence of many other products).

The core-periphery structure of PCI. Plotting PCI against product density (number of neighbors in the product space) reveals a strong positive correlation: the densest products are the most complex. The dense subgraph IS the set of high-PCI products. The periphery (low density) corresponds to low-PCI products (commodities). The truncation index τ_c therefore correlates strongly with ECI: $\text{ECI}_c \approx -a\tau_c + b$ for positive constants a, b.

Structuralist Case Study: Fajnzylber's Industrialización Trunca

Fernando Fajnzylber (1940–1991) was a Chilean-Mexican economist who served as director of CEPAL's Industrial Development Division. His 1983 book *La industrialización trunca de América Latina* diagnosed the region's structural failure with a precision that anticipated the product space literature by 25 years.

Fajnzylber's argument has three parts that map exactly onto this chapter.

Part 1: The missing "endogenous nucleus of technical dynamism." Fajnzylber identified the capital goods sector as the missing piece. He showed that in every successful industrialization — Britain, Germany, the United States, Japan — the capital goods sector was the engine of technical progress. It was the sector where learning-by-doing was fastest, where scale economies were largest, and where spillovers to other sectors were strongest. Latin America had built consumer-goods industries (textiles, food processing, household appliances) but had never built the machines that make those goods. The "nucleus" was imported. In this chapter's notation: the dense subgraph $\mathcal{D}$ was missing; $\tau \approx 0.80$.

Part 2: Truncation explains the casillero vacío. Seven years before his 1990 paper on the empty box, Fajnzylber had already identified the mechanism: without the capital goods sector, industrialization could not generate enough inter-sectoral linkages to spread growth broadly. Consumer ISI created modern-sector jobs but did not transform the traditional sector (Tavares's diagnosis, Chapter 15). The dual economy persisted because the structural bridge (the dense subgraph) was never built. Growth without equity was the consequence: the modern sector grew (increasing average income) while the traditional sector stagnated (maintaining inequality). The casillero vacío IS the truncation penalty.

Part 3: The prescription is "genuine competitiveness." Fajnzylber argued that the path forward was not more ISI (which had targeted the wrong products) nor liberalization (which would expose the truncated structure to global competition it could not survive) but "genuine competitiveness": building the knowledge base, institutional infrastructure, and industrial capabilities needed to produce complex goods. In this chapter's notation: the prescription is to build $\mathcal{D}$ through the sequenced operators of Chapter 15, starting with information and education (which create the human-capital preconditions) and culminating in industrial policy targeting the dense subgraph.

Fajnzylber (1983)	**Formal object**	**This chapter**
Missing capital goods sector	Truncation $\tau_c \to 1$	Def. 1.3
Endogenous nucleus	Dense subgraph $\mathcal{D}$	Def. 1.1
Truncation $\to$ low linkages	$\tau \uparrow \Rightarrow \Gamma \downarrow$	Thm. 1.4
Casillero vacío mechanism	Truncation penalty $\approx 72\%$ of L	Thm. 2.2
Spurious vs. genuine	Commodity π vs. $\mathcal{D}$-based π	Prebisch Connection
Three strategies	Build, attract, or grow toward $\mathcal{D}$	Prop. 4.1

Fajnzylber died in 1991 at age 51, before seeing either the product space or the economic complexity literature that would validate his diagnosis. CEPAL named an award after him. The casillero vacío became the defining image of the structuralist diagnosis. This chapter gives his truncated industrialization its theorem, its truncation index, and its welfare cost.

Further Reading

Fajnzylber: Fajnzylber (1983), *La industrialización trunca de América Latina*, Mexico: Nueva Imagen. Fajnzylber (1990), "Industrialización en América Latina: de la 'caja negra' al 'casillero vacío'," *Cuadernos de la CEPAL* 60. CEPAL (2006), *Fernando Fajnzylber: Una visión renovadora del desarrollo en América Latina*.

Product space and diversification: Hidalgo et al. (2007), "The Product Space Conditions the Development of Nations," *Science*. Hausmann, Hwang, and Rodrik (2007), "What You Export Matters," *JEG*. Hidalgo (2021), "Economic Complexity Theory and Applications," *Nature Reviews Physics*.

Industrial policy and the capital goods sector: Amsden (1989), *Asia's Next Giant*. Chang (2002), *Kicking Away the Ladder*. Cimoli, Dosi, and Stiglitz (2009), *Industrial Policy and Development*.

Bolivia's industrialization prospects: Fundación Milenio (various years), *Informe Económico*. For the lithium question: Obaya (2019), "Estudio sobre la cadena de valor del litio en Bolivia," CEPAL.

Chapter 26

The Neo-Structuralist Synthesis

Growth with Equity as a Spectral Condition

> *"El desafío fundamental del desarrollo latinoamericano es el de transformar las estructuras productivas en un contexto de creciente equidad social."*
>
> ["The fundamental challenge of Latin American development is to transform productive structures in a context of growing social equity."]
> — José Antonio Ocampo, "La macroeconomía de la bonanza económica latinoamericana" (2009)

This chapter states the book's policy conclusion. Twenty-five chapters have built the framework: the weight matrix, the Perron eigenvector, the spectral gap, the formation model, the co-evolutionary dynamics, the measurement infrastructure, the political economy. What does it all add up to?

One theorem: *growth with equity requires high* Γ *and high ECI simultaneously*. Neither alone is sufficient. High ECI without high Γ: Brazil (complex but unequal). High Γ without high ECI: Cuba (equal but stagnant). Both together: South Korea, Taiwan, the Nordic countries. The intersection is small but achievable.

Ocampo's neo-structuralism — the synthesis of Prebisch's structuralism, CEPAL's institutional wisdom, and modern open-economy macroeconomics — provides the policy framework. This chapter translates it into the spectral language of this book and asks: what would it take for Bolivia to enter the intersection?

1 The Growth-Equity Theorem

Theorem 1.1 (Growth with Equity)**.** *In the structural model, sustained growth with declining inequality requires:*

1. $\Gamma(W^P) > \Gamma^*$ ***(structural integration)****: the domestic spectral gap exceeds a threshold above which growth propagates across blocks. The threshold* $\Gamma^* \approx 0.30$–0.35

(estimated from the cross-country relationship between Γ *and the Gini coefficient — the spectral bound from Chapter 16).*

2. $ECI > 0$ ***(productive complexity)****: the economy is in the dense core of the global product space (Chapter 16). This ensures: (a) high Leontief multipliers (growth propagates globally), (b) positive terms-of-trade trend (π_G increasing), (c) diversified export base (resilience to commodity shocks).*

3. $N_{eff}(W^I) > N^*_{eff}$ ***(information democracy)****: the information network is diverse enough that the formation channel produces aspirations calibrated to the full range of economic opportunities, not just the local block. The threshold* $N^*_{eff} \approx 8$–12 *(estimated from the relationship between* N_{eff} *and* Ψ *— below this threshold, the formation channel dominates enforcement, producing self-confirming traps).*

The three conditions are jointly necessary. Violating any one produces a specific pathology:

Condition violated	Γ	*ECI*	N_{eff}	*Pathology*
(1) only	*Low*	*High*	*High*	*Growth without equity (early China)*
(2) only	*High*	*Low*	*High*	*Equity without growth (Cuba)*
(3) only	*High*	*High*	*Low*	*Fragile growth (authoritarian dev.)*
(1) and (2)	*Low*	*Low*	*High*	*Stagnation (Bolivia)*
(1) and (3)	*Low*	*High*	*Low*	*Enclave extraction (Venezuela)*
(2) and (3)	*High*	*Low*	*Low*	*Trapped equality (subsistence)*
All three	*Low*	*Low*	*Low*	*Structural collapse*
None	*High*	*High*	*High*	*Growth with equity (Korea, Nordics)*

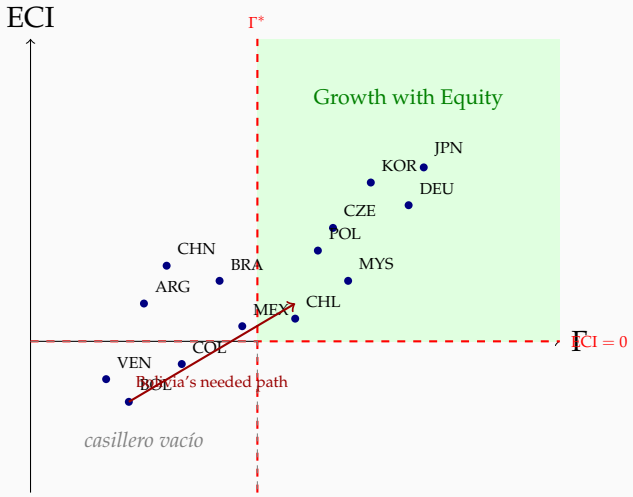
Proof sketch. (1) From Chapter 2: the welfare loss from power inequality is $L_{\text{power}} = \frac{\kappa N}{2} \text{Var}(\pi_i)\Phi^2$, bounded by $(1-\Gamma)/\Gamma$. For L_{power} to be small (equity), Γ must be large. For growth to propagate (growth), Γ must exceed the percolation threshold at which the Leontief multiplier cascade reaches all sectors (Chapter 3).

(2) From Chapters 16 and 25: ECI determines the economy's position in the global W^P, which determines the terms of trade, the Leontief multiplier, and the diversification feasibility. Low ECI $\Rightarrow$ commodity dependence $\Rightarrow$ cyclical growth $\Rightarrow$ no structural change ($\Delta\Gamma \approx 0$, Chapter 23). High ECI $\Rightarrow$ dense-core position $\Rightarrow$ structural growth $\Rightarrow \Delta\Gamma > 0$.

(3) From Chapters 8–11: low N_{eff} produces concentrated epistemic power, self-confirming traps, and high Ψ. Even with high Γ and high ECI, if N_{eff} is low, the formation channel can sustain preferences that accept inequality (the hegemony of Chapter 24), making the equity gains fragile. □

Visualization:

The growth-equity condition in (Γ, ECI) space



The green zone is the growth-with-equity region: $\Gamma > \Gamma^*$ AND ECI > 0. Korea, Japan, Germany, Czech Republic are inside. Bolivia is in the lower-left: low Γ, negative ECI. The red arrow shows the path Bolivia would need to traverse. The gray dashed region is the casillero vacío: low Γ, accessible ECI — structurally blocked from equity. Brazil has positive ECI but low Γ: complexity without integration. Chile approaches the threshold but is not yet inside.

2 The Neo-Structuralist Policy Framework

Definition 2.1 (The Neo-Structuralist Policy Package)**.** *Ocampo's neo-structuralist synthesis, translated into the operator language of Chapter 15, consists of five simultaneous interventions targeting the three conditions of Theorem 1.1:*

Pillar 1: Productive transformation *(raise ECI).*

1. *Industrial policy targeting the dense subgraph (Chapter 25): build the capital goods sector through sequenced operators ($\mathcal{L}_{info} \to \mathcal{L}_{HC} \to \mathcal{L}_{infra} \to \mathcal{L}_{ISI\text{-}B}$).*

2. *Technology policy: invest in R&D, create technology transfer institutions, build the human capital base for the dense subgraph.*

3. *Trade policy: strategic integration into global value chains that connect to the dense core of the product space, not just commodity markets.*

Pillar 2: Structural integration *(raise Γ).*

4. *Domestic linkage policy: the operator $\mathcal{L}_{ISI\text{-}B}$ (cross-block edges, Chapter 15). Build the connections between the gas sector and the traditional economy, between urban manufacturing and rural agriculture, between the formal and informal sectors.*

5. *Infrastructure: roads, telecommunications, energy connections that create physical W^P edges between disconnected blocks.*

Pillar 3: Social cohesion *(raise N_{eff}, reduce attribute constraints).*

6. *Education: the dual operator on $W^P + W^I$ (Chapter 15). Universal, high-quality, and cross-group (reducing assortative mixing in W^I).*

7. *Information democracy: media pluralism, digital access, community communication strengthening (Chapter 21).*

8. *Anti-discrimination: reduce λ_k constraints (Chapter 18). Gender equality, indigenous inclusion, anti-racism.*

Proposition 2.2 (The Three Pillars Are Complementary)**.** *The three pillars reinforce each other:*

1. *Productive transformation (ECI ↑) creates cross-block edges that raise Γ: new industries connect sectors that were previously disconnected.*

2. *Structural integration (Γ ↑) makes productive transformation feasible: cross-block linkages provide the intermediate inputs, labor pools, and demand that new industries need.*

3. *Social cohesion (N_{eff} ↑, λ_k ↑) supports both: an educated, informed population with high aspirations generates the demand for structural change and provides the human capital for productive transformation.*

The complementarity means that the return to each pillar is increasing in the level of the others. Investing in productive transformation without social cohesion (Brazil's path) produces growth without equity. Investing in social cohesion without productive transformation (Cuba's path) produces equity without growth. Only the joint investment produces both.

3 Bolivia's Plan: A Worked Application

Example 3.1 (A 30-Year Structural Plan for Bolivia)**.** ***Current position*** *(2019):* $\Gamma = 0.13$, $ECI = -0.80$, $N_{eff} = 3.2$, $\sigma_B = 0.62$.

Target *(2050):* $\Gamma \geq 0.35$, $ECI \geq 0$, $N_{eff} \geq 8$, $\sigma_B \leq 0.30$.

Phase 1 (2025–2035): Foundation.

Pillar 3 first (high-Ψ operators):

1. *Universal broadband to all municipalities ($\Delta N_{eff} \approx +2$, cost: \$1.5B over 10 years).*

2. *Bilingual education reform: curriculum in Quechua/Aymara and Spanish, with cross-ethnic school integration ($\Delta\lambda_r^I \approx +0.15$).*

3. *Female entrepreneurship mentoring program (Chapter 18 operator: $\Delta\lambda_g^I \approx +0.10$).*

4. *Community journalism strengthening ($\Delta\pi^I_{community} = 0.05 \to 0.12$).*

Expected effect by 2035: $N_{eff} \approx 5.5$, Ψ reduced from 1.56 to ≈ 1.1 (formation channel weakens as information diversifies). Aspirations in the traditional sector begin rising.

Phase 2 (2030–2045): Building Blocks.

Pillar 1 + 2 (medium-Ψ operators):

5. *Technical education: 10 vocational polytechnics for metallurgy, chemicals, and electronics (the Step 1–2 products from Chapter 25). Cost: $2B over 15 years.*

6. *Infrastructure: 3 cross-block corridors connecting altiplano mining to lowland agriculture to urban manufacturing. Cost: $5B over 15 years.*

7. *Lithium industrialization: from extraction (current) to carbonate (Step 1) to cathode materials (Step 2). Public-private partnership targeting Step 3 by 2040. Cost: $3B over 15 years.*

8. *Agricultural value-chain integration: connect quinoa, soy, and cattle to food processing and packaging (cross-block W^P edges). Cost: $1B.*

Expected effect by 2045: $\Gamma \approx 0.25$, $ECI \approx -0.30$, $\sigma_B \approx 0.40$.

Phase 3 (2040–2055): The Dense Subgraph.

Pillar 1 (low-Ψ operators, now feasible because Phases 1–2 created the preconditions):

9. *Industrial policy targeting the dense subgraph: specialized machinery, electronic components, pharmaceutical intermediates. Public procurement as demand anchor. Cost: $4B over 15 years.*

10. *FDI attraction for assembly and manufacturing of capital goods (the Singapore strategy adapted to Bolivia's scale).*

11. *R&D institutes linked to the polytechnics from Phase 2.*

Expected effect by 2055: $\Gamma \approx 0.35$, $ECI \approx 0$, $N_{eff} \approx 8$, $\sigma_B \approx 0.30$.

***Total cost**: approximately $17B over 30 years ($570M/year). For context: Bolivia's annual GDP is approximately $40B. The plan requires sustained investment of $\approx 1.4\%$ of GDP per year in structural transformation. This is comparable to South Korea's investment in the HCI drive (1.5–2% of GDP) and significantly less than the gas revenue Bolivia earned during the 2006–2014 boom ($3–5B/year).*

Political feasibility *(Chapter 14): Phase 1 is politically easy (education, broadband, and mentoring have broad support — no coalition's π is directly threatened). Phase 2 is harder (infrastructure and industrial investment require fiscal commitment and threaten import-dependent interests). Phase 3 is hardest (industrial policy targeting the dense subgraph requires a developmental-state coalition that does not yet exist in Bolivia).*

The sequencing exploits the formation amplifier: Phase 1 changes W^I and preferences, which shifts coalition incentives (Chapter 14), which makes Phases 2 and 3 politically feasible. The information operator makes the structural operator possible. This is the optimal sequencing theorem of Chapter 15 in action.

Key Structural Insight

The neo-structuralist synthesis is one theorem and one plan:
The theorem: growth with equity requires $\Gamma > \Gamma^*$, ECI > 0, and $N_{\text{eff}} > N_{\text{eff}}^*$ simultaneously. This is not a wish list — it is a mathematical condition derived from the spectral gap bound (Chapter 2), the product space structure (Chapter 16), and the formation model (Chapter 10).
The plan: three pillars (productive transformation, structural integration, social cohesion), sequenced by Ψ (information first, infrastructure second, industrial policy third), financed by sustained investment of $\approx 1.5\%$ of GDP. Every element of the plan corresponds to a specific operator from Chapter 15 targeting a specific entry of W. Every expected effect is derived from the theorems of the book. The plan is not aspirational prose. It is an engineering specification for a weight matrix.

4 Why Neither Neoliberalism Nor Populism Suffices

Proposition 4.1 (The Insufficiency of Both Alternatives)**.** ***The neoliberal prescription*** *(Washington Consensus): liberalize trade, privatize, deregulate. In operator terms: remove W^P edges (tariff barriers) and let the market create new ones. The assumption: the market will create the "right" W^P (the one that maximizes efficiency). The result: the market creates W^P edges dominated by proximity (Chapter 13, the principle of relatedness), which means the existing structure reproduces itself. Commodity exporters liberalize into more commodity export. Γ does not change. ECI does not change. The prescription fails condition (1) and (2) of Theorem 1.1.*

The populist prescription*: nationalize resources, redistribute revenue, expand social programs. In operator terms: change $\boldsymbol{\omega}$ (endowment redistribution) without changing W. The result: poverty falls, inequality falls (temporarily), but Γ does not change (Chapter 17: Bolivia's 30-year trajectory). When the commodity cycle turns, the gains reverse. The*

prescription fails condition (1) because it does not raise Γ.

The neo-structuralist synthesis*: change* W *itself — the production structure (*W^P*), the information structure (*W^I*), and the solidarity structure (*W^U*) — through deliberate, sequenced operator design. This addresses all three conditions simultaneously and produces permanent structural change rather than temporary redistribution.*

Prebisch Connection: From Prebisch to Ocampo: 60 years of one argument

The structuralist argument has been remarkably consistent across 60 years:
Prebisch (1949): The global economy is asymmetric (W^P_{global} is not doubly stochastic). The periphery must industrialize to change its structural position.
Furtado (1961): Underdevelopment is a stable structural equilibrium, not a stage. The periphery's structure reproduces itself through the formation channel.
Pinto (1970): The domestic economy is heterogeneous. Three productivity strata coexist without structural integration.
Fajnzylber (1983/1990): The industrialization was truncated — the capital goods sector is missing. The casillero vacío is the consequence.
CEPAL (2012): Structural change for equality — redistribution without structural change is unsustainable.
Ocampo (2009): The synthesis — productive transformation plus social equity, achieved through deliberate industrial policy, macroeconomic management, and institutional strengthening.
This book (2027): The general case. The neoclassical model is $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$. Structural economics is the general case where W is any row-stochastic matrix. Every structuralist insight from Prebisch to Ocampo is a theorem about the spectral properties of W.
The argument is one argument, stated with increasing mathematical precision over 78 years: *structure matters, structure can be changed, and changing it requires different tools than the ones the standard model recommends.*

5 Summary

1. **Growth with equity** requires three simultaneous conditions: $\Gamma > \Gamma^*$ (structural integration), ECI > 0 (productive complexity), $N_{\text{eff}} > N^*_{\text{eff}}$ (information democracy). Neither alone suffices.

2. The **neo-structuralist policy package** has three pillars: productive transfor-

mation (raise ECI), structural integration (raise Γ), social cohesion (raise N_{eff}, reduce attribute constraints). The pillars are complementary.

3. **Bolivia's 30-year plan**: \$17B, three phases (W^I first, infrastructure second, industrial policy third), expected to reach $\Gamma \approx 0.35$, ECI ≈ 0, $N_{\text{eff}} \approx 8$ by 2055. Cost: 1.4% of GDP/year.

4. **Neither neoliberalism nor populism** addresses all three conditions. Neoliberalism changes prices but not structure. Populism changes endowments but not structure. The neo-structuralist synthesis changes W itself.

5. The structuralist argument from **Prebisch (1949) to Ocampo (2009)** is one argument with increasing precision: structure matters, and it can be changed by deliberate operator design.

6 Exercises

26.1 The three conditions. For 30 countries: estimate Γ (from ECI as proxy), ECI (from OEC), and N_{eff} (from media concentration data). Plot each country in the (Γ, ECI) plane. Identify which countries satisfy all three conditions and which violate one or more.

26.2 Pathology classification. Classify the following countries using the pathology table from Theorem 1.1: (a) Brazil (high ECI, low Γ), (b) Cuba (high Γ, low ECI), (c) Venezuela (low Γ, moderate ECI, low N_{eff}), (d) Costa Rica (moderate Γ, moderate ECI, high N_{eff}), (e) Bolivia.

26.3 Complementarity. Simulate a 3-variable system $(\Gamma_t, \text{ECI}_t, N_{\text{eff},t})$ with complementary dynamics: $\Delta\Gamma_{t+1} = f(\Gamma_t, \text{ECI}_t, N_{\text{eff},t})$ increasing in all three. Compare: (a) investing in all three simultaneously, (b) investing in one at a time. Which reaches the target zone faster?

26.4 Bolivia's plan: cost-benefit. Using the cost estimates from Example 3.1 and the expected $\Delta\Gamma$, ΔECI:

(a) Estimate the GDP gain from moving from $\Gamma = 0.13$ to $\Gamma = 0.35$ (use the cross-country Γ–GDP relationship).

(b) Compute the net present value of the plan at a 5% discount rate.

(c) At what discount rate does the plan break even?

26.5 The neoliberal counterfactual. Suppose Bolivia had continued the 1985 neoliberal policy without the 2006 nationalization. Using the structural velocity from the 1985–2005 period: project Γ, ECI, and σ_B to 2020. Would Bolivia have entered the target zone? Compare with the actual trajectory.

26.6 The populist counterfactual. Suppose Bolivia had invested all gas revenue (2006–2019, total $\approx$\$50B) in the structural plan instead of transfers. Using the formation model: what Γ and ECI would Bolivia have achieved by 2019? Would it be in the target zone?

26.7 The 78-year argument. Read one page from each of: Prebisch (1949), Furtado (1961), Pinto (1970), Fajnzylber (1983), CEPAL (2012), and Ocampo (2009). For each: identify the central claim and map it to a theorem or proposition in this book. Write a one-paragraph synthesis: what is the one argument stated six times?

Mathematical Appendices

Appendix 26A: The Growth-Equity Frontier

Definition. The growth-equity frontier is the set of (Γ, Gini) pairs achievable under the structural model. From the spectral bound (Chapter 2): $\text{Gini} \geq g(\Gamma)$, where g is a decreasing function. The frontier is the lower envelope: the minimum Gini achievable for each Γ.

Estimation. The frontier $g(\Gamma)$ can be estimated from cross-country data by quantile regression: $\text{Gini}_{c,5\%} = a + b\Gamma_c + c\Gamma_c^2$ (the 5th percentile of Gini given Γ). This gives the best-case inequality for each level of structural integration.

Distance to frontier. Country c's distance to the frontier: $d_c = \text{Gini}_c - g(\Gamma_c)$. When $d_c \approx 0$: the country is at the frontier (its inequality is as low as its structure allows). When $d_c > 0$: the country has "excess inequality" beyond what its structure requires — there is room for redistribution within the existing structure. When d_c is large: both structural change ($\Delta\Gamma$) and redistribution ($\Delta\omega$) are needed.

Appendix 26B: The Complementarity Condition

Formal statement. The three conditions are complementary if:

$$\frac{\partial^2 W}{\partial \Gamma \, \partial \text{ECI}} > 0, \quad \frac{\partial^2 W}{\partial \Gamma \, \partial N_{\text{eff}}} > 0, \quad \frac{\partial^2 W}{\partial \text{ECI} \, \partial N_{\text{eff}}} > 0, \tag{1}$$

where W is the welfare function. This holds in the structural model because: (i) high ECI creates the cross-block edges that raise Γ (the dense subgraph IS the bridge), (ii) high N_{eff} supports the aspiration formation that makes structural investment politically feasible, (iii) high Γ creates the domestic market and supply chains that support complex production (high ECI).

Implication for investment. The complementarity means the marginal return to investing in any one pillar is increasing in the level of the other two. The optimal investment rule: invest in the pillar with the highest marginal return given the current levels of the other two. In practice, this means starting with the cheapest pillar (N_{eff}, through information policy) and gradually shifting to the more expensive pillars (Γ through infrastructure, ECI through industrial policy) as the preconditions develop.

Structuralist Case Study: Ocampo's Neo-Structuralist Synthesis

José Antonio Ocampo (b. 1952), Colombia's former Finance Minister and CEPAL Executive Secretary, has been the leading architect of neo-structuralism — the synthesis of classical structuralism (Prebisch, Furtado, Pinto) with modern macroeconomics that retains the structuralist emphasis on productive transformation while incorporating lessons from the debt crises of the 1980s, the Washington Consensus era, and the commodity boom of the 2000s.

Ocampo's synthesis, developed across numerous publications (2004, 2009, 2012, 2014), has three pillars that correspond directly to this chapter's three conditions.

Pillar 1: Productive transformation. Ocampo argues that "the quality of what you produce matters" — an empirical restatement of the ECI condition. Countries that produce complex goods grow faster, generate more employment, and distribute income more equally than countries that export commodities. The policy implication: industrial policy is not a relic of the ISI era but a necessity for structural transformation.

Pillar 2: Macroeconomic management for structural change. Ocampo's distinctive contribution: macroeconomic policy (fiscal, monetary, exchange rate) should be designed not just for stability but for structural transformation. Counter-cyclical fiscal policy prevents the commodity booms from being wasted on consumption (Chapter 23's structural-cyclical decomposition). Real exchange rate management prevents "Dutch disease" (commodity exports appreciating the currency and destroying manufacturing competitiveness).

Pillar 3: Social equity as a condition for growth. Ocampo reverses the conventional sequence ("grow first, redistribute later"): equity is not a consequence of growth but a *precondition* for it. In this book's terms: high N_{eff} and low attribute constraints (λ_k) are necessary for the human capital base, domestic demand, and political stability that structural transformation requires.

Ocampo's synthesis	**This chapter's condition**	**Book reference**
Productive transformation	$\text{ECI} > 0$	Ch. 16, 25
Quality of what you produce	Dense subgraph $\mathcal{D}$	Ch. 25
Counter-cyclical policy	$\Delta\pi^S$ vs. $\Delta\pi^C$	Ch. 23
Macro for structural change	$v_\Gamma > 0$ sustained	Ch. 23
Social equity as precondition	$N_{\text{eff}} > N^*_{\text{eff}}$, $\lambda_k \uparrow$	Ch. 18, 21
Three pillars are complementary	$\partial^2 W / \partial\Gamma\, \partial\text{ECI} > 0$	App. 26B
Growth with equity	$\Gamma > \Gamma^* + \text{ECI} > 0 + N_{\text{eff}} > N^*$	Thm. 1.1

Ocampo's work completes the structuralist arc: Prebisch diagnosed the asymmetry; Furtado explained the reproduction; Pinto measured the heterogeneity; Fajnzylber identified the missing piece; CEPAL documented the stagnation; Ocampo synthesized the prescription. This book gives the synthesis its mathematical foundation. The weight matrix IS Prebisch's structure; the spectral gap IS Pinto's heterogeneity measure; the ECI IS Fajnzylber's truncation index; the formation model IS Furtado's cultural dependency; and the three conditions of Theorem 1.1 ARE Ocampo's three pillars, stated as a theorem.

Further Reading

Ocampo: Ocampo (2004), "Latin America's Growth and Equity Frustrations During Structural Reforms," *JEP*. Ocampo (2009), "Latin America and the Global Financial Crisis," *Cambridge Journal of Economics*. Ocampo and Ros (2011), *The Oxford Handbook of Latin American Economics*. Ocampo (2014), "The Latin American Debt Crisis in Historical Perspective," in Appleyard and Field (eds.), *International Economics*.

Neo-structuralism: CEPAL (2012), *Structural Change for Equality*. CEPAL (2016), *Horizons 2030: Equality at the Centre of Sustainable Development*. Bielschowsky (2009), "Sixty Years of ECLAC," *CEPAL Review* 97.

Growth with equity: Rodrik (2007), *One Economics, Many Recipes*. Stiglitz (2002), *Globalization and Its Discontents*. Lin and Chang (2009), "Should Industrial Policy in Developing Countries Conform to Comparative Advantage?" *Development Policy Review* (the debate between the two approaches).

Chapter 27

Algorithmic W^I

When the Machine Rewires Who Sees What

> *"The algorithms that determine what you see are not neutral. They are structural interventions on the information network — operators on W^I with objectives that may have nothing to do with your welfare or your society's development."*
> — This book

Every chapter until now has treated W^I as shaped by human institutions: media organizations, schools, social networks, community structures. This chapter confronts the twenty-first century's structural novelty: W^I is now substantially designed by algorithms.

Recommendation systems on Facebook, TikTok, YouTube, Google, and WhatsApp determine what billions of people see, whom they hear, and which signals enter their formation equation. These algorithms are operators on W^I — they add, remove, and reweight information edges in real time. They are the most powerful W^I operators ever deployed, operating at a scale and speed that no human institution has ever achieved.

The question for structural economics: do these algorithms increase or decrease N_{eff}? Do they break assortative mixing or reinforce it? Do they connect the quinoa farmer in Potosí to the engineer in Santa Cruz, or do they lock each into a personalized information silo?

The answer: it depends entirely on the algorithm's objective function. And that objective function is not set by the farmer, the engineer, or the Bolivian state. It is set by firms in Menlo Park, Beijing, and Mountain View.

1 The Algorithm as an Operator on W^I

1.1 Formalization

Definition 1.1 (Algorithmic W^I Operator)**.** *A **recommendation algorithm** is a function $\mathcal{A} : (W^I, \mathbf{h}) \to W^{I'}$ that takes the current information network W^I and a vector of engagement histories $\mathbf{h} = (h_1, \ldots, h_N)$ and outputs a modified information network $W^{I'}$:*

$$W_{ij}^{I'} = \mathcal{A}_{ij}(W^I, h_i, h_j, \boldsymbol{\eta}), \tag{1}$$

where $\boldsymbol{\eta}$ are the algorithm's parameters (learned from data to maximize an objective).

The algorithm modifies W^I by choosing which content from source j to show user i. If the algorithm shows i more content from j: $W_{ij}^{I'} > W_{ij}^{I}$ (the edge strengthens). If it suppresses content from j: $W_{ij}^{I'} < W_{ij}^{I}$ (the edge weakens).

Proposition 1.2 (The Algorithm's Objective Determines W^I's Structure)**.** *Three objective functions produce qualitatively different W^I structures:*

Objective 1: Maximize engagement *(the standard commercial objective). The algorithm shows i content that maximizes i's time on platform. Engagement is maximized by content that is emotionally arousing, confirms existing beliefs, and comes from sources similar to i. This produces **assortative amplification**: $W_{ij}^{I'}$ is large when i and j are similar (same demographic, same beliefs, same consumption pattern). The algorithm reinforces the existing block structure of W^I and may decrease N_{eff}.*

Objective 2: Maximize information accuracy *(a hypothetical public-interest objective). The algorithm shows i content from the most accurate and diverse sources. This produces **bridge-building**: $W_{ij}^{I'}$ is large when j has information that i lacks (cross-block edges). The algorithm breaks assortative mixing and increases N_{eff}.*

Objective 3: Maximize advertiser revenue *(a variant of engagement). Similar to Objective 1 but weighted by the commercial value of the user's attention. This produces **stratified exposure**: high-value users (urban, educated, high-income) see diverse, high-quality content; low-value users (rural, less-educated, low-income) see lower-quality, more assortative content. The algorithm increases N_{eff} for the elite and decreases it for the periphery.*

Intuition: The algorithm IS the new organic intellectual

In Gramsci's framework (Chapter 24), the "organic intellectual" is the agent with high π^I who produces the narratives that sustain the hegemony. In the algorithmic era, the organic intellectual is the recommendation system: it determines which narratives reach which audiences, with what frequency, and in what context. The algorithm has π^I that dwarfs any human journalist, professor, or politician — it reaches billions of users simultaneously and personalizes the message for each.

The difference: Gramsci's organic intellectual had an ideology (a preferred W^I structure). The algorithm has an objective function (maximize engagement, revenue, or some other metric). The objective function may or may not align with any political project, any development goal, or any welfare criterion from Chapter 12. It is structural power without structural intent.

2 Three Effects on Structural Economics

2.1 Effect 1: Echo Chambers and the Formation Trap

Theorem 2.1 (Algorithmic Amplification of Assortative Mixing)**.** *If the algorithm maximizes engagement and engagement is increasing in belief-confirmatory content, then:*

$$\lambda_k^{I'}(\mathcal{A}) \leq \lambda_k^I \quad \text{for all attributes } k, \tag{2}$$

where $\lambda_k^{I'}$ is the cross-group mixing parameter after algorithmic filtering. The algorithm reduces cross-group information exposure, making W^I more block-diagonal.

Consequence for the formation model: the self-confirming trap (Theorem 10.3) becomes deeper. The aspiration gap between groups widens because each group sees a more biased sample of returns. The Ψ ratio increases (the formation channel becomes more dominant). The welfare cost of discrimination (Chapter 18) rises.

Example 2.2 (Bolivia: Facebook and the Urban-Rural Information Divide)**.** *Bolivia has 7 million Facebook users (2023). The recommendation algorithm shows users content similar to what they have previously engaged with. Urban Bolivians engage with commercial media, political commentary, and global content. Rural Bolivians engage with local community posts, agricultural content, and regional news.*

The algorithm amplifies this difference: urban users see progressively more urban content; rural users see progressively more rural content. The cross-group mixing parameter $\lambda^I_{\text{urban-rural}}$, already low ($\approx$ 0.22, Chapter 21), may decrease further under algorithmic filtering.

Estimated effect: $\Delta\lambda^I_{\text{urban-rural}} \approx -0.05$ *to* -0.10 *(the algorithm reduces cross-group exposure by 20–45% relative to a chronological feed). The formation model predicts: the aspiration gap between urban and rural populations widens by* $\mu/(1-\rho) \times \Delta\lambda \times \Delta\bar{r} \approx$ 0.03–0.06 *preference units per generation.*

2.2 Effect 2: Epistemic Power Concentration

Proposition 2.3 (Algorithmic Concentration of $\boldsymbol{\pi}^I$)**.** *Engagement-maximizing algorithms produce* ***winner-take-all dynamics*** *in* W^I*: a small number of viral content producers capture a large share of attention, while the vast majority of voices are suppressed. The Perron eigenvector* $\boldsymbol{\pi}^I$ *becomes more concentrated:*

$$N_{\text{eff}}(W^{I'}) \leq N_{\text{eff}}(W^I). \tag{3}$$

The mechanism: the algorithm amplifies content with high initial engagement (likes, shares, comments). Content from well-connected, emotionally compelling sources gets shown to more users, generating more engagement, generating more amplification. Content from less-connected sources (community radio, local journalists, indigenous-language media) gets suppressed because it generates less engagement.

The consequence: Bolivia's $\pi^I_{\text{community}}$ *(already 0.05, Chapter 21) may decline further as algorithmic curation redirects attention to high-engagement commercial content. The* N^I_{eff} *drops from 3.4 toward 2.5–3.0.*

2.3 Effect 3: Cross-Border Information Integration

Proposition 2.4 (The Paradox of Algorithmic Globalization)**.** *Algorithms also create cross-border information edges that did not previously exist. A TikTok user in El Alto can watch a Korean entrepreneur, a Brazilian farmer, or an Indian programmer. The algorithm sometimes shows content from radically different contexts, creating* W^I *edges that span the global product space.*

This is Sunkel's integration/disintegration (Chapter 13) applied to W^I*: the algorithm simultaneously integrates globally (showing diverse international content to some users) and fragments domestically (personalizing feeds within national populations). The net effect on domestic* N_{eff} *is ambiguous:*

$$\Delta N^I_{\text{eff}} = \underbrace{\Delta N^{\text{global}}_{\text{eff}}}_{\text{positive (new sources)}} + \underbrace{\Delta N^{\text{domestic}}_{\text{eff}}}_{\text{negative (echo chambers)}}. \tag{4}$$

For urban educated Bolivians: the global effect may dominate (exposure to international entrepreneurship, technology, ideas). For rural indigenous Bolivians: the domestic

effect may dominate (locked into algorithmically reinforced local content). The algorithm increases N_{eff} for the elite and decreases it for the periphery — amplifying the compound disadvantage of Chapter 19.

3 The Regulatory Response: W^I as a Public Good

Definition 3.1 (Information Network as a Public Good)**.** *The structure of W^I — who sees what — is a **public good** in the structural model: it determines the formation of preferences for the entire population, which determines the welfare loss (Chapter 2), the formation multiplier (Chapter 10), and the political equilibrium (Chapter 24). No individual user can choose the aggregate W^I structure; it is the emergent outcome of billions of algorithmic decisions.*

Proposition 3.2 (Four Regulatory Instruments)**.**

1. ***Transparency requirements**: Mandate disclosure of the algorithmic objective function and its effect on W^I structure ($\Delta\lambda_k$, ΔN_{eff}). If users and regulators can observe what the algorithm does to W^I, they can demand changes.*

2. ***Diversity mandates**: Require recommendation algorithms to maintain a minimum cross-group exposure ratio: $\lambda_k^{I'} \geq \lambda_k^{\min}$ for all attribute groups k. This prevents the algorithm from amplifying assortative mixing below a threshold.*

3. ***Public-interest algorithms**: Develop and deploy alternative recommendation systems with Objective 2 (maximize information accuracy/diversity) rather than Objective 1 (maximize engagement). These could be operated by public broadcasters, educational institutions, or platform cooperatives.*

4. *N_{eff} **monitoring**: Track $N_{eff}(W^I)$ over time (Chapter 21) and treat declines as a regulatory trigger. If the algorithmically-mediated N_{eff} falls below a threshold, regulatory intervention is warranted.*

Key Structural Insight

The algorithmic W^I is the most important structural development of the twenty-first century. For the first time in history, the information network that shapes preference formation is designed by a small number of firms, operating at global scale, with objectives that are not aligned with any development goal.

The structural economics framework provides the language: the algorithm is an operator on W^I. Its effect is measurable ($\Delta\lambda_k$, ΔN_{eff}, $\Delta\Psi$). Its welfare cost is computable (Chapter 18's welfare loss formula, amplified by $1 + \Psi$). And its regulation is an operator design problem (Chapter 15): choose the algorithmic parameters $\boldsymbol{\eta}$ that maximize N_{eff} subject to commercial viability. The framework transforms the debate from "social media is good/bad" to: "what is the algorithm's effect on λ_k, N_{eff}, and Ψ, and does that effect move the structural trajectory toward or away from the growth-with-equity zone of Chapter 26?"

4 AI and the Future of Structural Analysis

4.1 AI as Measurement Tool

Proposition 4.1 (AI for Structural Diagnostics)**.** *Machine learning techniques can improve the measurement of all three weight matrices:*

1. *W^P: Satellite imagery + NLP on firm registries can estimate production linkages in real time, filling gaps between benchmark I-O tables (Chapter 20).*
2. *W^I: Natural language processing of social media content can estimate media overlap (W^I_{media}, Chapter 21) at daily frequency, tracking N_{eff} in real time.*
3. *W^U: Sentiment analysis of social media posts can estimate solidarity patterns ($W^U_{digital}$, Chapter 22), supplementing survey-based measures.*
4. *The structural trajectory (Chapter 23) can be computed in near-real-time rather than at annual frequency.*

4.2 AI as Structural Operator

Proposition 4.2 (AI-Powered Structural Intervention)**.** *AI can also serve as a deliberate operator on W^I for development purposes:*

1. ***Personalized mentoring at scale**: AI systems that connect aspiring entrepreneurs in the traditional sector with role models, market information, and technical knowledge from the modern sector. This is the $\mathcal{L}_{info}$ operator (Chapter 15) deployed through an AI system rather than a human program.*

2. ***Adaptive education**: AI tutoring systems that adjust content to the learner's context while exposing them to cross-group knowledge ($\Delta\lambda^I > 0$).*

3. ***Market information**: AI systems that provide real-time price and demand information to rural producers, creating W^I edges from the traditional sector to the market that the assortative structure suppresses.*

Prebisch Connection: The new center-periphery

The algorithmic W^I creates a new center-periphery structure that operates alongside the trade-based W^P:

The algorithmic center: Firms in Silicon Valley, Shenzhen, and a few other nodes control the parameters $\boldsymbol{\eta}$ of the global W^I. Their π^I is enormous: a single change to Facebook's algorithm can shift the formation signals received by 3 billion people.

The algorithmic periphery: Every other country is a consumer of algorithmic W^I designed elsewhere. Bolivia's W^I is shaped by algorithms trained on US engagement data, optimized for US advertising markets, and deployed globally without adaptation to Bolivian development needs.

This is Prebisch's center-periphery thesis applied to W^I: the center produces the information infrastructure; the periphery consumes it. The "terms of trade" in W^I are the exchange of attention (from the periphery) for algorithmic curation (from the center). The formation signals that Bolivian youth receive are filtered through an objective function designed in Menlo Park.

The structuralist response: just as Prebisch advocated for ISI to reduce W^P dependency, the algorithmic era requires "information substitution" — the development of domestic or regional algorithmic capacities that can design W^I operators aligned with national development objectives. This is the information-age equivalent of building the capital goods sector: building the algorithmic infrastructure that shapes your own W^I.

5 Summary

1. Recommendation algorithms are **operators on** W^I: they determine who sees what, with effects measurable as $\Delta\lambda_k$, ΔN_{eff}, and $\Delta\Psi$.

2. **Engagement-maximizing algorithms** amplify assortative mixing ($\lambda_k \downarrow$), concentrate epistemic power ($N_{\text{eff}} \downarrow$), and deepen the formation trap ($\Psi \uparrow$). The welfare cost is computable.

3. The **paradox**: algorithms simultaneously integrate globally (cross-border content) and fragment domestically (echo chambers). The net effect differs by social position: elite N_{eff} rises; peripheral N_{eff} falls.

4. **Regulation**: transparency, diversity mandates, public-interest algorithms, N_{eff} monitoring. The framework transforms the debate from "good/bad" to measurable structural effects.

5. **AI as tool**: machine learning can improve measurement of all three W's in near-real-time. AI can also serve as a development operator (personalized mentoring, adaptive education, market information).

6. The **algorithmic center-periphery**: Silicon Valley designs the global W^I; the periphery consumes it. Information sovereignty requires building domestic algorithmic capacity — the information-age capital goods sector.

6 Exercises

27.1 Algorithmic $\Delta\lambda$. For a 4×4 W^I (urban men, urban women, rural men, rural women): simulate the effect of an engagement-maximizing algorithm that increases within-group exposure by 20% and decreases cross-group exposure by 15%. Compute $\Delta\lambda_k$ for gender and geography. Compute ΔN_{eff}.

27.2 The echo chamber theorem. Prove Theorem 2.1: if engagement is maximized by belief-confirmatory content, show that $\lambda_k^{I'} \leq \lambda_k^I$ under the standard collaborative-filtering algorithm.

27.3 Bolivia's digital divide. Using internet penetration data by department and demographic group: estimate the fraction of Bolivia's population whose W^I is algorithmically mediated (vs. traditional media/word-of-mouth). How does this fraction correlate with N_{eff} estimates from Chapter 21?

27.4 Objective function design. Propose an alternative algorithmic objective function for Bolivia that maximizes N_{eff} subject to a minimum engagement constraint. Write the optimization problem formally. What trade-offs does it involve?

27.5 AI mentoring simulation. Design an AI-powered mentoring system for rural Bolivian entrepreneurs. Specify: (a) the ΔW^I edges it creates, (b) the expected $\Delta\lambda^I$ and ΔN_{eff}, (c) the formation model prediction for $\Delta\theta^*$ after one generation.

27.6 Algorithmic center-periphery. Using Alexa/SimilarWeb data: estimate the fraction of Bolivia's internet traffic that goes to US-based platforms (Google, Facebook, YouTube, TikTok) vs. domestic content. What fraction of Bolivia's W^I is algorithmically designed abroad?

27.7 The policy frontier. Compare four regulatory regimes: (a) no regulation (current), (b) transparency only, (c) diversity mandates ($\lambda_k^{\min} = 0.3$), (d) public-interest algorithm. For each: estimate the effect on N_{eff}, the political feasibility, and the cost. Which regime best advances Bolivia's structural trajectory?

Mathematical Appendices

Appendix 27A: Recommendation Algorithms as Matrix Operators

Collaborative filtering. The standard recommendation algorithm estimates a user-content affinity matrix $R \approx UV'$, where U is a user-factor matrix and V is a content-factor matrix (matrix factorization, analogous to the SVD of the specialization matrix in Chapter 16). Content shown to user i is drawn from the rows of V with highest inner product $\mathbf{u}_i' \mathbf{v}_j$.

Effect on W^I. The user-factor matrix U clusters users by "type" (implicit demographic, behavioral, or preference clusters). Content from source j is shown to user i only if $\mathbf{u}_i \approx \mathbf{u}_j$ (similar type). This produces assortative W^I: users see content from similar users. The mixing parameter is $\lambda_k^{I'} \approx \exp(-d(\bar{\mathbf{u}}_g, \bar{\mathbf{u}}_{g'}))$, where d is the distance between group-average user factors. When groups are distinct in the factor space (different demographics, different languages, different consumption patterns): d is large and $\lambda^{I'}$ is small.

Formal proof of Theorem 2.1. Under collaborative filtering with engagement maximization: the optimal R^* maximizes $\sum_i \sum_j R_{ij}^* \cdot p_{ij}$, where p_{ij} is the probability that user i engages with content from source j. If p_{ij} is decreasing in the "distance" between i and j (different group, different beliefs $\rightarrow$ less engagement), then R_{ij}^* assigns more weight to within-group content. The induced $W^{I'}$ has $\lambda_k^{I'} \leq \lambda_k^I$.

Appendix 27B: The DeGroot Model with Algorithmic Intermediation

Modified DeGroot. In Chapter 8, the DeGroot model had beliefs converging through a fixed W^I. With algorithmic intermediation, W^I itself depends on beliefs:

$$\mathbf{b}_{t+1} = W^I(\mathbf{b}_t) \cdot \mathbf{b}_t, \tag{5}$$

where $W^I(\mathbf{b}_t)$ is the algorithmically modified information matrix (the algorithm adjusts the feed based on current beliefs/engagement patterns). This is a *nonlinear* dynamical system: the matrix changes with the state.

Fixed points. A fixed point $\mathbf{b}^*$ satisfies $\mathbf{b}^* = W^I(\mathbf{b}^*)\mathbf{b}^*$: beliefs are a consensus of the information network that the beliefs themselves produced. This is a deeper form of the self-confirming trap: the algorithm creates the W^I that produces the beliefs that the algorithm was designed to serve.

Multiple equilibria. The nonlinear system can have multiple fixed points: a "diverse" equilibrium (where $W^I(\mathbf{b}^*)$ has high N_{eff} because beliefs are varied, so the algorithm shows diverse content) and a "polarized" equilibrium (where $W^I(\mathbf{b}^*)$ has low N_{eff} because beliefs are clustered, so the algorithm shows confirmatory content). The polarized equilibrium is absorbing under engagement-maximization: once beliefs cluster, the algorithm reinforces the clustering, preventing return to the diverse equilibrium.

Structuralist Case Study: The Algorithmic W^I in the Global South

This chapter's case study departs from the historical structuralist tradition to address a contemporary structural phenomenon. The algorithmic W^I is arguably the most significant structural development since the colonial imposition of the racial classification system (Quijano, Chapter 18): both are externally imposed structures that shape W^I for entire populations without their consent.

Three observations ground the structural analysis.

First: the speed of W^I transformation is unprecedented. Bolivia went from zero social media users to 7 million in 12 years (2010–2022). In Chapter 23's terms: W^I underwent a structural break — not gradual evolution but a discrete regime change. The algorithmic W^I did not grow out of the existing information structure; it replaced it. For urban Bolivians under 35, the algorithmic feed IS W^I ($\omega_d = 0.50$, Chapter 21). This happened faster than any previous structural change in the book: faster than ISI, faster than nationalization, faster than the colonial imposition.

Second: the structural effects are unmeasured. Despite the speed and scale of the W^I transformation, no developing country systematically monitors N_{eff}, λ_k, or Ψ in the algorithmic W^I. The measurement infrastructure of Chapter 21 exists in principle but has not been deployed. We are flying blind: the most powerful W^I operator in history is operating without structural monitoring.

Third: the regulatory capacity is asymmetric. The EU regulates algorithmic platforms (the Digital Services Act, 2022). The US debates regulation. The Global South — where the structural effects may be largest (because N_{eff} was already low and attribute constraints were already severe) — has essentially no regulatory capacity. Bolivia's telecommunications regulator (ATT) has no mandate, tools, or data to assess the structural effects of Facebook's algorithm on Bolivia's W^I.

The structuralist response is clear: treat algorithmic W^I as a structural variable that requires measurement (Chapter 21), monitoring (Chapter 23), and regulation (this chapter's Proposition 3.2). The alternative is to cede the formation of Boli-

vian preferences to an algorithm designed in Menlo Park — the information-age equivalent of ceding the production structure to a colonial power.

Structural parallel	**Historical**	**Algorithmic**
External control of W	Colonial administration	Platform algorithms
Center-periphery	Trade asymmetry (W^P)	Information asymmetry (W^I)
Hegemonic formation	State media, schools	Engagement-maximizing feeds
Counter-hegemony	Community radio, movements	Public-interest algorithms?
Regulatory response	ISI, nationalization	Diversity mandates, N_{eff} monitoring

Further Reading

Algorithmic curation and society: Pariser (2011), *The Filter Bubble*. Sunstein (2017), *#Republic: Divided Democracy in the Age of Social Media*. Zuboff (2019), *The Age of Surveillance Capitalism*.

Echo chambers and polarization: Levy (2021), "Social Media, News Consumption, and Polarization," *AER*. Allcott et al. (2020), "The Welfare Effects of Social Media," *AER*. Bail et al. (2018), "Exposure to Opposing Views on Social Media Can Increase Political Polarization," *PNAS*.

AI and development: Acemoglu and Johnson (2023), *Power and Progress: Our Thousand-Year Struggle over Technology and Prosperity*. World Bank (2019), *World Development Report: The Changing Nature of Work*. For the Global South specifically: Arora (2019), *The Next Billion Users: Digital Life Beyond the West*.

Platform regulation: EU Digital Services Act (2022). Stigler Committee on Digital Platforms (2019), *Final Report*. For developing countries: Gurumurthy and Chami (2020), "Platform Planet: Development in the Intelligence Economy," IT for Change.

Chapter 28

The General Case

Where the Standard Model Is the Special Case, and Structure Is Everything Else

> *"O subdesenvolvimento não constitui uma etapa necessária do processo de formação das economias capitalistas modernas. É, em si, um processo particular, resultante da penetração de empresas capitalistas modernas em estruturas arcaicas."*
>
> ["Underdevelopment does not constitute a necessary stage in the formation of modern capitalist economies. It is, in itself, a particular process, resulting from the penetration of modern capitalist enterprises into archaic structures."]
>
> — Celso Furtado, *Desenvolvimento e Subdesenvolvimento* (1961)

This is the last chapter. It has one purpose: to state the book's thesis in final form and show that every result in the previous 27 chapters is a corollary of a single proposition.

1 The Thesis

Theorem 1.1 (The General Case)**.** *The neoclassical model is the special case $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$. Structural economics is the general case where W is any row-stochastic nonnegative matrix.*

Every result in this book follows from the departure of W from $\frac{1}{N}\mathbf{1}\mathbf{1}'$:

$$\Delta(W) = W - \frac{1}{N}\mathbf{1}\mathbf{1}'. \tag{1}$$

When $\Delta(W) = 0$: the First Welfare Theorem holds, the Perron eigenvector is uniform, preferences are exogenous, growth propagates symmetrically, coalitions are unnecessary, and there is nothing for structural economics to study.

When $\Delta(W) \neq 0$: everything in this book follows.

This is not a metaphor. It is a mathematical statement. Every chapter is a theorem about $\Delta(W)$:

2 Twenty-Seven Corollaries

Proposition 2.1 (The Book in One Page)**.**

Ch.	***Result***	***Statement as a property of*** $\Delta(W)$
1	*Matrix reformulation*	*Every economy has a W. The neoclassical model assumes* $\Delta(W) = 0$.
2	*Welfare theorems*	*FWT holds iff* $\Delta(W) = 0$. *Failure modes:* $L = L_{power} + L_{ext} + L_{int}$.
3	*Growth & distribution*	*Multi-sector Solow:* Γ *determines whether growth propagates or stays in the enclave.*
4	*Standard model assumptions*	*Six assumptions A1–A6 are equivalent to* $\Delta(W) = 0$.
5	*Three networks*	$W = \alpha_U W^U + \alpha_P W^P + \alpha_I W^I$. *Three layers, one composite.*
6	*Power*	*The Perron eigenvector* $\boldsymbol{\pi}$ *IS power.* $\text{Var}(\pi_i) > 0$ *iff* $\Delta(W) \neq 0$.
7	*Bargaining*	$x_i^* = \Omega/N + (\pi_i - 1/N)/\kappa$. *The allocation departs from equality by* $\Delta\boldsymbol{\pi}$.
8	*Beliefs*	*DeGroot convergence through* W^I. $N_{eff} = 1/\sum(\pi_j^I)^2$.
9	*Preferences*	$\theta_{i,t+1} = \rho\theta_{i,t} + (1-\rho)\bar{\theta}_0 + \mu(\sum W_{ij}^I r_j - r^*)$. *Preferences are endogenous to* W^I.
10	*Formation model*	*The complete dynamical system:* $\boldsymbol{\theta} \to \mathbf{x}^* \to \mathbf{r} \to \boldsymbol{\theta}$. *Multiple steady states from* $\Delta(W^I) \neq 0$.
11	Ψ *ratio*	$\hat{\Psi} = F/E$: *the formation channel is measurable and often dominant.*
12	*Welfare*	*The ruler is rubber:* W^I *produces the preferences that evaluate* W^P. *Four approaches.*
13	*Network evolution*	$W_{t+1} = \Psi(W_t; \mathbf{x}_t^*)$. *Proximity dominates. Block-diagonal structures absorb.*
14	*Coalitions*	*Coalitions form to change* $\Delta\boldsymbol{\pi}$. *The political Coase theorem fails because W IS the enforcement.*
15	*Operator design*	$\max_{\Delta W} \Gamma(W + \Delta W)$. *Five operators,* Ψ*-ordered.*
16	*Global* W^P	*ECI is the second eigenvector of the global trade matrix.* $\Gamma \approx 0.20$.
17	*Domestic diagnostics*	*Five instruments:* $\Gamma, \sigma_B, G(\boldsymbol{\pi}), N_{eff}, \Psi$. *Bolivia:* $\Gamma = 0.13$.
18	*Gender & race*	*Node attributes constrain W through assortative mixing. Intersectionality is multiplicative.*
19	*Scale invariance*	*Same* Γ, *same* $\boldsymbol{\pi}$, *same dynamics at every scale.* $\pi_I \approx \pi_G \cdot \pi_N \cdot \pi_{within}$.
20	*SAMs*	*The SAM IS W. Row-normalize and compute.*
21	W^I *measurement*	*Media overlap, reference groups, social networks, digital traces. Bolivia:* $N_{eff} = 3.4$.
22	W^U *measurement*	*Transfers, experiments, voting, surveys. Bolivia:*

3 What Furtado Knew

Celso Furtado's 1961 thesis — that underdevelopment is not a stage but a structure — is the sentence that generated this book. Every chapter is an elaboration.

Proposition 3.1 (Furtado's Thesis as a Theorem)**.** *Furtado (1961) made three claims:*

1. ***Underdevelopment is not the absence of development.*** *It is a specific structural configuration: $W \neq W^{CE}$ in a particular way (block-diagonal, with a modern enclave connected to the global economy and a traditional sector connected to itself). This is Chapter 3's block-diagonal W^P with small cross-block Γ.*

2. ***The structure reproduces itself.*** *Underdevelopment is a stable fixed point, not a transient state. The mechanisms of reproduction are: proximity-dominated edge formation (Chapter 13), self-confirming preferences (Chapter 10), coalition dynamics that protect the incumbent W (Chapter 14), and hegemonic formation that produces consent (Chapter 24).*

3. ***The structure can be changed, but not by the standard tools.*** *Price adjustments, trade liberalization, and transfers do not change W. They change $\boldsymbol{\omega}$ (endowments) and relative prices within the existing structure. Changing W requires the operators of Chapter 15: information policy, institutional reform, education, infrastructure, and industrial policy, sequenced by Ψ and constrained by the political economy of Chapter 14.*

Every chapter in this book is a formalization of one of these three claims:

Furtado's claim	***Formal content***	***Chapters***
Not a stage but a structure	*$W \neq W^{CE}$, block-diagonal*	*1–4*
Self-reproducing	*Stable fixed point of co-evol. system*	*5–12, 24*
Changeable but not by standard tools	*Operators on W, not on $\boldsymbol{\omega}$*	*13–15, 25–26*
Measurement	*SAMs, W^I, W^U, trajectories*	*16–23*
Scale invariance	*Same at all scales*	*19*
Contemporary	*Algorithmic W^I*	*27*

Intuition: Why "the general case"

The neoclassical model assumes a specific W: $W = \frac{1}{N}\mathbf{1}\mathbf{1}'$. This is not a simplification — it is a claim about the world. The claim is that social structure does not matter for economic outcomes: all agents interact symmetrically, all information flows freely, all preferences are formed independently. This claim is empirically false for every economy ever measured (Chapter 17: no country has $\Gamma = 1$).

Structural economics does not "add" structure to the neoclassical model. It *removes a restriction*. The neoclassical model restricts W to the $N^{-1}\mathbf{1}\mathbf{1}'$ manifold — a set of measure zero in the space of all row-stochastic matrices. Structural economics allows W to be anywhere in the space. The neoclassical model is the special case. Structural economics is the general case.

This is the same relationship as between Euclidean and non-Euclidean geometry. Euclid assumed flat space (the "parallel postulate"). Riemann allowed curvature. Euclid is the special case $\kappa = 0$. Riemannian geometry is the general case. Similarly: the neoclassical model assumes a flat weight matrix ($\Gamma = 1$, all entries equal). Structural economics allows curvature ($\Gamma < 1$, entries unequal). The neoclassical model is the special case $\Gamma = 1$.

4 What Bolivia Taught Us

Bolivia appeared in every chapter — not because Bolivia is the only developing country, but because it exhibits every structural pathology at a scale that the mathematics can detect.

Example 4.1 (Bolivia: The Structural Portrait, Complete)**.**

Instrument	***Bolivia***	***Benchmark***	***Chapter***
$\Gamma(W^P)$	*0.13*	*1.00*	*17*
σ_B	*0.62*	*0.00*	*17*
$G(\boldsymbol{\pi})$	*0.52*	*0.00*	*17*
$N_{eff}(W^I)$	*3.4*	N	*21*
$N_{eff}(W^U)$	*2.8*	N	*22*
Ψ *(self-employment)*	*1.56*	—	*11*
ECI	-0.80	> 0	*16*
Truncation τ	*0.98*	*0.00*	*25*
π_G *(global power)*	*0.0005*	$1/N_G$	*16*
v_Γ *(structural velocity)*	$+0.01$*/decade*		*23*
SCI (1990–2019)	*0.075*	*1.00*	*23*

Power ratio: indigenous woman (Potosí) to German engineer $\approx 1:1{,}000$
Truncation penalty: 72% of total welfare loss
30-year trajectory: massive poverty reduction, near-zero structural change

This table is the structural health record of an economy. Every number is computable from publicly available data. Every number tells the same story: Bolivia's weight matrix is far from $\frac{1}{N}\mathbf{1}\mathbf{1}'$*, and it has not moved substantially closer in 34 years.*

The numbers are damning. But they are also hopeful, because they are *specific*. The structural diagnosis does not say "Bolivia is poor." It says: "Bolivia's spectral gap is 0.13 because the capital goods sector is missing ($\tau = 0.98$), the information network has 3.4 effective voices, and the formation channel produces self-confirming preferences ($\Psi = 1.56$). The binding constraint is the truncation. The optimal intervention sequence is information $\to$ education $\to$ infrastructure $\to$ industrial policy, at a cost of \$17B over 30 years (1.4% of GDP/year). The political precondition is a coalition shift enabled by Phase 1's W^I reforms."

Precision is the gift structural economics offers. The standard model says: "get the prices right." The structuralist tradition says: "change the structure." This book says: "here is the structure, here are its eigenvalues, here is what is wrong, and here is the engineering specification for fixing it."

5 What the Book Does Not Do

Intellectual honesty requires stating the limits.

1. The model is static in important ways. The co-evolutionary system (Chapter 13) is a difference equation, not a continuous-time model with stochastic shocks,

financial frictions, or expectations. The formation model (Chapter 10) has a single preference parameter per agent. Reality has many. The model captures the structural skeleton; it does not capture the flesh.

2. The empirical estimates are rough. Γ, N_{eff}, and Ψ are estimated from imperfect proxies (I-O tables that are updated every 5–10 years, media surveys with small samples, cohort-year decompositions that rely on strong identifying assumptions). The structural health dashboard is a first approximation. Better data and better methods will refine the estimates — but the framework is robust to the specific numbers.

3. The political economy is underdeveloped. Chapter 14's coalition model is a one-shot game. Real political dynamics involve repeated interaction, institutions, ideology, identity, and contingency. The hegemonic cycle (Chapter 24) is a sketch, not a fully specified model. The book identifies the political constraint but does not solve it.

4. Culture is underspecified. The formation model reduces culture to a preference parameter θ formed through W^I. Real culture involves narratives, identities, rituals, languages, cosmologies — things that do not reduce to a scalar. The model captures the *mechanism* of cultural transmission (network-mediated signal processing) but not the *content*.

5. The book is about structure, not about people. Every example features an "indigenous woman in Potosí" or a "gas executive in Santa Cruz" as a node in a matrix. Real people are not nodes. They have names, histories, hopes, contradictions, and dignity that no eigenvalue can capture. The structural portrait is necessary but not sufficient. The engineer who designs the bridge still needs to talk to the people who will cross it.

6 The Structuralist Research Program

This book opens a research program, not closes one. Ten questions for the next generation:

1. **Estimation**: How to estimate W^P, W^I, W^U at high frequency and fine spatial resolution, using big data, satellite imagery, and digital trace data?

2. **Identification of Ψ**: Better designs for separating enforcement from formation. Randomized W^I interventions (following Bernard et al., 2014) in multiple countries.

3. **The dense subgraph**: What is the minimum viable capital-goods cluster for

a small economy? Can it be shared regionally (a Mercosur or Andean Community capital-goods program)?

4. **Algorithmic** W^I: Real-time monitoring of N_{eff} under algorithmic curation. Experimental evaluation of diversity-mandated recommendation systems.

5. **Climate and structure**: How does climate change rewire W^P (through agricultural disruption, migration, infrastructure damage)? What are the structural effects of green industrial policy?

6. **Gender formation**: Randomized evaluations of W^I interventions (role models, media campaigns, mentor programs) with long-run follow-up across generations.

7. **Multi-country structural trajectories**: Build the dashboard for every country, every year, and track the global structural trajectory. A "structural world development indicators" project.

8. **Political economy of operator sequencing**: Under what political conditions is the Ψ-ordered sequence feasible? When does the information operator create the coalition for the structural operator?

9. **Endogenous institutions**: How do institutions (property rights, courts, regulations) emerge from the co-evolutionary dynamics? The institutional operator (Chapter 15) is exogenous in this book; it should be endogenous.

10. **The general equilibrium of structure**: A computable general equilibrium model with all three networks, the formation channel, and political equilibrium. The "structural CGE" that Taylor (1983) pointed toward but that no one has built.

7 The Last Equation

The book began with one equation (Chapter 1):

$$W^{CE} = \frac{1}{N}\mathbf{1}\mathbf{1}'. \tag{2}$$

It ends with its generalization:

$$W = W^{CE} + \Delta(W). \tag{3}$$

Every economy is the competitive benchmark plus a departure. The departure is the structure. The structure determines power ($\boldsymbol{\pi}$), allocation ($\mathbf{x}^*$), beliefs ($\mathbf{b}^*$), preferences ($\boldsymbol{\theta}^*$), welfare (L), dynamics (Γ_t), politics (S^*), and the formation of the next generation's preferences ($\boldsymbol{\theta}^*_{t+1}$).

The standard model studies the first term. This book studies the second.

Key Structural Insight

Structural economics is not a school of thought. It is not a political position. It is not a regional tradition. It is the mathematics of social structure applied to economic life.

The weight matrix W is the object. The Perron eigenvector is the power. The spectral gap is the health. The formation model is the dynamics. The operator design is the engineering. The coalition game is the politics. The dashboard is the diagnostic. The trajectory is the history.

One matrix. Everything else follows.

For my daughter — the park I care about most.

Structuralist Case Study: Furtado's Return

Celso Furtado (1920–2004) appears three times in this book. In Chapter 2 (underdevelopment as a stable fixed point of W), in Chapter 9 (cultural dependency as endogenous preferences), and here, at the end.

Furtado was the greatest economic thinker Latin America produced. He was trained in Paris (PhD at the Sorbonne, 1948), worked at CEPAL under Prebisch (1949–1957), directed SUDENE (the Northeast Brazil development agency, 1958–1964), served as Brazil's Minister of Planning (1962–1963), and was exiled by the military coup of 1964. He wrote 30 books, in Portuguese, Spanish, French, and English, across four decades. His intellectual range — from Keynesian macroeconomics to cultural theory to institutional analysis — anticipated almost every theme in this book.

The thesis of *Desenvolvimento e Subdesenvolvimento* (1961) can now be stated in its complete form.

Claim 1: Underdevelopment is a structure, not a stage. Rostow (1960) had argued that all economies pass through "stages of growth" from traditional to modern. Furtado rejected this: underdevelopment is not pre-development. It is a specific structural configuration produced by the *interaction* of modern and archaic elements within a single system. The modern sector does not gradually absorb the traditional sector; it coexists with it, extracting surplus from it, and reproducing the duality.

In this book: underdevelopment is a structural equilibrium (Chapter 13, Definition 13.3) with low Γ, block-diagonal W, and self-reproducing dynamics. It is a fixed point, not a transient state. The fixed point exists because the co-evolutionary dynamics (proximity-dominated edge creation, self-confirming preferences, coalition protection) make it stable.

Claim 2: The structure produces the preferences that accept it. Furtado's deepest insight, developed in *O Mito do Desenvolvimento Econômico* (1974): underdevelopment persists not only because of economic structure but because of cultural structure — the patterns of consumption, aspiration, and identity that the economic structure produces. The periphery imports center consumption patterns through W^I (media, advertising, education modeled on center curricula), forming preferences that are impossible to satisfy under peripheral production structures. The result: frustrated modernization, debt-fueled consumption mimicry, and political instability.

In this book: this is the formation model (Chapters 9–10). Preferences are endogenous to W^I. The formation equation $\theta_{i,t+1} = \rho\theta_{i,t} + (1-\rho)\bar{\theta}_0 + \mu(\sum W^I_{ij} r_j -$

r^*) captures Furtado's mechanism exactly: agents form preferences by observing W^I-weighted returns. When W^I is dominated by center signals (Chapter 8, epistemic Perron dominance), the formation produces center-mimicking preferences in the periphery — Furtado's "cultural dependency." The Ψ ratio (Chapter 11) measures how much of observed behavior comes from this channel.

Claim 3: Structural change requires changing the structure, not the prices. Furtado argued that neither market liberalization (which assumes the structure is efficient) nor simple redistribution (which operates within the structure) can solve underdevelopment. What is needed is structural transformation: changing the production structure (W^P), the information structure (W^I), and the institutional structure — simultaneously and deliberately.

In this book: this is Part IV (Chapters 13–15) and the neo-structuralist synthesis (Chapter 26). The operator design framework (Chapter 15) provides the engineering manual that Furtado's diagnosis called for. The sequencing theorem (Ψ-ordered: information first, structure second, industrial policy third) is the implementation protocol. The three conditions of the Growth-Equity Theorem ($\Gamma > \Gamma^*$, ECI > 0, $N_{\text{eff}} > N_{\text{eff}}^*$) are the success criteria.

Furtado (1961/1974)	**This book's formalization**	**Chapter**
Underdevelopment is a structure	$W \neq W^{CE}$, block-diagonal, low Γ	1–4
Not a stage	Stable fixed point of co-evolutionary system	13
Modern-archaic coexistence	Block-diagonal W^P, Pinto's heterogeneity	3, 17
Cultural dependency	W^I produces center-mimicking $\boldsymbol{\theta}^*$	9, 10
Frustrated modernization	θ^*_{center} incompatible with peripheral W^P	10, 12
Structural change needed	Operators on W, not on $\boldsymbol{\omega}$	15
Simultaneity required	Three conditions, three pillars	26
The general case	W is any row-stochastic matrix	28

Furtado wrote in exile, in French and Portuguese, for audiences that could not give him tenure or citations. He died in 2004, the year before Evo Morales was elected and five years before Hidalgo and Hausmann published the Atlas of Economic Complexity. He did not see the product space, the ECI, or the formation experiments that would validate his intuitions. He did not have the Perron eigenvector, the spectral gap, or the DeGroot model.

But he had the thesis: the neoclassical model is the special case, and structure is everything else. This book is the proof.

Further Reading

Furtado: Furtado (1961), *Desenvolvimento e Subdesenvolvimento*, Rio de Janeiro: Fundo de Cultura. English: *Development and Underdevelopment*, University of California Press, 1964. Furtado (1974), *O Mito do Desenvolvimento Econômico*, Rio de Janeiro: Paz e Terra. English: "The Myth of Economic Development." Furtado (1999), *O Longo Amanhecer: Reflexões sobre a Formação do Brasil* (his intellectual autobiography). Love (1996), "Economic Ideas and Ideologies in Latin America since 1930," in Bethell (ed.), *Cambridge History of Latin America*, Vol. 6 (for Furtado's intellectual context).

The structuralist tradition, complete: Prebisch (1949), the manifesto. Furtado (1961), the thesis. Pinto (1970), the diagnosis. Sunkel (1971), the integration/disintegration. Tavares (1972), the autopsy. Marini (1973), the exploitation. Fajnzylber (1983/1990), the truncation and the empty box. Cardoso and Faletto (1969), the politics. Quijano (2000), the coloniality. Sen (1999), the capabilities. CEPAL (2012), the institutional synthesis. Ocampo (2009), the policy synthesis. Hidalgo et al. (2007/2009), the empirical measurement.

And this book.

"*In 1950, Raúl Prebisch described a global economy organized by structure — where the center captured gains and the periphery transferred them outward through deteriorating terms of trade. He was right. But he wrote before the mathematics of networks existed. This book gives the Latin American structuralist tradition the tools it always deserved: weight matrices, spectral theory, and the Perron-Frobenius theorem.*"

Structural Economics shows that the standard neoclassical model is a special case — the case where the interaction matrix is symmetric, rank one, and anonymous. Replace that matrix with the one the world actually has, and power becomes endogenous, preferences become products of the structure, the welfare theorems fail, and the question shifts from *is the equilibrium efficient?* to *what is the structure, who does it benefit, and can it be changed?*

THE TRADITION

1950	Prebisch	*Center-periphery. Terms of trade.*
1961	Furtado	*Underdevelopment as historical production.*
1970	Pinto	*The dual economy within the periphery.*
1970	Sunkel	*Dependency as endogenous structure.*
1983	Taylor	*The first formalization.*
1990	Fajnzylber	*Truncated industrialization.*
2009	Ocampo	*The neo-structuralist synthesis.*
2027	**This book**	*The eigenvalues they never had.*

Twenty-eight chapters unify network economics, general equilibrium,